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Limiting Contractions for Starting Simple Ramp-Type Scramjet Intakes with Overboard Spillage

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A method for obtaining the limiting contraction for supersonic intake-starting via overboard spillage is demonstrated for a simple ramp-type intake family. The strong-shock design principle is proposed on the basis of comparison of the limiting contraction line with the Kantrowitz (self-starting) lines of a few particular ramp intakes. Predicted starting characteristics compare favorably with two-dimensional inviscid numerical simulations.

I. Introduction

THE air intake is an important component of hypersonic airbreathing engines. It is essentially a converging duct decelerating and compressing airflow and supplying the compressed air to the engine's combustor. For efficient engine operation the intake must be "started" (i.e., all incoming supersonic flow must be captured). In the started mode, steady supersonic flow in the intake decelerates toward its exit. For some intake geometries, the started flow may contain subsonic pockets or even become entirely subsonic (e.g., due to the presence of strong oblique shocks and/or Mach reflections). For the purpose of the present study, such intakes are considered started, as long as all incoming flow remains captured.

The quasi-one-dimensional Kantrowitz theory [1,2], of intake-starting is based on two main assumptions:

1) The intake is fully enclosed; that is, the freestream velocity is normal to the entry plane (Fig. 1a).

2) The flow is quasi-one-dimensional and considered as quasi-steady; that is, the freestream velocity changes so slowly that there is ample time for the flow inside the intake to adjust itself to the variation in the freestream conditions.

Under these assumptions the Kantrowitz theory leads to three distinct regions on the intake-area-ratio/freestream-Mach-number diagram (Fig. 2a). Below the isentrope curve, steady supersonic adiabatic flow in the intake (i.e., started flow) is not possible because its existence would require a decrease in entropy or, in other words, such a steady flow would pass through the area less than its sonic (critical) area. In this region, the only steady solution is the nonstarted one. A bow shock in front of the intake necessitates partial overboard spillage; flow throughout the intake is subsonic.

Above the Kantrowitz line the intake will start spontaneously; that is, steady supersonic flow in it can be established by quasi-steady acceleration of the intake from zero velocity to the required Mach number. However, the intakes belonging to this region are of little practical importance because the maximum achievable contraction (entry-to-exit area) ratio is too low to give adequate performance to a scramjet engine.

In the area between the two curves, both the started and nonstarted flow configurations are possible. An entry into this area from above the Kantrowitz line, either by a Mach number or an area ratio decrease, will maintain started flow. An entry from below the isentrope (as would happen at the takeoff of a scramjet) will result in nonstarted flow. Inside this region, a transition from nonstarted flow to started flow (shock swallowing) will not occur spontaneously because the shock, moving into the intake, would be ingesting too much mass flow for the exit to pass. A normal shock that has somehow passed into the intake would return to its upstream external position (nonstarted flow configuration) to permit the excessive mass to be spilled overboard. This constitutes the well-known intake-starting problem. The compression-ratio requirements of scramjet engine thermodynamic cycles call for high-contraction intakes, which are well below the Kantrowitz line (in fact, rather close to the isentrope) and thus do not start spontaneously when a fixed-geometry engine is gradually accelerated to its cruising speed.

Let us briefly outline the known ways of intake-starting [3–12]. In fact, if we consider that the aforementioned two assumptions of the Kantrowitz theory hold, then the only general way to start an intake would be to somehow bring it to the region above the Kantrowitz line, where it would start spontaneously and then, by another quasi-steady process, transfer it to the desired operational point above the isentrope. It is possible to do so by increasing the freestream Mach number M_∞, and this way of starting is called overspeeding (Fig. 2b). Another possibility is to change area ratio A_e/A_i by either increasing exit area A_e or decreasing entry area A_i (Fig. 2b). The exit area can be increased simply by opening the exit up with a variable geometry design. The general/classical problems on intake starting by overspeeding and exit-area enlargement in an enclosed duct are discussed in [3–6]. An alternative is to use holes or perforations in the intake walls, thereby effectively increasing the exit area [7–11]. The reduction of intake entry area can be attained with a strategically profiled movable central body (e.g., SR-71 aircraft).

In [12] it is proposed to use unsteady effects, thus circumventing the Kantrowitz limitations. It is demonstrated that an intake can be started using high intake-acceleration values when the induced flow is no longer quasi-steady. Another interesting option suggested in [12] is to attain intake-starting by inducing unsteady flow in the intake by rupturing strategically placed diaphragms. Both of these techniques rely on unsteady flow to circumvent the area-ratio limitation imposed by the Kantrowitz quasi-steady theory.

One of the simplest known intake-starting techniques is starting via so-called overboard spillage. The intake is designed in such a way that it is not fully enclosed (Fig. 1b). There is no wall on one side of the flow in the external compression section, where flow spillage takes place, during the starting process: that is why it is termed an

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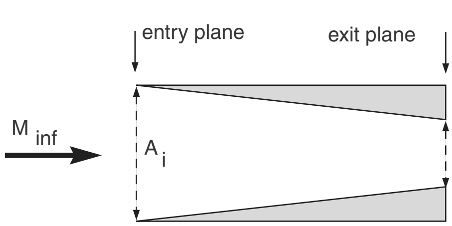
overboard spillage technique. The incoming flow is first subjected to external compression, after which it enters the fully enclosed internal compression section. The starting characteristics of the intake can now be determined by applying the Kantrowitz theory to the internal (fully enclosed) portion of the flow only (Figs. 1b and 2b). This leads to effective reduction of the Kantrowitz limit for the overall intake so that higher overall contractions can be started (see Fig. 2b). It is to be noted that in the present paper, the term *Kantrowitz line* is used to designate the self-starting condition for both nonspilling (the now-classical case considered by Kantrowitz) and spilling intakes.

