

$$\Rightarrow P_1 = 1.117 \times 10^3 \text{ Pa.}$$

$$T_1 = 226.98 \text{ K.}$$

$$\rho = 0.01715 \text{ kg/m}^3.$$

$$u = 7620 \text{ m/sec.}$$

Assume $\theta = 3 = 1.29$, $= 48^\circ$

$$\cancel{u_{n2} = 0}$$

$$u_{n2} = 0$$

$$u_{n1} = u \sin \theta$$

$$= 5662.76 \text{ m/sec.}$$

$$P_1 + \rho_1 u_{n1}^2 = P_2 + \rho_2 u_{n2}^2$$

$$1.117 \times 10^3 + 0.01715 \times 7620^2 = \cancel{P_2}$$

$$1.117 \times 10^3 + 0.01715 \times 5662.76^2 = P_2$$

$$\Rightarrow P_2 = 551063 \text{ Pa}$$

$$= 5.47.$$

$$h_1 = h_{01} = C_p T_1 + \frac{u_1^2}{2} = h_{02} \quad \therefore u_{n2} = 0$$

$$h_2 = 16.26 \text{ MJ/kg}$$

From Chart for h_2 & P_2 .

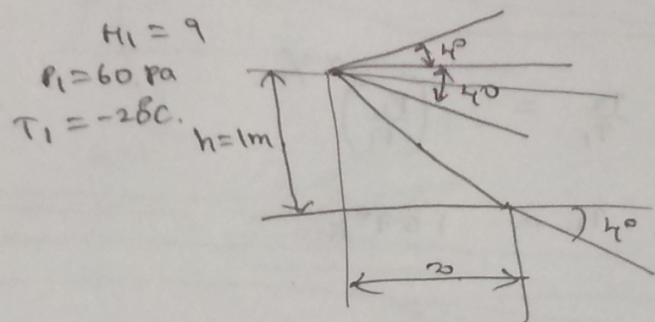
$$Z = 1.324.$$

$$T = 6650 \text{ K.}$$

$u_{n2} = 0$

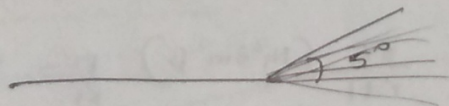
1. Air flows down a wide tunnel at mach number of 9.8 with a pressure of 120 Pa and a temperature of -45°C . The upper wall of this channel turns through an angle of 5° away from the flow leading to the generation of an expansion wave. Find the pressure, mach number and temperature behind this expansion wave.

2. Consider the fig below.



Q. (1)

$M = 9.8$ $p_1 = 120 \text{ Pa}$ $T_1 = -45^{\circ}\text{C}$

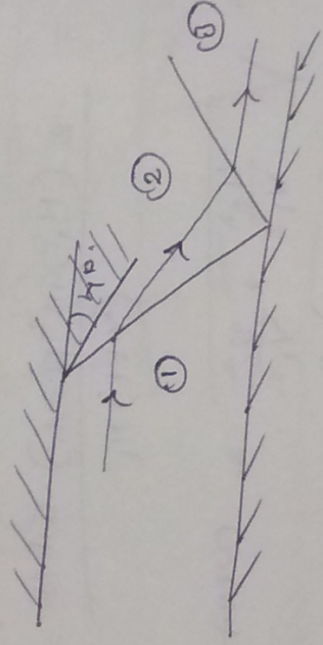


$$\theta = \frac{2}{\gamma-1} \left[\frac{1}{M_1} - \frac{1}{M_2} \right]$$

$$0.0872 = \frac{2}{0.4} \left[\frac{1}{9.8} - \frac{1}{M_2} \right]$$

$$M_2 = 11.82$$

(15) Air flowing with a Mach number of 7 with a pressure of 120 Pa and a temper of -60°C passes over a wedge which turns the flow through an angle of 40° leading to the generation of an O.S. wave. The O.S. wave impinges on a flat wall, which is parallel to the flow upstream of the wedge and is reflected from it. Find the pressure, Mach and velocity behind the reflected shock wave.



$$\begin{aligned}
 \frac{P_2}{P_1} &= \frac{\gamma+1}{\gamma} + \sqrt{\left(\frac{\gamma-1}{\gamma}\right)^2 + \frac{1}{M_1^2 \sin^2 \beta}} \\
 &= \frac{2.4}{1.4} + \sqrt{\left(\frac{2.4}{1.4}\right)^2 + \frac{1}{7^2 \times 0.698^2}} \\
 \frac{P_2}{P_1} &= 4.832 + 2.732 \\
 &= \frac{0.1907}{0.3372} \text{ rad} = 10.929^\circ \\
 \Rightarrow P_2 &= P_1 \left(1 + \frac{2\gamma}{\gamma+1} (M_1^2 \sin^2 \beta - 1)\right) \\
 &= 120 \times \frac{2 \times 1.4}{2.4} \left(7^2 \times \sin^2 10.929^\circ - 1\right)
 \end{aligned}$$

1st iteration. $\frac{P_2}{P_5} = 0.1$ Approximate.

$P_5 = 988.8 \text{ atm}$

$h_5 = 4.46 \text{ MJ/kg}$

$Z = 1.009$

$T = 2420 \text{ K}$

$\rho_5 = 9.086 \text{ kg/m}^3$

2nd iteration.

$P_5 = 102.082$

$h_2 = 5.14 \text{ MJ}$

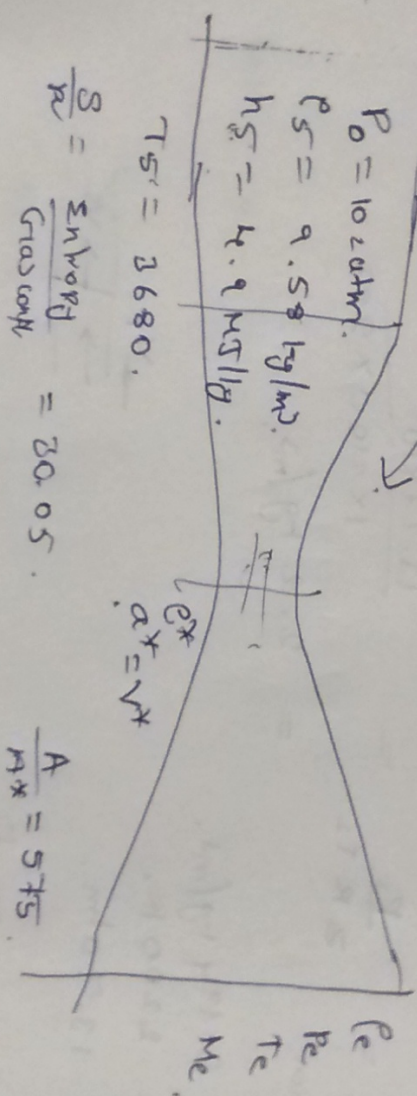
$Z = 1.01$

$T = 3340 \text{ K}$

$\rho_5 = 9.56 \text{ kg/m}^3$

Q.

Shock tunnel.



