Estimation of Lift Coefficient

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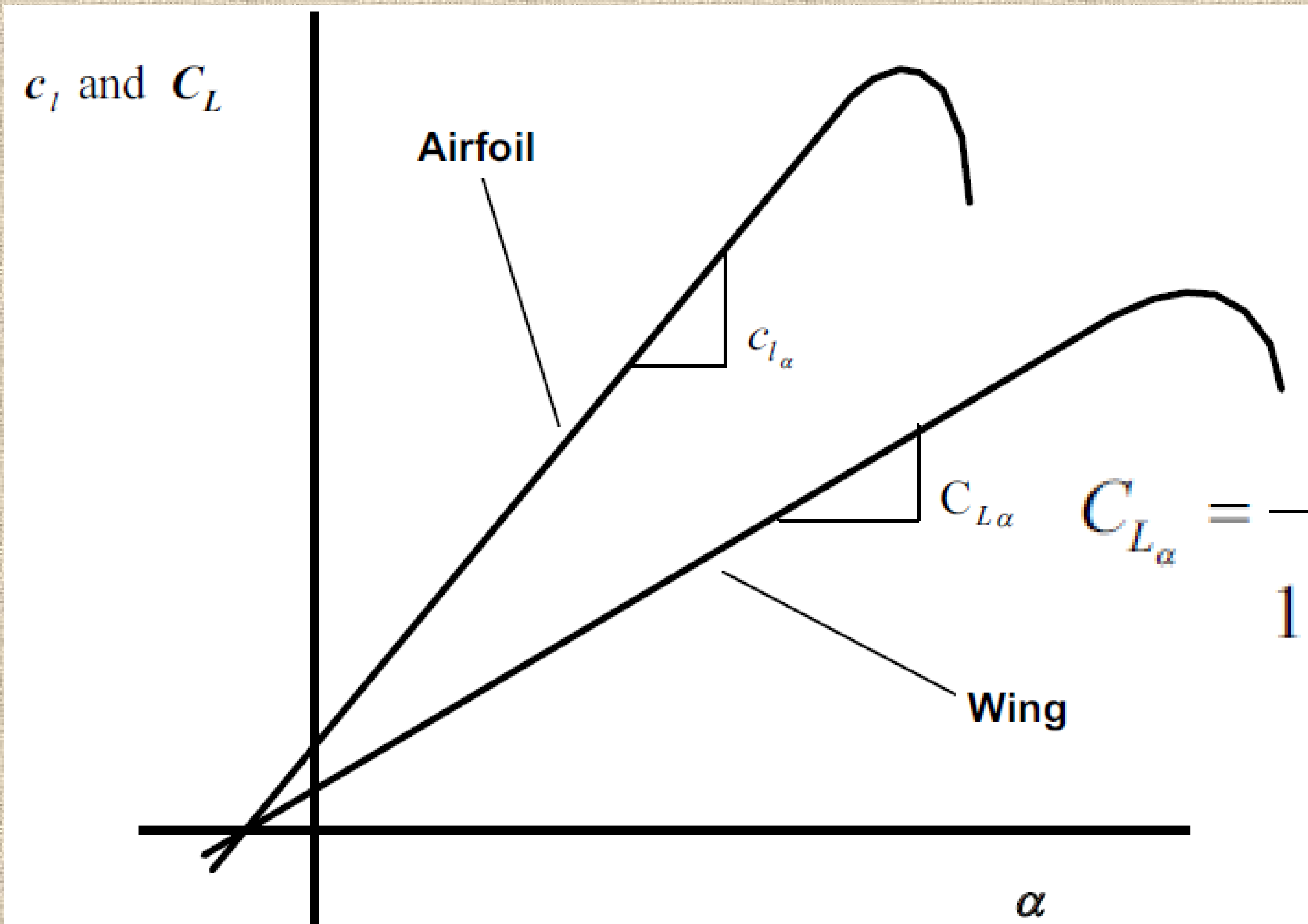
2-D and 3-D Lift Coefficient



$$C_{L_\alpha} = \frac{c_{l_\alpha}}{1 + \dfrac{57.3\, c_{l_\alpha}}{\pi\, e\, AR}}$$