The main advantage of the overboard spillage approach for starting lies in its simplicity: the intake starts spontaneously, without the help of any additional arrangements or mechanisms, such as are needed for variable geometry or perforation techniques. That is why many modern intake designs have provisions for overboard spillage [13–15]. It is obviously of interest to get the *highest allowable* contraction for starting via overboard spillage. It may not be possible to carry out such analysis for arbitrary intake geometry, but it can be performed for a given type of intake geometry.

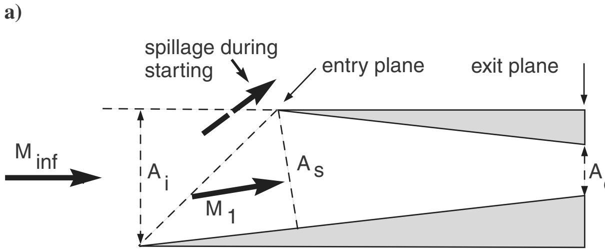
We have chosen to present an analysis of overboard-spillage-aided starting of a ramp-type family of intakes (Sec. II). This geometry was chosen because it is one of the simplest possible intake geometries with overboard spillage and as such it is fully amenable to a simple theoretical analysis. This intake is, by far, not a very high-efficiency intake. However, the presented intake-starting principles are directly applicable to more useful intakes as well. Having derived the absolute limiting contraction ratio for the intake family, we proceed in Sec. III with the study of ramp intakes corresponding to particular started flow patterns. It turns out that the Kantrowitz line for one such intake is quite close to the aforementioned limiting contraction ratio. The implications of this finding for the design of intakes with improved starting characteristics are discussed in Sec. IV, where the *strong-shock design principle* is proposed. Section V illustrates our theoretical finding with numerical simulations. Conclusions are given in Sec. VI.

II. Limiting Contractions for a General Ramp Intake

Consider the family of planar (nonaxisymmetrical) intakes schematically shown in Fig. 3a. The intakes consist of a compression ramp and a cowl. The ramp angle δ_1 is such that, at the design Mach number M_∞, the *weak* oblique shock *attached* to the ramp leading edge comes to the leading edge of the cowl. This is a necessary condition to avoid overboard spillage when the intake is started. We also assume, for the reasons explained next, that the line issued from the leading edge of the cowl perpendicularly to the ramp surface always hits the surface (or its trailing edge as a limiting case). The shock angle σ_1 serves as a parameter defining the set of intakes for the given M_∞ and area ratio A_e/A_i.



a)



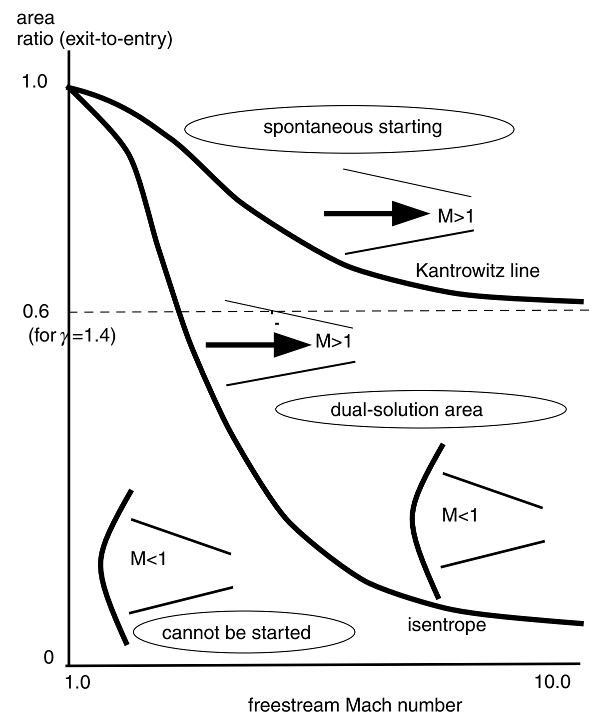
b)

Fig. 1 Air intakes: a) fully enclosed and b) with overboard spillage.

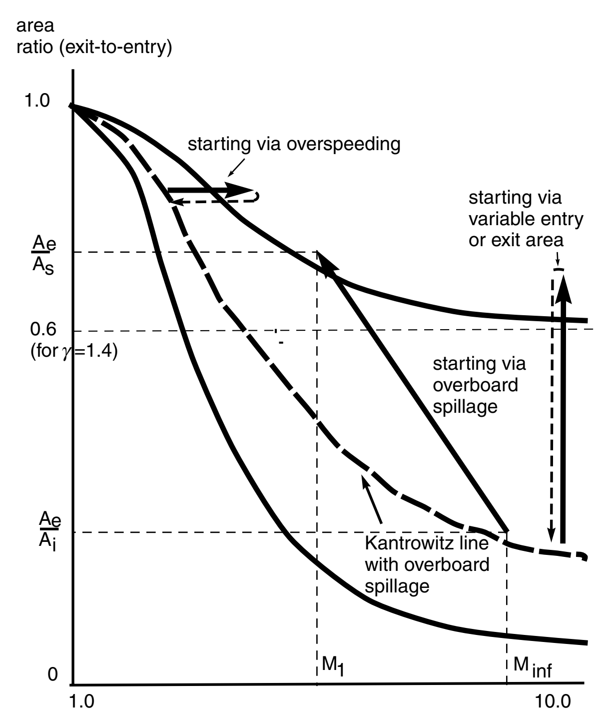
The following purely geometrical relation can be written for such intakes (see Fig. 3a):

$$\frac{A_e}{A_i} = \frac{A_e}{A_s}\frac{\sin(\sigma_1 - \delta_1)}{\sin \sigma_1} \tag{1}$$

The preceding equation relates A_i and A_s, with A_e formally added to both sides to form area ratios. Thus, it is valid for arbitrary geometry of the intake duct downstream of cross section A_s (e.g., even if the cowl is not horizontal).



a)



b)

Fig. 2 Diagrams for supersonic air intakes: a) possible flow regimes and b) intake-starting techniques.