$\frac{S}{R} = \frac{Z h_{\text{prop}}}{C_{\text{gas}} \text{ const}} = 20.05$

$\frac{A}{A^*} = 575$

$T ds = dh - v dp$

For a diabatic flow. $h_0 = \text{const}$

$\Rightarrow dh + v dv = 0$

$dp = -\rho v dv$

$v dx = -\frac{dp}{\rho} = -\frac{dp}{\rho} = -\frac{dp}{\rho}$

sf. volume

$$P_{02} = 9 \text{ kPa} \text{ Answer}$$

2. Air flow over a blunt body. The air flow in the free stream, ahead of the body has a Mach number of 9 and static pressure of 100 Pa. Find the pressure acting on the front of the body using hypersonic relation and compare it with the solution from exact ~~N.S~~ N.S relation. Sketch the flow pattern near the nose.

$$M_1 = 9. \quad P_1 = 100 \text{ Pa.}$$

P_{02}

$$\frac{P_2}{P_1} = \frac{2\gamma}{\gamma+1} M_1^2$$

$$= 94.5$$

$$\frac{u_2}{u_1} = 1 - \frac{2}{\gamma+1}$$

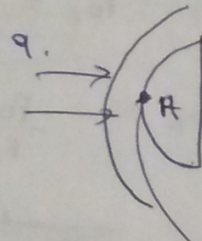
$$= 0.167 \checkmark$$

$$u_2 = 0.167 u_1$$

$$v_1 = M_1 a_1$$

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M_1^2$$

$$= 15.75$$

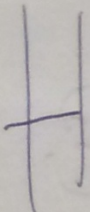
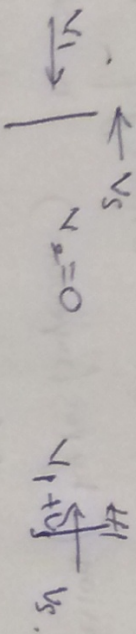


$$P_A = P_2 = P_{02}$$

It is assumed to be N.S.

Still deep up to 18th, Air sheet of formulae.
 0916 known.

A. Air is flowing out of a duct at a velocity of 250 m/s with a temp of 0°C and a pr of 101325 Pa. A valve at the end of the duct is suddenly closed. Find the pressure acting on the valve immediately after the valve closure.



$$M_{\text{down}} = \sqrt{\frac{\gamma T_1}{T_2}} \left[\gamma p_1 - \frac{\gamma V_1}{\alpha_1} \right]$$

$$M_1 = \frac{V_1 + V_5}{\alpha_1} \quad M_2 = \frac{V_5}{\alpha_2}$$

$$= \frac{V_1}{\alpha_1} + \frac{\gamma p_1 \times \alpha_2}{\alpha_2 \alpha_1}$$

$$M_1 = \frac{V_1}{\alpha_1} + M_2 \cdot \sqrt{\frac{T_2}{T_1}}$$

$$= \frac{250}{331.19} + M_2 \sqrt{\frac{T_2}{T_1}}$$

$$M_1 = 0.75485 + M_2 \sqrt{\frac{T_2}{T_1}}$$

$$C_d \rightarrow 1$$

$$D_{\text{drag}} = (P - P_0) \pi a^2$$

$$C_d = \frac{26389}{26389}$$

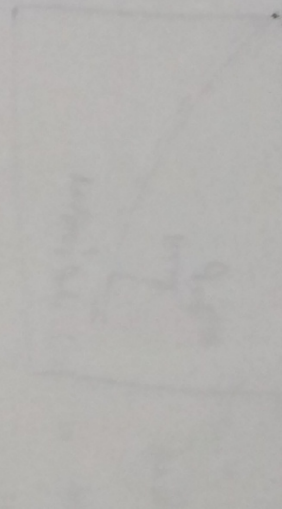
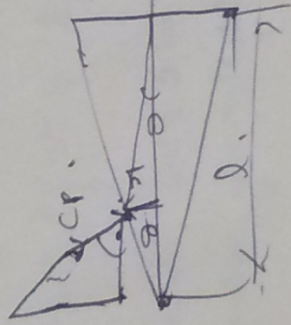
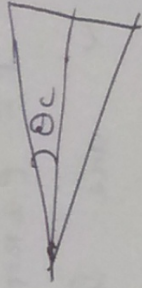
$$D_{\text{drag}} = 26389 \text{ N} = 26.38 \text{ kN}$$

(5) Find the C_b for cone (C_c) using Newtonian Approximation.

We have to take
Projected area

$$C_d = 1$$

$$D_{\text{drag}} = \frac{1}{2} \rho V^2 C_d A_p$$



Assume ~~the~~ $V_e = 2000 \text{ m/sec}$.

$$h = 2.9 \text{ MJ/1g.}$$

$$P = 10.4 \text{ atm.}$$

$$Z = 1$$

$$C_e = 1.47$$

$$C_e V_e = 2.942 \neq 9.9.$$

$$V_e = 3040 \text{ m/sec.}$$

$$h = 3.4 \text{ MJ}$$

$$P_e = 303 \text{ Pa.}$$

$$P = 310 \text{ k.}$$

$$T_e = 310 \text{ K.}$$

$$C_e = 3.41 \times 10^3 \text{ J/m}^3.$$

$$C_e V_e = 10.386.$$

Page No - 617 J.D Anderson.

Ex 8.1 Quasi 1D Flow

$\Rightarrow P_L = 4112 \text{ Pa.}$

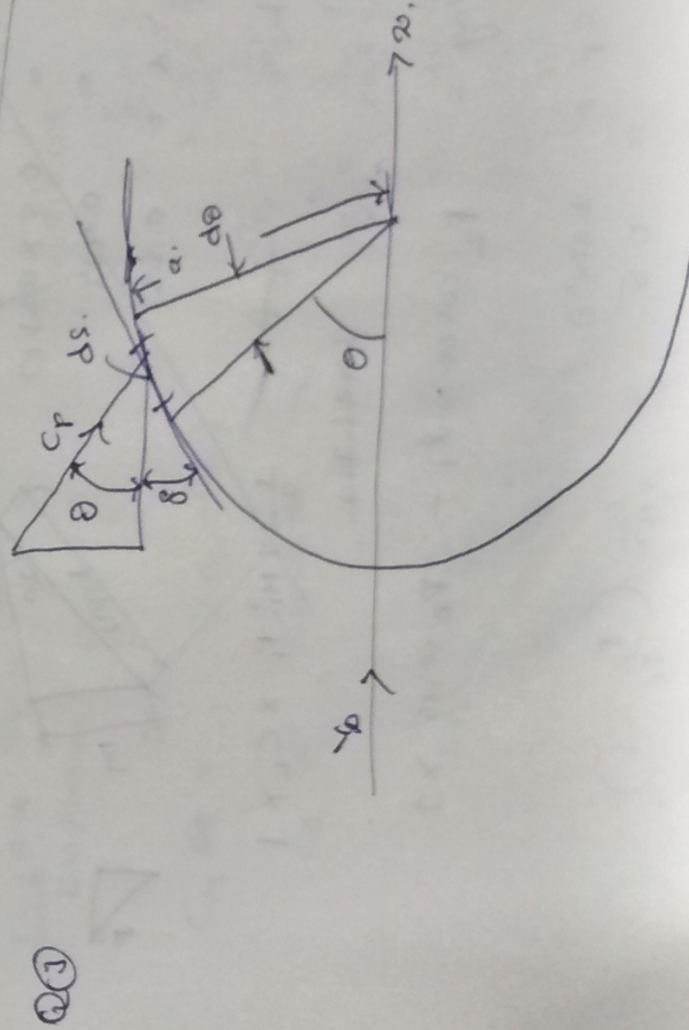
$D_{\text{drag}} = 3890 \text{ N}$



$\sin 11.5 = \frac{0.5}{l}$
 $l = 1.97$

(3) Find the C_D for circular cylinder using Newtonian Approximation.

