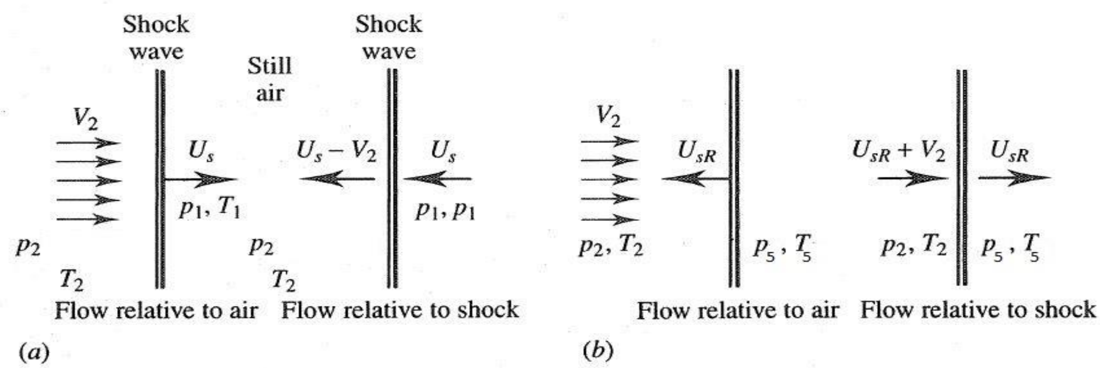
A normal shock wave, across which the pressure ratio is 1.45, moves down a duct into still air at a pressure of 100 kPa and a temperature of 20^0C. Find the pressure, temperature and velocity of the air behind the shock wave. If the end of the duct is closed, find the pressure acting on the end of the duct after the shock is reflected from it.

Solution:



(*a*) Initial shock wave; (*b*) reflected shock wave.

First consider the shock wave before the reflection. For this wave:

$p_2/p_1 = 1.45$, $p_1 = 100$ kPa, $T_1 = 20^0\text{C} = 293$ K

Consider the flow relative to the shock wave as shown in figure above.

Flow with velocity U_s (velocity of the shock) is passing through the normal shock with pressure ratio 1.45.

From normal shock relation, for pressure ratio 1.45

$M_1 = 1.1772$, $M_2 = 0.8567$, $T_2/T_1 = 1.1137$

Therefore: $p_2 = 1.45 \times 100 = 145$ kPa

$T_2 = 1.1137 \times 293 = 326.3$ K

Also,

$M_1 = U_s/a_1$ and $M_2 = (U_s - V_2)/a_2$

(Note: M_1 and M_2 are relative to shock, not the actual Mach number of the flow. Flow is stationary before the shock, hence Mach number will be zero for actual flow upstream.)

$V_2 = M_1a_1 - M_2a_2$

$$V_2 = 1.1772 \times \sqrt{1.4 \times 287 \times 297} - 0.8567 \times \sqrt{1.4 \times 287 \times 326.3}$$

V_2 = 93.8 m/s (actual velocity of the flow after the shock has passed.)

Therefore, the pressure, temperature and velocity behind the initial shock wave are 145 kPa, 53.3^{0}C, and 93.8 m/s.

Now consider the wave that is “reflected” off the closed end. The strength of this wave must be such that it brings the flow to rest. Hence, the Mach numbers of the air flow upstream and downstream of the reflected wave relative to this wave are:

$$M_{up} = \frac{V_2 + U_{sR}}{a_2} \quad \text{and} \quad M_{down} = \frac{U_{sR}}{a_5}$$

Where U_{sR} is the velocity of the reflected wave and a_5 is the speed of sound in the flow downstream of the reflected wave.

$$a_2 = \sqrt{1.4 \times 287 \times 326.3} = 362.1 \text{ m/s}$$

hence:

$$M_{up} = \frac{93.8}{362.1} + \frac{U_{sR}}{a_2} = 0.259 + \frac{U_{sR}}{a_2}$$

$$M_{down} = \frac{U_{sR}}{a_5} = \frac{U_{sR}}{a_2} \times \frac{a_2}{a_5} = \frac{U_{sR}}{a_2}\sqrt{\frac{T_2}{T_5}}$$

$$M_{down} = (M_{up} - 0.259)\sqrt{\frac{T_2}{T_5}} \qquad \text{----- eqn (A)}$$

Now the values of M_{up} and M_{down} are related by the normal shock equations, these equations also relating T_2/T_5 to M_{up}. These equations together, therefore allow M_{up} to be found. A simple way to find the solution is by guessing a series of values of M_{up} and then for each of these values to find M_{down} and T_2/T_5 from shock relations or tables and then to derive the value of M_{down} from eqn (A). The correct value of of M_{up} is that which has value of M_{down} as given directly by normal shock table equal to that given by the above equation.

M_{up} (guessed)	**T_5/T_2** (shock table)	**M_{down}** (shock table)	$M_{up} - 0.259)\sqrt{\frac{T_2}{T_5}}$ (calculated)
1.0	1	1.0	0.741
1.1	1.065	0.912	0.802
1.2	1.128	0.842	0.886
1.17	1.109	0.861	0.863

For M_{up} = 1.17 the values of M_{down} given by the shock relations and by the above equation are approximately same. For this M_{up}, from shock table,

P_5/p_2 = 1.4304 (from shock table for M=1.17)

P_5 = 207 kPa