The deflection angle δ_1 is determined from the shock angle σ_1 with the oblique shock relation for a perfect gas:

$$\tan\delta_1 = \frac{(M_\infty^2 \sin^2\sigma_1 - 1)\cot\sigma_1}{[(\gamma+1)/2]M_\infty^2 - M_\infty^2\sin^2\sigma_1 + 1} \tag{2}$$

The shock Mach number M_1 after the oblique shock is calculated as follows:

$$M_1^2 = \frac{1}{\sin^2(\sigma_1 - \delta_1)} \times \frac{2 + (\gamma-1)M_\infty^2\sin^2\sigma_1}{2\gamma M_\infty^2 \sin^2\sigma_1 - (\gamma - 1)} \tag{3}$$

The first step in the determination of the limiting contraction value for an intake to be started, by overboard spillage effect alone, is to establish the external and internal compression sections. For the preceding ramp intake, the freestream flow, processed by the attached weak oblique shock, is directed along the ramp surface. The internal compression (or fully enclosed) intake section begins from the aforementioned normal line issued from the cowl leading edge (Fig. 3a). In this case, the flow entering the internal compression section is normal to its entry plane, as required by the underlying assumptions of the Kantrowitz theory. Such delineation of the external and internal compression sections for ramp intakes is also meaningful from the gasdynamic point of view: the inviscid numerical simulations [10] show that at the contraction ratios close to those required for intake self-start, the bow shock is straight, normal to the ramp surface, and located close to the cowl leading edge.

According to the quasi-one-dimensional Kantrowitz theory, flow in the internal compression section starts spontaneously if its area ratio A_e/A_s is equal to or higher than the following K value, determined with the Mach number M_1:

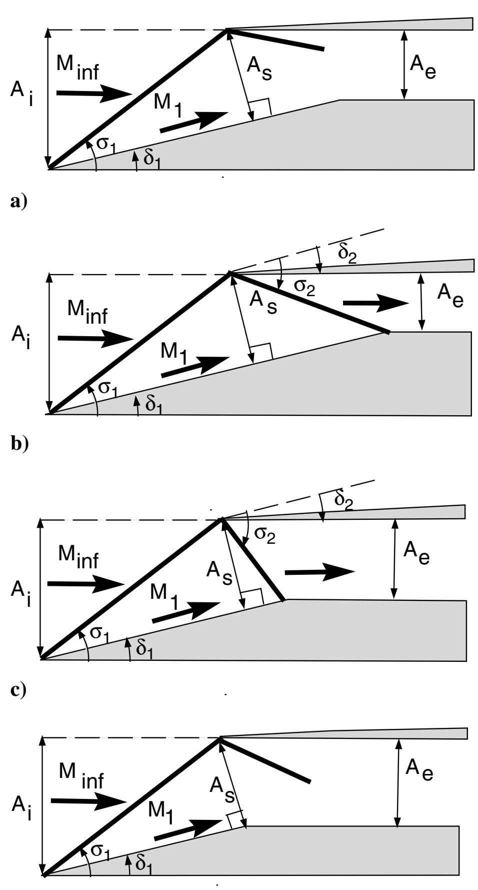


Fig. 3 Intake schematics: a) general ramp intake, b) two-shock intake with the second weak shock, c) two-shock intake with the second strong shock, and d) normal-line design.

$$\left(\frac{A_e}{A_s}\right)_k = \left[\frac{\gamma-1}{\gamma+1} + \frac{2}{(\gamma+1)M_1^2}\right]^{\frac{1}{2}} \left[\frac{2\gamma}{\gamma+1} - \frac{\gamma-1}{(\gamma+1)M_1^2}\right]^{\frac{1}{\gamma-1}} \tag{4}$$

This condition corresponds to the stationary normal shock of Mach number M_1 at the entry to the fully enclosed section A_s and choked exit A_e. For this condition to be applicable, the normal line issued from the cowl leading edge is required to hit the ramp surface or its trailing edge. Otherwise, the flow upstream of the normal shock would be nonuniform due to the expansion fan at the ramp trailing edge.

Using Eqs. (1–3), the limiting area ratio for the overall intake, $(A_e/A_i)_k$, can be expressed as a rather bulky function of the freestream Mach number M_∞ and the shock angle σ_1:

$$\left(\frac{A_e}{A_i}\right)_k = f(M_\infty, \sigma_1) \tag{5}$$

Equation (5) is shown graphically in Figs. 4a and 4b as an elevated area-ratio surface on the M_∞–σ_1 plane; above this surface, the intake starts spontaneously.

In Fig. 4, the lower limit of shock angle σ_1 is equal to the Mach angle value for the respective freestream Mach number. In this limiting case δ_1 approaches zero, A_s tends to A_i (M_1 to M_∞), thus recovering the classical Kantrowitz line on the A_e/A_i–M_∞ plane. In Fig. 4b, the upper limit of σ_1 is equal to the value resulting, for the respective freestream Mach number, in sonic flow downstream of the oblique shock at the ramp (i.e., $M_1 = 1$). For subsonic postshock flow, condition (4), implying steady normal shock at A_s, cannot be used.

Further constraints on the upper values of shock angle σ_1 may arise, depending on the geometry of the cowl. If we consider a straight horizontal cowl, as shown in Fig. 3a, the deflection angle of the reflected shock is equal to δ_1. The detachment of the reflected shock would result in a bow shock at the cowl leading edge and a certain amount of spillage. Such flow cannot be considered as being "started," according to the preceding definition. (It is to be noted that in case of small standoff distances, the overboard spillage may be negligible for practical purposes.) Thus, for the geometry shown in Fig. 3a, shock angle σ_1 is limited by the value resulting in the postshock flow Mach number M_1 for which the detachment angle is equal to δ_1. The surface [Eq. (5)] subjected to this constraint on σ_1 is shown in Fig. 4a. If the inside surface of the cowl is not horizontal, the constraint would allow higher maximum shock angles σ_1. In fact, Fig. 4b corresponds to the case when the cowl is initially inclined at δ_1.