(4) Find the C_D for ~~hemisphere~~ sphere using Newtonian approximation and then find the drag for sphere of $\rho_{\text{air}} = 1.208 \text{ kg/m}^3$ at mach 10, using Newtonian Approximation $T_{\infty} = 200 \text{ K}$, $\rho_{\infty} = 1.208 \text{ kg/m}^3$, Radius = 1m



(5)

Calculate the flow downstream of a normal shock wave
 concave, cohesion velocity $u_1 = 4572 \text{ m/sec}$ at an
 altitude of 45.72 km .

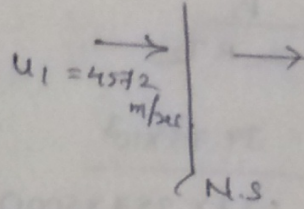
$$\frac{\gamma}{\gamma_{SL}} = 1.40628.$$

$$P_1 = 142 \text{ N/m}^2.$$

$$T_1 = 266 \text{ K} \quad a_1 = 327 \text{ m/sec}$$

$$\rho_1 = 1.895 \times 10^{-3} \text{ kg/m}^3.$$

$$M_1 = \frac{u_1}{a_1} = 13.98 \approx 14$$



$$u_2 = \rho_1 / \rho_2 \cdot u_1$$

$$P_2 = P_1 + \rho_1 u_1^2 - \rho_2 u_2^2$$

$$h_2 = h_1 + \frac{u_1^2}{2} - \frac{u_2^2}{2}$$

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2.$$

$$T_{01} = 10693^\circ \text{K.} \quad \text{Temp is large enough to have dissociation.}$$

$$h_1 = C_p T_1 = 267.33 \times 10^3 \text{ J/kg.}$$

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \quad u_2 = 0. \quad \text{assume iteration.}$$

$$267.33 \times 10^3 + \frac{4572^2}{2} = h_{02}.$$

$$h_{02} = 10718.9 \text{ kJ/kg.}$$

$$P_2 = 142 + 1.895 \times 10^{-3} \times 4572^2 = 39.75 \text{ kPa.}$$

$$\frac{P_2}{P_1} = 1.89 = 1.91466$$

$$\frac{P_2}{P_1} = \frac{(\gamma+1) M_1^2 \sin^2 \beta}{(\gamma-1) M_1^2 \sin^2 \beta + 2}$$

$$= 1.563$$

$$\frac{u_2}{v_1} = 0.9866 = 1 - \frac{2 \sin^2 \beta}{\gamma+1}$$

$$\frac{v_2}{v_1} = \gamma \cdot 0.0298 = \frac{\sin^2 \beta}{\gamma+1} \quad 0.067 \checkmark$$

$$= \frac{2 (M_1^2 \sin^2 \beta - 1) \cot \beta}{(\gamma+1) M_1^2}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{u_2^2}{v_1^2} + \frac{v_2^2}{v_1^2}} = 0.987$$

$$M_2 = \cancel{6.24}$$

$$\frac{P_2}{P_1} = 1.8885, \quad \frac{P_2}{P_1} = 1.5632$$

$$\frac{T_2}{T_1} = \frac{P_2/P_1}{\rho_2/\rho_1} = 1.208$$

$$M_2 = 6.3, \quad \beta_2 = 11.8^\circ$$

$$\frac{P_3}{P_2} = 1.769, \quad \frac{P_3}{P_2} = 1.4953$$

$$\frac{T_3}{T_2} = 1.183$$

$$T_3 = 304 \text{ K}, \quad P_3 = 400 \text{ Pa}$$

$$M_3 = 5.688$$

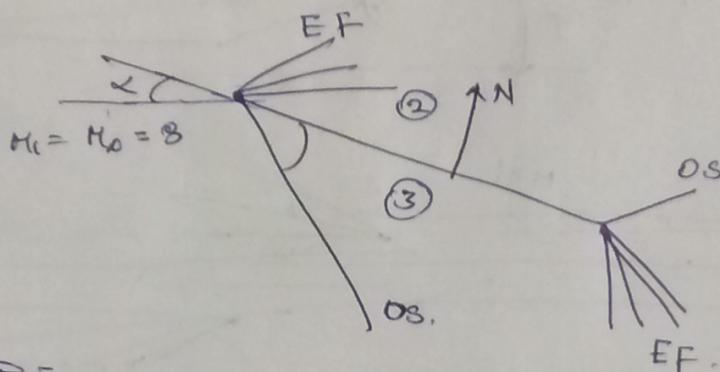
compute $\frac{P_{2-3}}{\rho}$ = 2.97

$$\frac{P_3}{\rho_2}, \quad \frac{v_3}{v_2} =$$

Tutorial-3

25/1/16

- 1) Consider an infinitely thin flat plate at angle of attack of 15° in a Mach 8 flow. Assume inviscid flow. Calculate the pressure coeff on the top and bottom surface of the plate, the lift and drag coefficients and the lift to drag ratio using
- Exact shock wave and expansion wave theory
 - Newtonian theory. Compare results.



1-2
Expansion fan

$$\frac{P_2}{P_1} = 0.0203$$

$$P_3/P_1 = 9.443$$

$$= \frac{2}{\gamma M^2} \left(\frac{P_3}{P_1} - 1 \right)$$

$$C_{P3} = \frac{2}{\gamma M^2} \left(\frac{P_3}{P_1} - 1 \right)$$

$$= \frac{2}{\gamma M^2} \left(9.443 - 1 \right)$$

$$C_{P2} = \frac{2}{\gamma M_1^2} \left(\frac{P_2}{P_1} - 1 \right) = +0.1888$$

$$= -0.02186$$

$$C_n = C_{P3} - C_{P1}$$

$$C_d = C_n \cos \alpha$$

$$\rightarrow dh - \frac{dp}{\rho} - v dv = dh - v dp = 0$$

$$\int \left[T ds = 0 \right]$$

Isentropic flow.

$$\frac{dp}{\rho} = (M^2 - 1) \frac{dv}{v}$$

Assume

$$v^* = 1200 \text{ m/sec.}$$

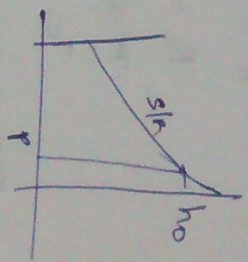
$$h_0 - \frac{(1200)^2}{2} = h^*$$

$$h_0 = h_5 = 49 \text{ kJ/kg.}$$

$$h^* = 48.28 \text{ kJ/kg}$$

From chart, $h_0 \approx 5 \text{ kJ/kg}$.

$$T^* = 3270 \text{ K, } Z = 1$$



$$v^* = \frac{h_0}{e} = \frac{\gamma D}{\gamma D - \gamma + 1}$$

$$(\gamma = 1.4)$$

$$v^* = 1.28$$

$$D = \frac{h}{Z c_p T}$$

$$c_p = 1005 \text{ J/kg}$$

$$\alpha^* = \sqrt{\gamma^* p/p_0} = \sqrt{\gamma^* Z R T^*} = 1099 \text{ m/sec.}$$

At the nozzle exit $\alpha^* = v^*$.