Source: Brandt et al., Intro. To Aeronautics: A Design Perspective, 2^{nd} ed, AIAA Education Series, 2004, pp 96

Estimation of span efficiency factor *e*

$$e = \frac{2}{2 - AR + \sqrt{4 + AR^2 (1 + \tan^2 \Lambda_{t_{max}})}}$$

AR = Wing Aspect Ratio

$\Lambda_{t\,max}$ = sweep of maximum thickness line
= sweep at 30% of chord for low speed aircraft
= sweep at 50% of chord for high speed aircraft

$$AR\, Tan\, \Lambda_n = AR\, Tan\, \Lambda_0 \; - 4n\, \frac{1-\lambda}{1+\lambda}$$

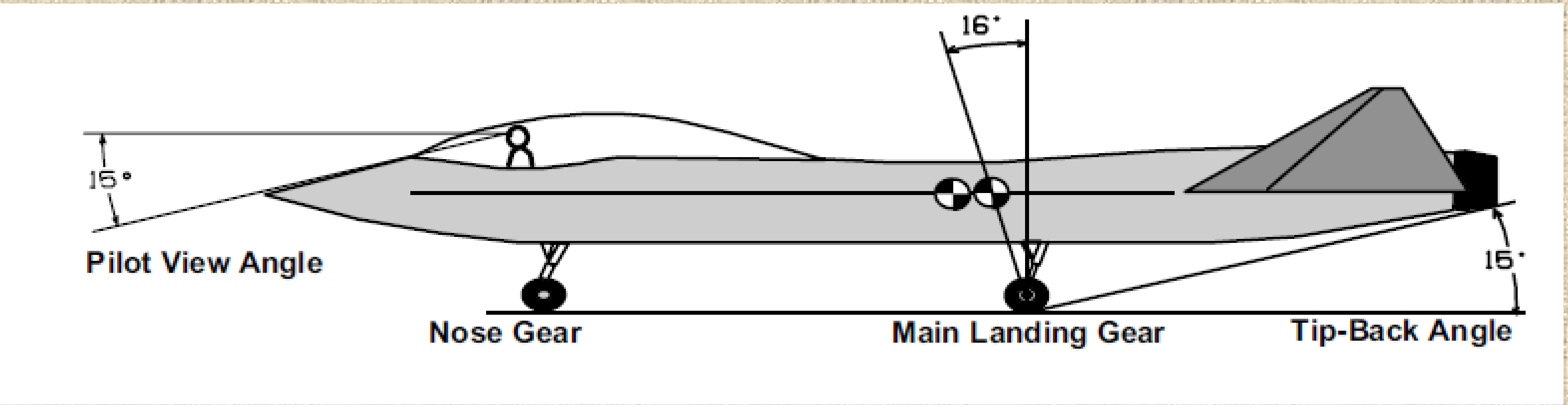
Where

Λ_0 = Leading Edge Sweep
Λ_n = Sweep at any fractional location n
λ = Wing Taper Ratio

Source: Brandt et al., Intro. To Aeronautics: A Design Perspective, 2nd ed, AIAA Education Series, 2004, pp 107

Concept of Absolute AoA

- It is difficult to keep track of $\alpha_{L=0}$ in design
  - It is affected by aerofoil camber and twist distribution
- Hence, we define Absolute AoA (α_a)
- $\alpha_a = \alpha - \alpha_{L=0}$
  - When Lift = 0, $\alpha_a = 0$
- Max. AoA α_{max} limited to ~15 deg
  - Take-off or Landing Considerations
- Thus $\alpha_{a\ max} = (\alpha_{max} - \alpha_{L=0}) = (15 - \alpha_{L=0})$