It is seen in Fig. 4 that for any given freestream Mach number, there is a minimum value of $(A_e/A_i)_k$ corresponding to a certain value of σ_1. This minimum area ratio corresponds to the maximum contraction $(A_i/A_e)_k$ at which this type of intake would start spontaneously, assisted by the overboard spillage effect alone. In other words, for a given freestream Mach number, there is an optimal intake geometry providing the best starting characteristics via overboard spillage for the given intake family.

To explain the existence of the optimal design for starting via overboard spillage, let us consider ramp intakes at the Kantrowitz surface with a constant area ratio of 0.4 and vary the shock angle σ_1 (and therefore, the freestream Mach number as well) from a minimum value to a maximum one. (Similar considerations would apply when considering a fixed value of freestream Mach number.) This is equivalent to proceeding along one of the constant-area-ratio curves shown in Fig. 4b on the M_∞–σ_1 plane. Figure 5 shows how the internal area ratio A_e/A_s and Mach number M_1 of the flow entering the internal compression section vary with the shock angle σ_1. It is seen that the internal area ratio increases continuously and, hence, internal contraction decreases, which is favorable for spontaneous starting. However, Mach number M_1 decreases continuously, and that is unfavorable for starting. These two opposite effects compete and an optimal point exists at a certain shock angle.

The minimum $(A_e/A_i)_k$ values are shown on the area-ratio/freestream-Mach-number diagram (Fig. 6) as a bold line. The uppermost line corresponds to the Kantrowitz line for fully enclosed ducts [Eq. (4)] and the lowest line is the isentrope below which steady started flow is impossible. It is seen that overboard spillage improves the intake-starting characteristics considerably. The bold line indicates the *limiting* contraction beyond which overboard spillage is not sufficient to start the intakes subjected to the preceding geometrical constraints.

III. Limiting Contractions for Particular Types of Ramp Intakes

The ramp intake with the maximum allowable contraction (minimum area ratio) for overboard spillage starting does not seem to correspond to any particular wave pattern or geometrical design. The only limitations imposed so far are that the leading weak oblique shock should be attached to the ramp and it should hit the cowl leading edge, and the line issued from the cowl leading edge perpendicularly to the ramp surface should hit the surface or its trailing edge, as a limiting case.

It is of interest to examine the Kantrowitz (self-starting) lines of ramp intakes that either attain a prescribed shock pattern when started (because these patterns are used as a basis for the intake's design) or adhere to a prescribed geometrical configuration and compare them with the derived limiting Kantrowitz line for the whole intake family. First, consider the two-shock intakes in which uniform exit flow is achieved by deflecting the flow along the ramp back to the freestream direction via the second oblique shock. The second shock can be either weak (Fig. 3b) or strong (Fig. 3c). If it is strong, the exit flow is subsonic, however, all freestream flow is captured so that the intake flow is still termed *started*. Secondly, the so-called normal-line-design intake (Fig. 3d) is considered. This design is purely geometrical: the line issued from the cowl leading edge perpendicularly to the ramp surface hits the ramp trailing edge; in this case, the exit flow pattern may be complicated because the second shock is not canceled at the ramp trailing edge. For all intakes studied, the cowl is considered to be horizontal.

The preceding three cases, to be analyzed in detail, do not exhaust all possibilities. For instance, one may consider an optimized two-shock intake in terms of total pressure recovery. It is known that with two shocks, the highest total pressure recovery is obtained when the total pressure ratio across the two shocks is equal. For equal total pressure ratios, $M_\infty \sin\sigma_1 = M_1 \sin\sigma_2$ (see Fig. 3). However, because in this case $\delta_1 \neq \delta_2$, the design would have a slightly offaxis exit flow.

The derivation of the Kantrowitz line equation for the two-shock intake is common for the weak and strong second-shock cases. The weak or strong solution is chosen by selecting the appropriate shock angle σ_2. For both cases, we have the following geometrical relation (Figs. 3b and 3c):

$$\frac{1 - A_e/A_i}{\tan(\delta_1)} = \frac{1}{\tan(\sigma_1)} + \frac{A_e/A_i}{\tan(\sigma_2 - \delta_2)} \tag{6}$$