After many expansions.

$$T^* = 3310 \text{ K.}$$

$$p^* = 4.96 \text{ bar.}$$

$$\rho^* = 5.22 \text{ kg/m}^3.$$

$$A^* v^* \rho^* = \rho_e v_e A_e.$$

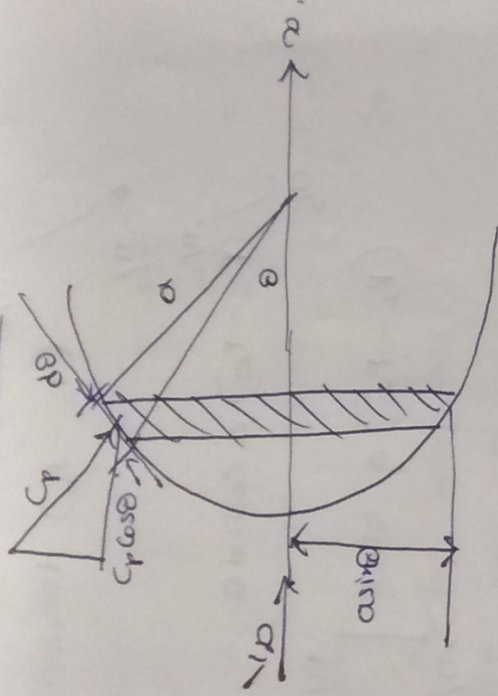
$$\rho_e v_e = 9.99$$

Q.14.

Full sphere.

~~hemispherical~~

(Area integration)



$$ds = a d\theta.$$

(dA), Area of strip = $ds \times 2\pi a \sin\theta$

dA

$$D_{\text{net}} = \int_{-\pi/2}^{\pi/2} (P_2 - P_0) \cos\theta \cdot (dA)$$

$$= 2 \int_{-\pi/2}^{\pi/2} (P - P_0) \cos\theta \times [a \sin\theta \cdot (2\pi a \sin\theta) d\theta]$$

$$= \int_{-\pi/2}^{\pi/2} (P - P_0) 2\pi a^2 \sin\theta \cos\theta d\theta$$

$$= \int_{-\pi/2}^{\pi/2} (P - P_0) \cdot \pi a^2 \sin 2\theta d\theta$$

$$= (P - P_0) \pi a^2 \left[-\frac{\cos 2\theta}{2} \right]_{-\pi/2}^{\pi/2}$$

$$M_{up} > M_{down} = (M_{up} - 0.259) \sqrt{\frac{T_2}{T_1}}$$

Iterative procedure, by solve

M_{up}	$\frac{T_2}{T_1}$	M_{down}	$(M_{up} - 0.259) \sqrt{\frac{T_2}{T_1}}$
1.2344	1.235	0.75	1
1.02	1.128	0.9422	0.986
1.165	1.166	0.965	0.961
1.12	1.164	0.965	0.965

$$\frac{T_2}{T_1} = 1.109$$

$$T_2 = 362 \text{ K}$$

$$\frac{P_2}{P_1} = 1.43$$

$$P_2 = 207.35 \text{ Pa}$$

$$P_2 = \frac{P_1}{R T_2} = 1.995 \times 10^3 \text{ kg/m}^3$$

AC

$$\frac{P_2}{P}$$

$$\frac{P_2}{P_1} = \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}$$

$$1.575 = \frac{1 + 0.2 \times 9^2}{1 + 0.2 \times M_2^2}$$

$$M_2 = 0.678$$

$$\frac{P_{02}}{P_2} = \left(1 + 0.2 \times 0.678^2 \right)^{3.5}$$

$$P_{02} = 1.36 P_2$$

$$P_{02} = \frac{P_{02}}{P_2} \times \frac{P_2}{P_1} \times P_1$$

$$1.36 P_2 = \frac{P_{02}}{P_1} \times P_1$$

$$\frac{U_2}{U_1} = \frac{V_2}{V_1} = 1 - \frac{2}{\gamma+1} = 0.167$$

$$\frac{M_2}{M_1} = \frac{V_2}{V_1} \sqrt{\frac{T_1}{T_2}}$$

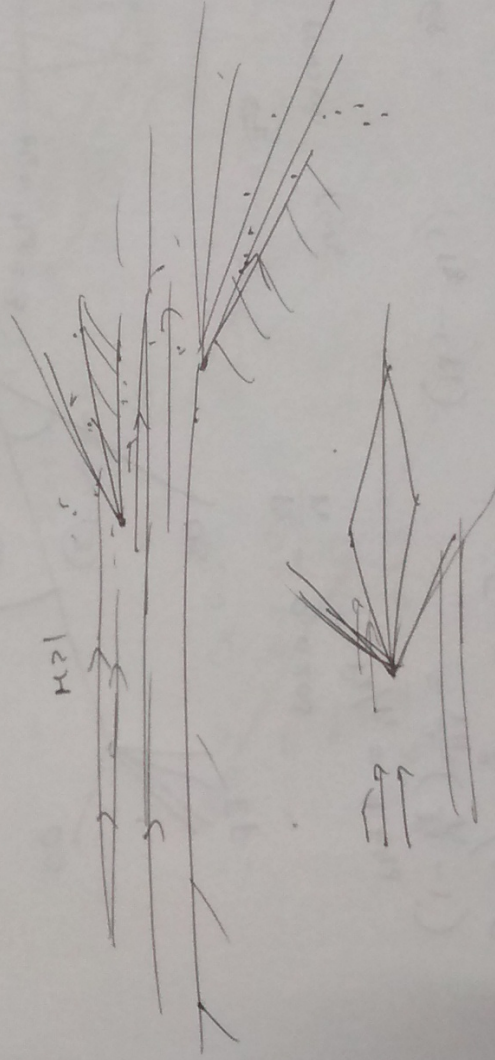
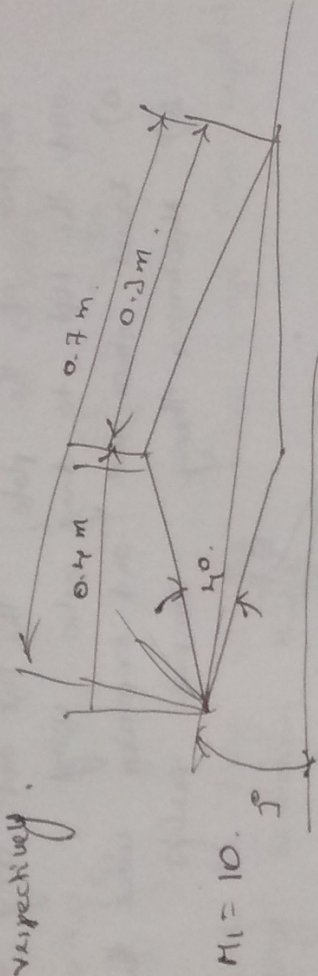
$$M_2 = 0.388 \sqrt{\gamma-1}$$

$$\frac{P_{02}}{P_2} = \left(1 + \frac{\gamma-1}{2} M_2^2 \right)^{\frac{\gamma}{\gamma-1}}$$

$$= 1.109$$

Q. 13.

Find the lift per meter span for the wedge shaped airfoil shown in fig. Also sketch the flow pattern about the airfoil. The mach number and the pressure and the pressure ahead of the airfoil are 10.6 and 400pa, respectively.