ESTIMATION OF $C_{L,MAX}$

Drivers of Max. Lift Coefficient

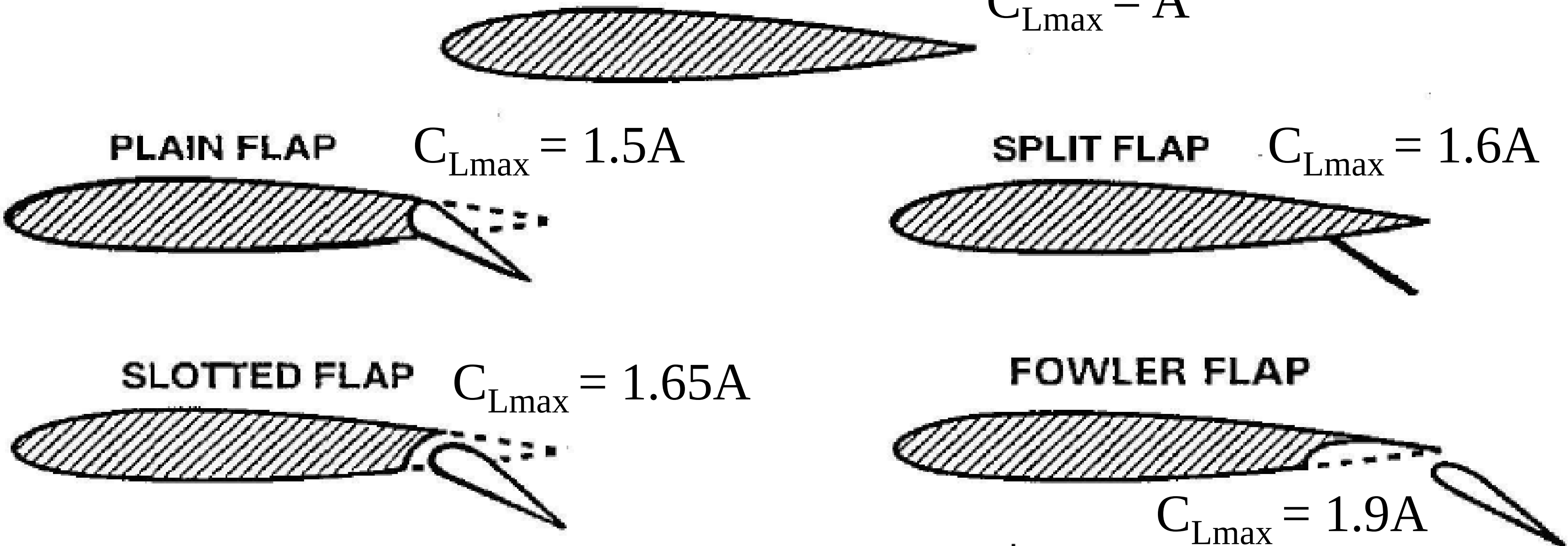
- Wing geometry
  - Increase in Λ reduces C_{Lmax}
  - Increase in AR increases C_{Lmax}
- Airfoil shape
  - Increase in t/c and L. E. radius increase C_{Lmax}
- Reynolds Number
- Surface Texture
- Interference from Fuselage, Pylons, Nacelles
- T. E. Flap and/or L.E. Slat Geometry & Span
  - Larger chord and Span increase C_{Lmax}
  - Swept Flaps have lower C_{Lmax}