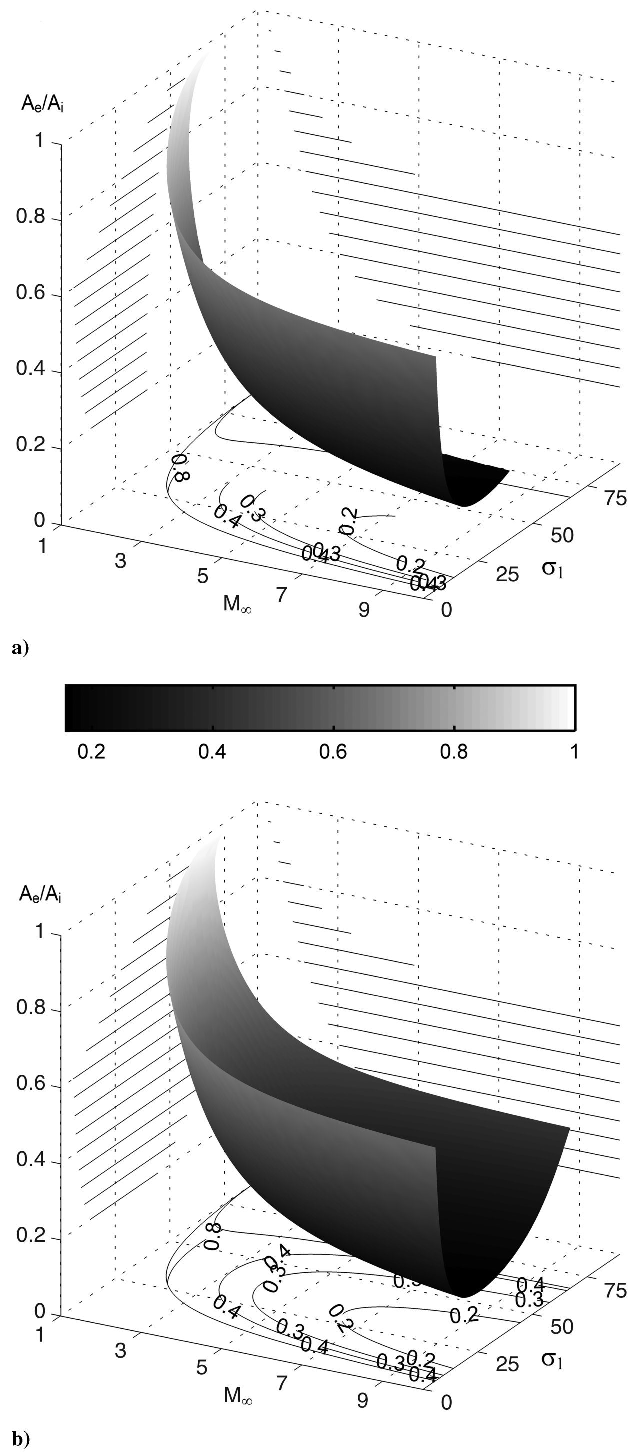


Fig. 4 The Kantrowitz surface, Eq. (5), for ramp intakes. The limiting area ratio for spontaneous starting, $(A_e/A_i)_k$ is shown as an elevated surface on the M_∞–σ_1 plane and the upper boundary of σ_1 corresponds to a) detachment of the reflected shock at a horizontal cowl and b) sonic flow downstream of the weak oblique shock at the ramp.

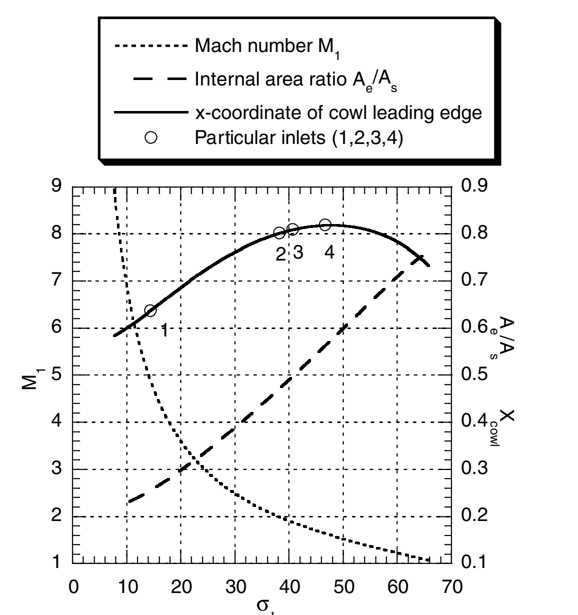


Fig. 5 Mach number M_1 behind the first oblique shock, internal area ratio A_e/A_s, and x coordinate of the cowl leading edge for ramp intakes with overall area ratio $A_e/A_i = 0.4$ as functions of shock angle σ_1. The open circles correspond to the following intakes: 1 denotes the two-shock intake with a weak second shock, 2 denotes the two-shock intake with a strong second shock, 3 denotes the ramp intake with the best starting characteristics (for the given area ratio), and 4 denotes the ramp intake with normal-line design.

which results in the following expression for the overall area ratio:

$$\frac{A_e}{A_i} = \frac{\cot(\delta_1) - \cot(\sigma_1)}{\cot(\sigma_2 - \delta_2) + \cot(\delta_1)} \tag{7}$$

where δ_2 is the deflection angle for the second shock, and σ_2 is the shock angle for the second shock (either weak or strong), given by

$$\tan\delta_2 = \frac{(M_1^2 \sin^2\sigma_2 - 1)\cot\sigma_2}{[(\gamma+1)/2]M_1^2 - M_1^2\sin^2\sigma_2 + 1} \tag{8}$$

with M_1 from Eq. (3). For the two-shock intakes with a horizontal cowl inner surface $\delta_1 = \delta_2$ by design, which leads to the following relation to determine σ_2:

$$\frac{(M_\infty^2 \sin^2\sigma_1 - 1)\cot\sigma_1}{[(\gamma+1)/2]M_\infty^2 - M_\infty^2\sin^2\sigma_1 + 1} - \frac{(M_1^2 \sin\sigma_2^2 - 1)\cot\sigma_2}{[(\gamma+1)/2]M_1^2 - M_1^2\sin^2\sigma_2 + 1} = 0 \tag{9}$$

Equation (1) still holds because it is valid for the whole family of intakes. Therefore, we arrive, with Eqs. (1) and (7), at the following relation for the area ratio A_e/A_s for the fully enclosed part of the two-shock intake:

$$\frac{A_e}{A_s} = \frac{\cot(\delta_1) - \cot(\sigma_1)}{\cot(\sigma_2 - \delta_2) + \cot(\delta_1)} \frac{\sin\sigma_1}{\sin(\sigma_1 - \delta_1)} \tag{10}$$