$$u_{n1} = V_1 \sin \beta$$

$$f_1 u_{n1} = f_2 u_{n2}$$

$$\tan(\beta - \theta) = \frac{V_{n2}}{V_{n1}} \tan \beta$$

$$\boxed{\theta = 0}$$

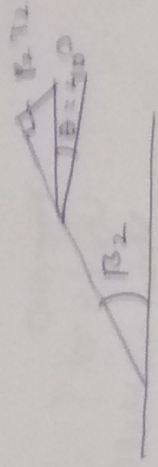
Problem:-

$$V_D = 25000 \text{ ft/sec}$$

$$\text{at alt} = 100,000 \text{ ft}$$

$$= 30.48 \text{ km}$$

$$V_D = 7620 \text{ m/sec}$$



$$\left(\frac{P}{\rho_{SL}}\right)_{30.48} = 1.1813 - \frac{(1.1813 - 1.0177) \times 0.48}{1}$$

$$= 1.10277 \times 10^{-2}$$

$$\frac{T}{V} = 226.5 + \frac{(227.5 - 226.507) \times 0.48}{1}$$

$$= 226.98 \text{ K}$$

$$\frac{f}{\rho_{SL}} = \frac{1.5029 - (1.5029 - 1.2891) \times 0.48}{1}$$

$$= 1.4 \times 10^{-2}$$

$$a = 301.71 + (302.37 - 301.71) \times 0.48$$

$$= 302 \text{ m/sec}$$

$$P_2 = \rho_2 Z R T_2$$

$$551063 = P_2 \times 1.324 \times 287 \times 6650$$

$$P_2 = 0.218 \text{ kg/m}^3$$

$$\rho_1 u_{n1} = \rho_2 u_{n2}$$

$$u_{n2} = 445.48 \text{ m/sec}$$

0.6

$$\tan(\beta - \theta) = \frac{u_{n2}}{u_{n1}} [\tan \beta]$$

$$\beta = 44.43 / 85.6$$

~~B~~ ~~ts~~ ~~rs~~

$$\beta = 44.43^\circ$$

2nd iteration

$$u_{n1} = 5334.28 \text{ m/sec}$$

$$u_{n2} = 445.329 \text{ m/sec}$$

$$1.117 \times 10^3 + 0.01715 \times 5334.28^2 = P_2 + 0.218 \times 445$$

$$P_2 = 4.45 \times 10^5 \text{ Pa}$$

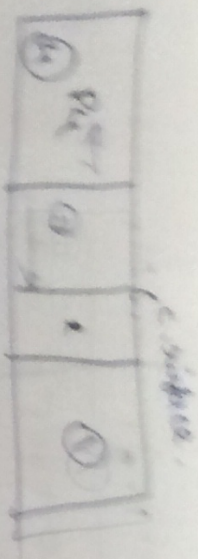
$$P_2 / P_a = 4.4 \checkmark$$

~~$$h_2 = h_1 =$$~~

~~$$h_2 + \frac{\rho_2 u_{n2}^2}{2}$$~~

$$h_2 + \frac{445.329^2}{2} = 1.005 \times 10^3 \times 226.98 + \frac{5334.28^2}{2}$$

$$h_2 = 14.35 \text{ MJ/kg}$$



From $M = 3$

$$\frac{T_{12}}{T_1} = 2.639$$

$$T_{12} = 803.74 \text{ K}$$

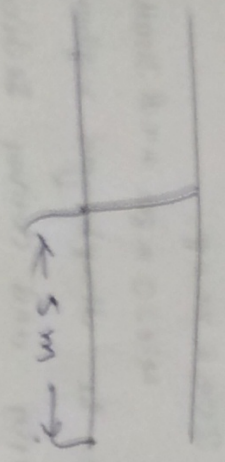
$$Q_{12} = 568.27 \text{ m/sec}$$

$$M_2 = M_1 \frac{a_1}{a_2} = \frac{V_2}{a_2}$$

$$0.4752 = 3 \sqrt{\frac{1.4}{2.639}} - \frac{V_2}{568.27}$$

$$V_2 = 741.53 \text{ m/sec} = a_3$$

$$a_3 = 347.2 \text{ m/sec}$$



$$t = \frac{5}{424.34}$$

$$= 0.0117 \text{ sec}$$

$$\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_4}{T_3}$$

$$\Rightarrow \left(\frac{14420}{10021}\right)^{\frac{0.4}{1.4}} = \frac{T_4}{\frac{300}{1.10}}$$

$$T_3 = 198.78 \text{ K}$$

$$a_3 = 282.61 \text{ m/sec}$$

$$C_n = 0.21$$

$$C_L = 0.208$$

$$C_d = C_n \sin \theta = 0.05435$$

$$L/D = C_L / C_d = 3.74$$

Newtonian.

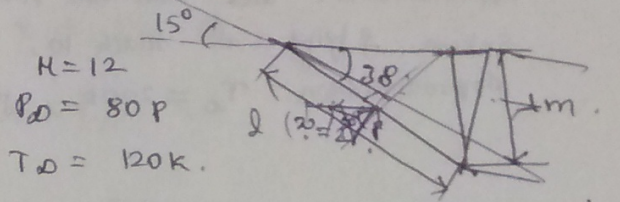
$$C_{P2} = 0, \quad C_{P3} = 2 \sin^2 \theta = 0.134$$

$$C_L = C_n \cos \theta = 0.1294$$

$$C_d = C_n \sin \theta = 0.034$$

$$\frac{C_L}{C_d} = 3.74$$

2Q. Find the drag on the below wedge (base height 1m) using Newtonian theory.



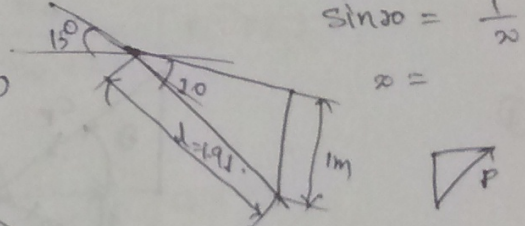
$H = 12$
 $P_2 = 80 \text{ P}$
 $T_2 = 120 \text{ K}$

$$C_L = C_n \cos \theta$$

$$= 0.5 \times \cos 30$$

$$= 0.433$$

$$C_d = 0.25$$



$$L = \frac{1}{2} \rho V^2 \times H^2 \times P_1 \times C_L \times 1$$

$$= 3491712$$

$$\text{Drag} = P_2 \cos 60 \times l_1 - P_1 \cos 15 \times 1$$

$$C_P = 2 \sin^2 \theta = 0.5 = \frac{2}{\gamma M^2} \left(\frac{P_2}{P_1} - 1 \right)$$

From page 28 for h_2 & P_2

$$z_2 = 1.23, \quad T_2 = 5000 \text{ K.}$$

$$\begin{aligned} \rho_2 &= \frac{P_2}{z_2 R T_2} \\ &= \frac{39.75 \times 10^3}{1.23 \times 287 \times 5000} = 0.0225 \text{ kg/m}^3. \end{aligned}$$

Iteration 2

$$u_2 = \frac{\rho_1 u_1}{\rho_2}$$

$$u_2 = 385 \text{ m/s}$$

$$P_2 = 36.418 \text{ kPa}$$

$$\frac{\rho_2}{\rho_0} = 0.3594$$

$$\begin{aligned} h_2 &= 10718 \times 10^3 - \frac{385^2}{2} \\ &= 10.64 \text{ MJ/kg} \end{aligned}$$

$$T_2 = 4970 \text{ K.}$$

$$z_2 = 1.228.$$

$$\rho_2 = 0.021 \text{ kg/m}^3$$

III Iteration $u_2 = 412.6 \text{ m/s}$

$$P_2 = 36.17 \text{ kPa}$$

$$\begin{aligned} h_2 &= 10718 \times 10^3 - \frac{412.6^2}{2} \\ &= 10.63 \text{ MJ} \end{aligned}$$

$$\frac{\rho_2}{\rho_0} = 0.357$$

$$P_{02} = \frac{P_2}{\gamma_2} \times \frac{\gamma_2}{\gamma_1} \times \gamma_1$$