Flaps as High Lift Devices

- Landing Setting

- T

Types of Flaps



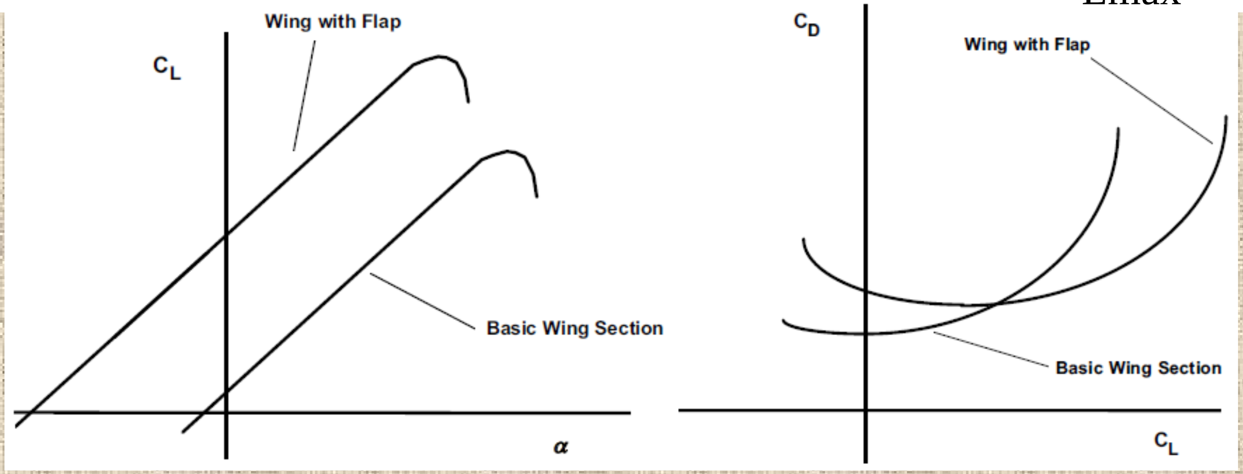
Basic section: $C_{Lmax} = A$

Plain flap: $C_{Lmax} = 1.5A$

Split flap: $C_{Lmax} = 1.6A$

Slotted flap: $C_{Lmax} = 1.65A$

Fowler flap: $C_{Lmax} = 1.9A$



Source: Brandt et al., Intro. To Aeronautics: A Design Perspective, 2^{nd} ed, AIAA Education Series, 2004, pg. 102

Types of Flaps- 1/4

High-lift devices	*Increase of maximum lift*	*Angle of basic aerofoil at max. lift*	*Remarks*
 Basic aerofoil	–	15°	Effects of all high-lift devices depend on shape of basic aerofoil.
 Plain or camber flap	50%	12°	Increase camber. Much drag when fully lowered. Nose-down pitching moment.
 Split flap	60%	14°	Increase camber. Even more drag than plain flap. Nose-down pitching moment.
 Zap flap	90%	13°	Increase camber and wing area. Much drag. Nose-down pitching moment.

Types of Flaps-2/4

 Slotted flap	65%	16°	Control of boundary layer. Increase camber. Stalling delayed. Not so much drag.
 Double-slotted flap	70%	18°	Same as single-slotted flap only more so. Treble slots sometimes used.
 Fowler flap	90%	15°	Increase camber and wing area. Best flaps for lift. Complicated mechanism. Nose-down pitching moment.

High lift devices

Note. Since the effects of these devices depend upon the shape of the basic aerofoil, and the exact design of the devices themselves, the values given can only be considered as approximations. To simplify the diagram the aerofoils and the flaps have been set at small angles, and not at the angles giving maximum lift.

Types of Flaps-3/4

High-lift devices	*Increase of maximum lift*	*Angle of basic aerofoil at max. lift*	*Remarks*
 Double-slotted Fowler flap	100%	20°	Same as Fowler flap only more so. Treble slots sometimes used.
 Krueger flap	50%	25°	Nose-flap hinging about leading edge. Reduces lift at small deflections. Nose-up pitching moment.
 Slotted wing	40%	20°	Controls boundary layer. Slight extra drag at high speeds.
 Fixed slat	50%	20°	Controls boundary layer. Extra drag at high speeds. Nose-up pitching moment.