The spontaneous starting boundary (i.e., the Kantrowitz line) is defined by Eq. (4), so that we have at the boundary

$$\frac{\cot(\delta_1) - \cot(\sigma_1)}{\cot(\sigma_2 - \delta_2) + \cot(\delta_1)} \frac{\sin\sigma_1}{\sin(\sigma_1 - \delta_1)} = \left[\frac{\gamma-1}{\gamma+1} + \frac{2}{(\gamma+1)M_1^2}\right]^{\frac{1}{2}} \left[\frac{2\gamma}{\gamma+1} - \frac{\gamma-1}{(\gamma+1)M_1^2}\right]^{\frac{1}{\gamma-1}} \tag{11}$$

For a given freestream Mach number M_∞, Eq. (11), combined with Eqs. (2), (3), (8), and (9), contains the single unknown σ_1 defining the two-shock intake at the Kantrowitz line. Having found the first shock angle σ_1, it is possible to get the respective overall area ratio A_e/A_i with Eq. (7). Upon repeating the procedure for the whole range of freestream Mach numbers, we arrive at the Kantrowitz curves for the two-shock intakes with the second weak and strong shocks shown in Fig. 6. It is seen that the Kantrowitz line for the two-shock intake with the second weak shock lies approximately halfway between the classical (for fully enclosed duct) Kantrowitz line and the limiting contraction line for the family of intakes under consideration. The Kantrowitz line for the two-shock intake with the second strong shock is very close to the limiting contraction line.

A very similar procedure leads us to the Kantrowitz curve for the normal-line-design intake (Fig. 3d). The geometrical relation for such an intake is:

$$\frac{A_e}{A_s} = \cos\delta_1 \tag{12}$$

Then the Kantrowitz condition (4) gives:

$$\cos\delta_1 = \left[\frac{\gamma-1}{\gamma+1} + \frac{2}{(\gamma+1)M_1^2}\right]^{\frac{1}{2}} \left[\frac{2\gamma}{\gamma+1} - \frac{\gamma-1}{(\gamma+1)M_1^2}\right]^{\frac{1}{\gamma-1}} \tag{13}$$

Equation (13), in combination with Eqs. (2) and (3), represents the equation for finding σ_1 and, also using Eq. (7), A_e/A_i. The resulting curve is shown in Fig. 6 as well. It is seen that this design possesses self-starting characteristics that are slightly worse than those for the strong-shock intake but noticeably better than those for the weak shock design.

Because the two-shock and normal-line-design intakes both belong to the ramp family considered in Sec. II, all the curves in Fig. 6 belong to the Kantrowitz surface for ramp intakes shown in Fig. 4. This is demonstrated in Fig. 7, which is a top view of the Kantrowitz surface of Fig. 4 with added curves corresponding to the particular intakes. These are the same curves as in Fig. 6. The detachment line for the second (reflected) shock at a horizontal cowl inner surface is shown in Fig. 7 as well. Above that line, the resulting intake flow cannot be considered as being started, according to our definition, due to the presence of spillage induced by a detached shock at the cowl leading edge. The intake in the region above the second-shock detachment line can be started only with an inclined internal cowl surface and the required angle of inclination increases with the shock angle σ_1. It is seen that the ramp intake with normal-line design (Fig. 3d) cannot be started with a horizontal cowl at all: the respective line is above the detachment line for all freestream Mach numbers. At the same time, both two-shock intake lines are below the detachment line for all Mach numbers, and hence these intakes can be operated with a horizontal cowl. The limiting contraction line for low Mach numbers lies above the detachment line.

Overboard spillage, in general, is achieved by displacing the cowl leading-edge downstream relative to the leading edge of the ramp. This is equivalent to a single large perforation at the beginning of the upper intake wall, which assists starting, as would multiple wall perforations. It would be of interest to investigate exactly how the relative displacement of the cowl leading edge influences the starting characteristics. We again consider the ramp intakes at the Kantrowitz surface with a constant area ratio of 0.4. The origin of coordinates is established at the ramp leading edge and the x axis is directed along the freestream. Then all the intakes are proportionally scaled to the same ramp length of 1.0 or, in other words, the x coordinate of the ramp trailing edge is made to be 1.0 for all the intakes for the sake of comparison. The x coordinate of the cowl leading edge is shown in Fig. 5 as a function of shock angle σ_1. The location of all particular intakes on the diagram is indicated with open circles. The intake corresponding to the detachment of the second, reflected shock at a horizontal cowl is situated between points 2 and 3 (Fig. 5). The following observations can be made. The maximum displacement of the cowl leading edge relative to the ramp leading edge is achieved with the normal-line design. However, this is not the best design from the point of view of starting characteristics. This is due to the fact that

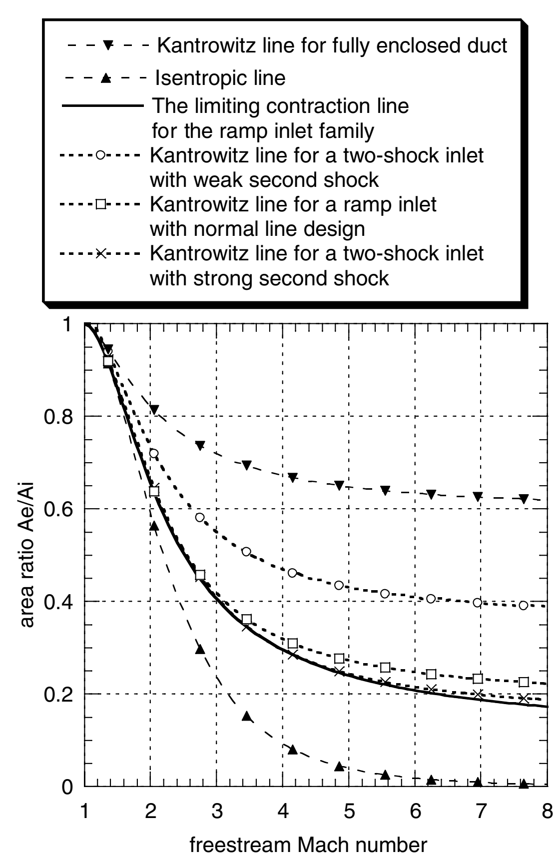


Fig. 6 Diagram for different ramp intakes.

starting properties are determined not only by geometrical characteristics but by flow gasdynamics as well. However, restricting ourselves to intakes with lower first shock angles (low σ_1), it is seen that a larger relative displacement of the cowl leading edge results in better starting characteristics.