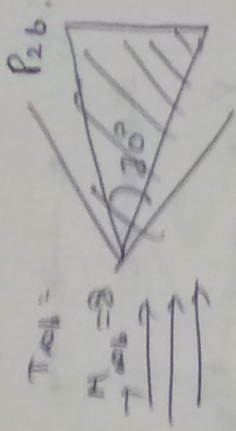
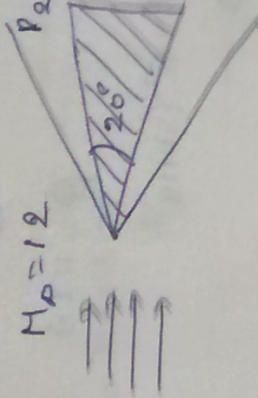
$$P_{02} = 10.4833 \text{ kPa}$$

$$\Rightarrow \frac{P_{02}}{P_{01}} = 0.00476$$

2(4)

$$P_{2b}, \text{ if } P_{0b} = P_{0a}/2 \text{ and } T_{0a} = T_{0b}$$

$$T_{0a} = 12$$



$$\frac{P_2}{\gamma_1} = \frac{2 \times 12}{\gamma + 1} \gamma_1^2 \sin^2 \beta$$

$$\frac{P_{2a}}{\gamma_{0a}} = \frac{P_{2b}}{\gamma_{0b}}$$

$$\frac{P_{2a}}{P_{0a}} = \frac{P_{2b}}{P_{0b}} = 2$$

Taking $\beta = 1.20$ for case 'a'.

$$= 1.2 \times 10 = 12^\circ$$

$$\Rightarrow \frac{P_2}{\gamma_1} = \frac{2 \times 12}{2.4} \times 12^2 \times \sin^2 12$$

$$P_2 = 7.26 \text{ kPa}$$

$$P_1 = 688.5 \text{ kPa}$$

$$\frac{P_{2a}}{P_{0a}} = \frac{2 \times 12}{\gamma + 1} \gamma_1^2 \sin^2 \beta$$

$$= \frac{2 \times 12}{2.4} \times 12^2 \times \sin^2 12$$

$$= 7.26$$

$$\Rightarrow P_{2a} = 5/7.26 = 0.6887 \text{ kPa}$$

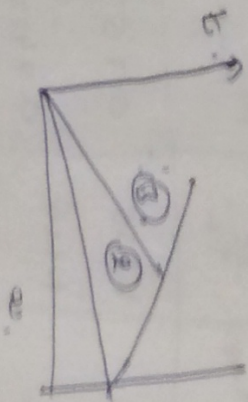
Acoustic Impedance. (Z) - Its relevance to
Shock propagation.

$$Z = \rho a$$

for gas $\rho =$ assumed to be constant.

$$R = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2$$

→ skew R in ppt for notes ...



→ Z - Determining whether a shock wave will be reflected or not.

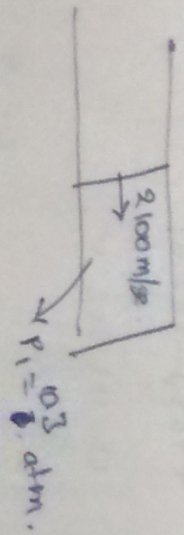
Qs. A quick valve of a shock tube, filled with He on the driver side and air on the driver side operated open generating an incident shock wave of Mach-3. The driver gas was initially at a temp and pr of 300K and 930 kPa. The driver gas initial conditions were 21k Pa pressure & 300K. Determine the wave system generated at the end of the tube when the wave interact with the contact surface.

$$t_{\text{time}} = \frac{\text{Distance}}{V_B - a_B} = \frac{55}{41.5500 - 22.611} = 0.0015 \text{ Sec.}$$

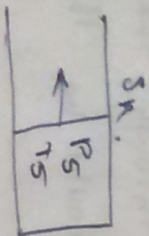
Q30 The archerive the tailored condition, in which the reflected shock pass through the contact surface unaltered, what should be the velocity of the shock wave.
 Driven gas: helium, driven gas: air $T_2 = T_1 = 300K$, $P_2 = 10 \text{ atm}$.

Tutorial 2.

Shock tube



Find $p_2 = ?$ $\gamma_2 = ?$ Conditions behind reflected shock



$$p_1 \gamma_2 = p_2 (\gamma_2 - \gamma_1)$$

$$p_1 + \rho_1 \gamma_1^2 = p_2 + \rho_2 (\gamma_2 - \gamma_1)^2$$

$$h_1 + \frac{\gamma_1^2}{2} = h_2 + \frac{(\gamma_2 - \gamma_1)^2}{2}$$

Let $a_1 \cdot \frac{\rho_2}{\rho_1} = 6$. (For hypersonic flow).

$$\gamma_2 - \gamma_1 = \frac{\rho_1}{\rho_2} \cdot \gamma_1$$

$$p_2 = p_1 + \rho_1 \gamma_1^2 \left[1 - \frac{\rho_1}{\rho_2} \right]$$

$$\gamma_2 - \gamma_1 = \frac{\gamma_1}{6}$$

$$0.88 \gamma_2 = \gamma_1$$

$$\gamma_2 = 1.02 \gamma_1$$

$$\Rightarrow \gamma_1 = 1750 \text{ m/sec}$$

From chart $z = 1.28$ for $P_2 = 3 \text{ h}_2$

$$z = 1.28$$

$$T = 6280 \text{ K}$$

$$P_2 = \rho_2 z R T_2$$

$$\rho_2 = 0.192 \text{ kg/m}^3$$

$$\cancel{\rho_1 + \rho_1 u_{n1}^2} = \cancel{\rho_2 + \rho_2 u_{n2}^2}$$

$$\rho_1 u_{n1} = \rho_2 u_{n2}$$

$$u_{n2} = 476 \text{ m/sec}$$

$$\beta = 45.06 / 84.9$$

IIIrd iteration

$$\beta = 45.0423$$

$$T_2 = 6350 \text{ K}$$

$$z = 1.288$$

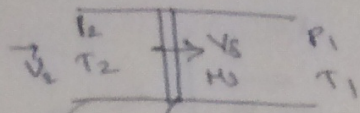
$$\rho_2 = 0.194 \text{ kg/m}^3$$

$$u_{n2} = 475.118 \text{ m/sec}$$

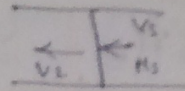
$$P_2 = 4.49 \text{ atm}$$

$$v_2 = 5408.2 \text{ m/sec}$$

Find v_2 using $\left[\frac{P, \theta}{u_{n2}} \right]$



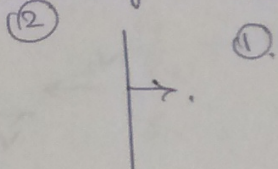
Incident shock.
Lab co-ordinate system.
(moving co-ordinate system).



Shock co-ordinate system.

- Normal shock relations can be used.
- please read from book → don't rely on cliff.

A Normal shock across which Pr ratio is 1.45 moves down a duct into still air at Pr of 100 kPa and Temp of 20°C. If the end of the duct is suddenly closed, find the pressure acting at the end of the duct behind reflected shock.



$$\frac{P_2}{P_1} = 1.45$$

$$M_1 = \frac{V_1}{a_1}$$

$$M_2 = \frac{V_1 - V_2}{a_2}$$

From tables,

$$M_2 = 1.1772$$

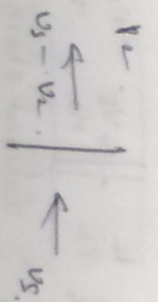
$$M_1 = 0.9567$$

$$\frac{T_2}{T_1} = 1.1137 \Rightarrow T_2 = 326.3 \text{ K}$$

$$a_2 = 362 \text{ m/sec}$$

$$a_1 = 341.11 \text{ m/sec}$$

$$H_2 = \frac{V_3 - V_2}{a_2}$$



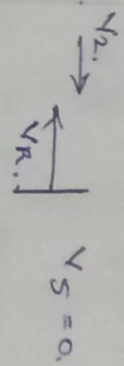
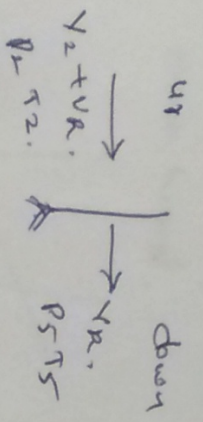
$$V_3 = a_1 x_{H_1} = 403.9 \text{ m/s}$$

$$0.8567 = \frac{403.9 - V_2}{862}$$

$$V_2 = 93.77 \text{ m/s}$$

$$P_2 = 145 \text{ Pa}$$

Reflected shock



$$M_{up} = \frac{V_2 + V_R}{a_2}$$

$$= \frac{V_R}{a_2} + \frac{V_R}{a_2} \cdot \frac{a_5}{a_2}$$

$$H_{down} = \frac{V_R}{a_5} = \frac{V_R}{a_2} + H_{down} \cdot \sqrt{\frac{T_5}{T_2}}$$

$$M_{up} = \frac{93.77}{862} + \frac{V_R}{a_2}$$

$$= 0.259 + H_{down} \sqrt{\frac{T_5}{T_2}} \quad \text{--- (1)}$$

$$P_2 = 0.21 \times 1.01325 \times 10^5 + 3.53 \times 0.353 \times \sqrt{2} \left[1 - \frac{P_2}{P_1} \right]$$

$$P_2 = 30397.5 + 0.2941 \times 2.100^2$$

$$h_2 = h_1 + \frac{V_2^2}{2} \left[1 - \left(\frac{P_2}{P_1} \right)^2 \right]$$

$$= 1005 \times 200 + \frac{2100^2}{2} \left[1 - \left(\frac{P_2}{P_1} \right)^2 \right]$$

From chart

From P_2 8 h₂

(Chart are not possible) for $\frac{P_2}{P_{atm}} > 10$

$$\frac{P_2}{P_{atm}} = 13, \quad Z = 1, \quad T_2 = 2100K$$

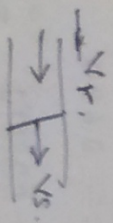
$$e_2 = \frac{P_2}{Z R T_2} = \frac{13.27 \times 10^5}{1 \times 2100 \times 287} = 2.2 \text{ kg/m}^3$$

From 2nd iteration

$$e_2 = 2.194 \text{ kg/m}^3$$

$$T_2 = 2200K$$

$$P_2 = 13.7 \text{ atm}$$



~~$$P_2 = P_2 + P_2$$~~

$$\left[\sqrt{h_1 + V_1^2} \right] e_2 = V_1 \times \rho_2$$

$$P_2 = P_2 + P_2 \left(\sqrt{h_1 + V_1^2} \right)^2 \left[1 - \frac{P_2}{P_1} \right]$$

$$h_2 = h_1 + \frac{(V_1 + V_2)^2}{2} \left[1 - \left(\frac{P_2}{P_1} \right)^2 \right]$$

$$V_1 = \frac{V_2}{\left[\frac{P_2}{P_1} - 1 \right]}$$

$$\frac{P_2}{P_1} = \left[\frac{1 + \gamma \left(\frac{v_1}{c} \right)^2}{1 + \gamma \left(\frac{v_2}{c} \right)^2} \right]^{\frac{\gamma}{\gamma-1}}$$

$$M_2 = M_1 \checkmark$$

$$\frac{P_2}{120} = \left[\frac{1 + 1.4 \times 0.2 \times 9.8 \times 0.0872}{1 + 1.4 \times \left(\frac{2.8}{0.4} \right)^2} \right]^{\frac{1.4}{1.4-1}}$$

$$P_2 = 32.3 \text{ Pa.}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = 1157^\circ \text{K.}$$

Q.

$$M_1 = 9$$

$$P = 1 \text{ atm}$$

$$= 1.01325 \times 10^5 \text{ Pa}$$

$$\theta = 0.06 \text{ rad} = 4^\circ$$

$$\tan 4^\circ = \frac{1}{20}$$



$$\frac{P_2}{P_1} = \frac{e^{\gamma} (M_1^2 \sin^2 \theta)}{1 + \gamma} = \frac{2.8}{1.4} \times 9^2 \times (\sin 4^\circ)^2 = 0.16$$

$$P = 9.2^\circ$$

$$\rho = \frac{1}{\text{Temp.}}$$

$$= 6.17 \text{ m}$$

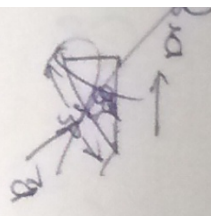
$$\frac{P}{\rho} = \frac{v_1}{4} + \sqrt{\left(\frac{v_1}{4} \right)^2 + \frac{1}{M_1^2 \delta^2}}$$

$$\Rightarrow P = 9.2^\circ$$

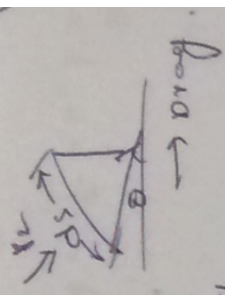
$$P_2 = 144.93 \text{ Pa} \checkmark$$

$$= 134.93 \text{ Pa} \checkmark$$

$$C_p = \frac{2 \sin^2 \theta}{\cos^2 \theta} \quad \delta = 90^\circ - \theta$$



$$\text{Drag force} = \int_{-\pi/2}^{\pi/2} (p_2 - p_1) ds \cos \theta$$



$$= \int_{-\pi/2}^{\pi/2} (p_2 - p_1) a \cos \theta d\theta$$

$$= (p_2 - p_1) a \sin \theta \Big|_{-\pi/2}^{\pi/2}$$

$$\text{Drag} = 2a (p_2 - p_1)$$

$$= C_p \times \rho \times 2a$$

~~$$C_D = \frac{2a (p_2 - p_1)}{\rho \times 2a}$$~~

$$C_D = \int_0^{\pi/2} \left(\frac{p_2 - p_1}{\rho V^2} \right) \cos \theta d\theta$$

$$C_p = \frac{2 \sin^2 \theta}{\cos^2 \theta}$$

$$= \int_0^{\pi/2} 2 \cos^2 \theta d\theta$$

$$= \frac{4}{3}$$

M_1	$\sqrt{T_2/T_1}$	M_2	M_1
1.02	1.0064	0.9805	1.73
1.2	1.06	0.8422	1.64
1.4	1.12	0.7397	1.58
1.5	1.1499	0.7011	1.56
1.54	1.16	0.6874	1.55
1.56	1.167	0.6869	1.55

Ans. $M_1 = 1.557$ get $P_2/P_1 \Rightarrow P_2 = 26.5 P_1$

Q. In a shock tube, driver gas-air and driven gas-O₂. After the diaphragm ruptured, the shock moved into the driven section with a Mach number of 3. Pressure and temp of driven gas are 1 atm 300 K resp. Press. and temp of driver gas 4420 kPa and 300K. Calculate the driver gas temp after the diaphragm ruptured. Assuming the driver section is open and driven section is very long. Calculate the time for the tail of EF from the driver section to reach the open end of the driven section. Length of the driven section = 5m.