IV. Strong-Shock Design Principle

The Kantrowitz line for the two-shock intake with the second strong shock is just above and very close to the limiting contraction line for this family of intakes. Therefore, it is possible to use this strong-shock topography to design an intake with the best possible starting characteristics (starting by overboard spillage only). Such a wave pattern, with the strong second shock, can be obtained in practice only if the proper pressure value is maintained at the back. The strong shock does not provide a flow suitable for scramjet application. Lowering the backpressure will cause the strong shock to revert to the weak variety, with the flow downstream of it becoming supersonic as the entry flow remains unaltered and on-design. The weak second shock would face the nonuniform flow caused by the expansion emanating from the trailing edge of the ramp.

The resulting nonuniform exit flow is the price to be paid for obtaining a spontaneous start at the contraction ratios close to the theoretical limit for the intake family under study. Because the second weak shock and the centered expansion would have a tendency of canceling each other, the nonuniformity may not be too severe. Furthermore, it may be conjectured that the exit flow nonuniformity may be potentially useful in facilitating efficient air/fuel mixing in the combustor.

The ramp intakes are chosen for the present investigation because of their relative simplicity for analysis and illustration. These intakes are not particularly efficient and hence not necessarily practical. However, it may be conjectured that our design principle, to ensure spontaneous starting, is applicable to more complex and more capable intakes as well. Indeed, in more practical intakes, such as Prandtl–Meyer, Oswatitch, Busemann, and others, most of compression is external and isentropic (as opposed to shock compression via the first oblique shock in ramp intakes), which improves the intake efficiency. Nevertheless, all these intake flows contain a terminal shock wave of some kind in the internal compression section, which produces the uniform exit flow parallel to the intake axis. According to our findings, the replacement, for design purposes, of this shock by a strong oblique shock might be beneficial for starting via overboard spillage. Because the terminal shock is responsible for a relatively low portion of overall compression, such a replacement would not lead to a substantial increase in total pressure loss, even if the intake would operate with the backpressure supporting the strong shock. With low backpressures, the terminal shock will be weak, resulting in an exit flow with some degree of nonuniformity. It may be conjectured that such exit flow nonuniformity may be quite tolerable in view of significant gains in starting characteristics. This line of investigation will be pursued and reported elsewhere.

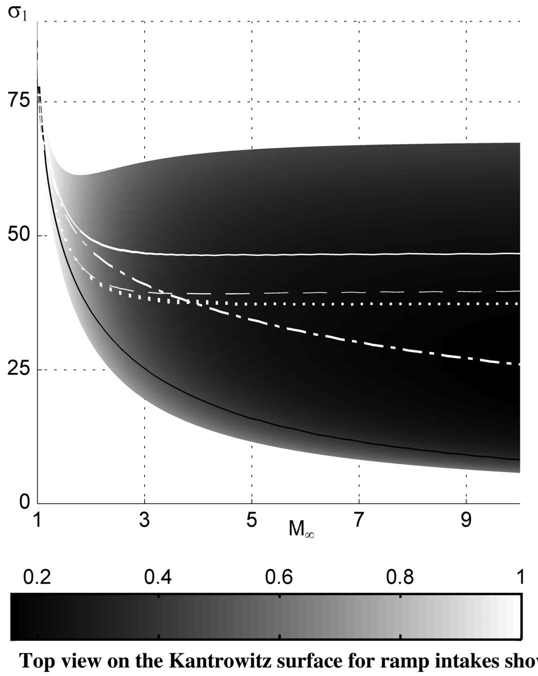


Fig. 7 Top view on the Kantrowitz surface for ramp intakes shown in Fig. 4. The following lines are shown: Kantrowitz line for a two-shock intake with weak second shock (solid, black); Kantrowitz line for a two-shock intake with strong second shock (dotted, white); Kantrowitz line for a ramp intake with normal-line design (solid, white); the limiting contraction line for the ramp intake family (dash-dotted, white); the line of second-shock detachment for a horizontal cowl (dashed, white).

V. Numerical Simulations

In this section, the preceding theoretical (Kantrowitz) starting predictions are verified by numerical simulation. Flow in six two-shock intakes, designed with strong second shock, is simulated for freestream Mach numbers 3, 5, and 7. For each Mach number, one intake is just above the Kantrowitz line and another is just below it. The six design points are shown in Fig. 8.

The Euler equations with the volumetric source term, accounting for intake acceleration, were used as the governing equations (see [12]). The numerical simulations were performed using the code SolverII [16]. The numerical kernel used for the present simulations is the second order in space and time, locally adaptive, unstructured, Godunov-type (MUSCL-Hancock with the exact Riemann solver, where MUSCL stands for monotone upstream-centered scheme for conservation laws), finite volume solver. The numerical scheme (and the code) has been extensively benchmarked against both analytical solutions and experimental data for steady and unsteady flows with shock waves (see [17] and the extensive list of references there).

At the initial time moment, the intake and the surrounding gas are at rest. In the course of computation, the intake of 1-m length is accelerated from zero velocity to the velocity corresponding to the design Mach number (3, 5, or 7) with the acceleration of $a = 100\ g$ at sea level. As discussed in [12], under such conditions the flow may be, with confidence, considered as quasi-steady. Indeed, with a

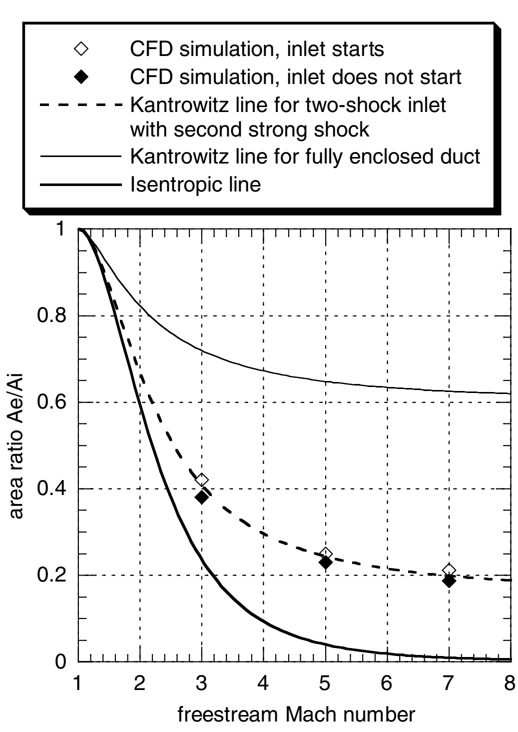


Fig. 8 Diagram indicating the two-shock intakes designed with second strong shock, which were subjected to numerical starting tests.

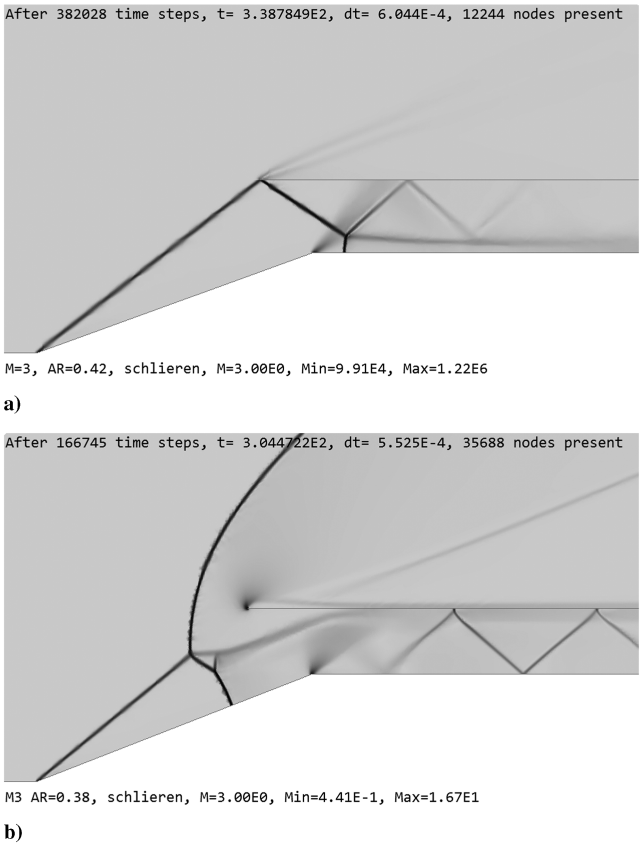


Fig. 9 Numerical schlieren images for the two-shock intake designed with second strong shock. At area ratio 0.42, intake starts (top) and at area ratio 0.38, intake does not start (bottom).

characteristic intake size L of 1 m and freestream speed of sound c_∞ of 345 m/s, the nondimensional acceleration $a/(c_\infty^2/L) = 0.0085$. Therefore, the acceleration term in the governing equations is negligibly small, and the quasi-steady condition is maintained in the course of computation [12]. Note that the numerically simulated flow evolves in time, from the initial state to the final steady state, in a time-accurate sequence. The quasi-steady assumption is implicit only in the Kantrowitz theory.

The boundary fluxes were computed via the solution of Riemann problems corresponding to physical boundary conditions at the respective boundaries. This guarantees proper accounting for in- and outgoing disturbances. For subsonic exit boundaries, the specified pressure boundary condition was used with a low pressure value equal to the freestream pressure.

A grid convergence study was done to prove that the obtained outcomes are grid-independent. Typical started (area ratio 0.42) and nonstarted (area ratio 0.38) flowfields for a freestream Mach number of 3 are shown in Fig. 9.

Note that the numerical values behind the incident shock and the reflected weak shock exactly correspond to the respective analytical solution. Other numerical experiments were carried out for the started cases (Fig. 9, top). The exit pressure was raised according to the procedure described and illustrated in [18], so that the flow eventually reverted to the steady solution with a strong second shock. For this flow, the postshock numerical values also coincide with analytical predictions. All these comparisons serve as more evidence of the validity of the employed numerical code.

It is seen that in the nonstarted case (Fig. 9, bottom), the bow shock provides the mechanism to spill the excessive amount of mass. In the started case (Fig. 9, top), all the flow is captured, the faint wave above the cowl being due to the smearing of the first oblique shock inherent to the shock-capturing approach. Because a low pressure value is maintained at the intake exit, the second (reflected) oblique shock is weak. It is seen that its interaction with the centered expansion at the trailing edge of the ramp eventually results in fairly uniform pressure distribution at some distance downstream.

The results for freestream Mach numbers 5 and 7 are qualitatively similar. The numerical intake-starting simulation leads to starting flows for area ratios higher than the Kantrowitz limit and to nonstarted flows if the area ratio is lower than the Kantrowitz limit (as graphically indicated in Fig. 8), in very good correspondence with theoretical predictions.

VI. Conclusions

For a simple ramp-type intake, the Kantrowitz starting theory was applied to obtain the limiting contraction for intake-starting via overboard spillage. The strong-shock design principle is then suggested. It results in a ramp intake with the Kantrowitz line very close to the established limiting values. Analytical self-starting predictions were well confirmed by numerical simulation. It is expected that the application of this approach to more practical intake designs will result in high-contraction as well as high-efficiency intakes that will start spontaneously.

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