→ N 305 → 2 clapp rubber.
 → non newtonian → tutorial

1. Air is expanded from a large reservoir in which the pressure and temp are 1 MPa and 35°C through the variable area duct. A normal shock occurs at a point in the duct where the Mach number is 8. Find the pressure and temperature in the flow just downstream of the shock wave. Downstream of the shock wave, the flow is brought to rest in another large reservoir. Find the pressure and temp in the reservoir. Assume that the flow is one-dimensional and isentropic everywhere except through the shock wave.

$$P_0 = 1 \text{ MPa}, \quad T_1 = 35^\circ\text{C} = 308 \text{ K}.$$

$$M = 8. \quad P_2 = ? \quad T_2 = ?$$

$$\frac{P_2}{P_1} = \frac{2\gamma}{\gamma+1} (M_1^2 \sin^2 \beta)$$

for N.S. $\beta = 90^\circ$

$$\therefore M_1 \rightarrow 0$$

$$= \frac{2 \times 1.4}{1.4 + 1} \times 8^2$$

$$P_2 = 74.67 \text{ MPa}$$

$$\frac{P_{02}}{P_1} = (1 + 0.2 \times 8^2)$$

$$P_1 = 10^6 \text{ Pa}$$

$$P_{02} = 7648 \text{ Pa}$$

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M_1^2 \sin^2 \beta$$

$$= \frac{2 \times 1.4 \times 0.4}{(2.4)^2} \times 8^2$$

$$T_2 = 3832 \text{ K}$$

$$\frac{p_{02}}{p_2} = \frac{1}{1 + \frac{\gamma-1}{2} M_2^2}$$

$$\frac{u_2}{v_1} = 1 - \frac{2 \sin^2 \beta}{(\gamma+1)}$$

$$= 1 - \frac{2}{2.4}$$

$$u_2 = 0.1667 v_1$$

$$v_1 = M_1 a_1$$

$$= 8 \times \sqrt{1.4 \times 287 \times 303}$$

$$= 2814.3 \text{ m/sec}$$

$$u_2 = 469.14 \text{ m/sec}$$

$$M_2 = \frac{u_2}{a_2} = \frac{469.14}{\sqrt{1.4 \times 287 \times 383.2}}$$

$$M_2 = 0.378$$

$$\frac{p_{02}}{p_2} = \frac{T_{02}}{T_2}$$

$$T_{02} = T_2 + \frac{u_2^2}{2c_p}$$

$$= 303 + \frac{469.14^2}{2 \times 1000} = 588.55 \text{ K} \cdot 394.1 \text{ K}$$

$$\frac{p_{02}}{p_2} = \left(\frac{T_{02}}{T_2} \right)^{\frac{\gamma}{\gamma-1}}$$

10/17

using isentropic similarity parameter

$$(k) = M \cdot 1$$

$$\Rightarrow \text{Taking } M_1 = M_2 = 1.2 \theta_2 = k$$

$\therefore \frac{P_2}{P_1}$ depend on only k .

$$k_1 = 1.2 \times \frac{10 \times \pi}{4 \times 180}, \quad 8 \times 20 \times \frac{\pi}{10}$$

$$P = 1.2 \theta = 1.2 \times 15 = 18$$

$$\frac{P_{2b}}{P_{2a}} = \frac{2 \times 1.4 \times 8^2 (\sin 15)^2}{2.4} = 7.13$$

$$\frac{P_{2a}}{P_{0a}} = \frac{P_{2b}}{P_{0b}} \quad P_{2b} = P_{0b} \times 7.13$$

$$P_{0b} = \frac{P_{2a}}{7.13} = \frac{0.6387}{7.13} = 0.0896$$

$$P_{2b} = \frac{P_{1b} \times P_{2a}}{P_{1a}} = \frac{P_{0b} P_{0a} T_{0b}}{P_{0a} R T_{0a}} \times P_{2a} = 2500 \text{ Pa}$$

1) For $N=3$.

$\frac{T_2}{T_1} = 2.679$ get $a_2 = ?$

$T_2 = 800 \text{ K}$

$a_2 = 568.027 \text{ m/sec}$ $e_2 = ?$

2) Find a_3 .

Find P_2/P_1 for $N=3$, get P_2 . Across contact surface $P_2 = P_3$.

3) Analyse the division side :: Driver gas encounter on EF. Any expansion (by per (supersonic)) is isobaric.

$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^{N-1}$ $\gamma = 1.667$

$\frac{P_2}{P_1} = 10.333 \Rightarrow P_2 = 21 \times 100 \text{ J}$

$P_2 = 2166.9133 \text{ kPa}$

$\frac{R}{216.93} = \frac{T_1}{T_2}$

$\frac{930}{216.933} = \left(\frac{300}{T_2}\right) \frac{1.667}{0.667}$

$T_2 = 167.5 \text{ K}$

$a_3 = \sqrt{\gamma R T_2} = 762 \text{ m/sec}$

So $a_2 < a_3$ there will be EF ✓

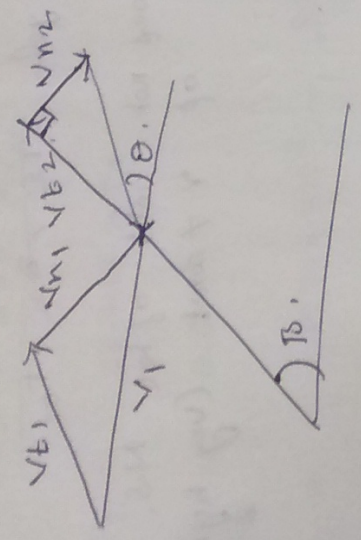
$R = 2078.5 \text{ J/kgK}$

$a_3 = 1077.33$

Tutorial

→ $\theta - \beta - \mu$ relationship with varying r , not valid for real gap.
 → solving problems with varying r ' value.

$\theta - \beta - \nu$ - relation.



$V_{t2} = V_{t1}$

$\tan(\beta - \theta) = \frac{V_{n2}}{V_{t2}} = \frac{V_{n2}}{V_{t1}} = \frac{V_{n2}}{V_{n1}} \tan \beta$

$\tan(\beta - \theta) = \frac{V_{n2}}{V_{n1}} \tan \beta$

$p_1 + p_1 u_{n1}^2 = p_2 + p_2 u_{n2}^2$

$h_1 + \frac{u_{n1}^2}{2} = h_2 + \frac{u_{n2}^2}{2}$

$p = \rho_2 n T$

For total enthalpy

$h_{01} = h_{02}$ for equilibrium flows.

1) Corresponding h_2, P_2 get $\frac{z_2}{R}$ from charts.

$$\frac{P_{01}}{P_1} = \left(\frac{T_{01}}{T}\right)^{\gamma/\gamma-1}$$

2) get h_{01} from previous cal.
match match. $h_{02} = h_{01}$ and get P_{02}

3) From P_{02} & h_{02} get T_2, z_2, T_{02} .

Isentropic relations does not hold after N.S.
use only charts if $\gamma \neq \text{const.}$ (very high $z > 1$)

$$4) C_p = \frac{P_{02} - P_1}{\frac{\gamma}{2} P_1 M_1^2}$$

8th quiz.