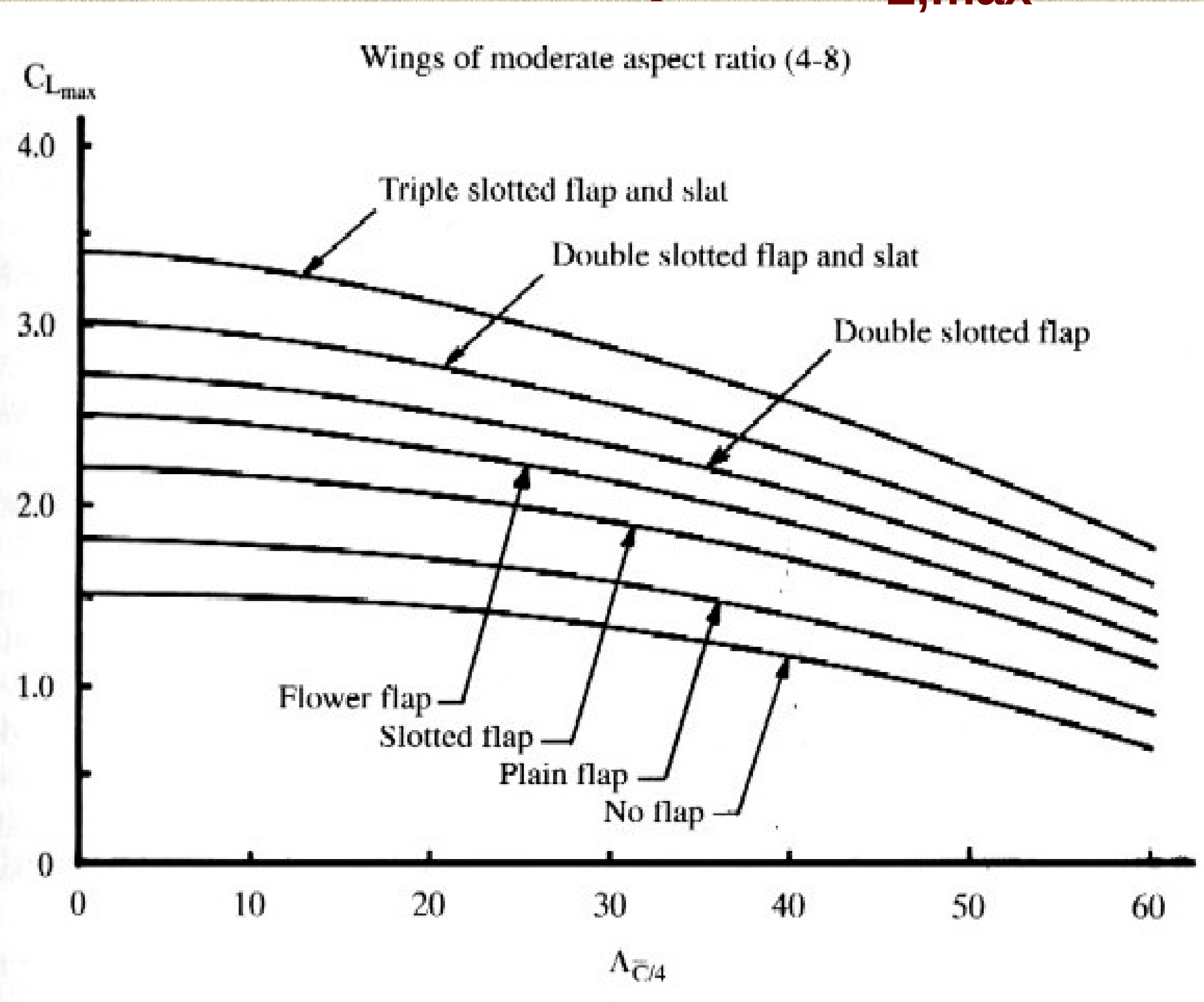
Types of flaps- 4/4

 Movable slat	60%	22°	Controls boundary layer. Increases camber and area. Greater angles of attack. Nose-up pitching moment.
 Slat and slotted flap	75%	25°	More control of boundary layer. Increased camber and area. Pitching moment can be neutralized.
 Slat and double-slotted Fowler flap	120%	28°	Complicated mechanisms. The best combination for lift; treble slots may be used. Pitching moment can be neutralized.
 Blown flap	80%	16°	Effect depends very much on details of arrangement.
 Jet flap	60%	?	Depends even more on angle and velocity of jet.

Typical Values of Max. Lift Coefficient

- Unflapped wings
  - Swept wing ($\Lambda_{0.25c} = 60^o$) 0.75
  - Swept wing ($\Lambda_{0.25c} = 45^o$) 1.00
  - Unswept wing 1.50
- Flapped Wings
  - Plain Flap 1.75
  - Slotted Flap 2.25
  - Fowler Flap 2.50
  - Double Slotted Flaps 2.75
  - Double Slotted Flaps and Slats 3.00
  - Triple Slotted Flaps and Slats 3.50
  - Blown Flaps ≈ 5.00

Effect of Sweep on $C_{L,max}$



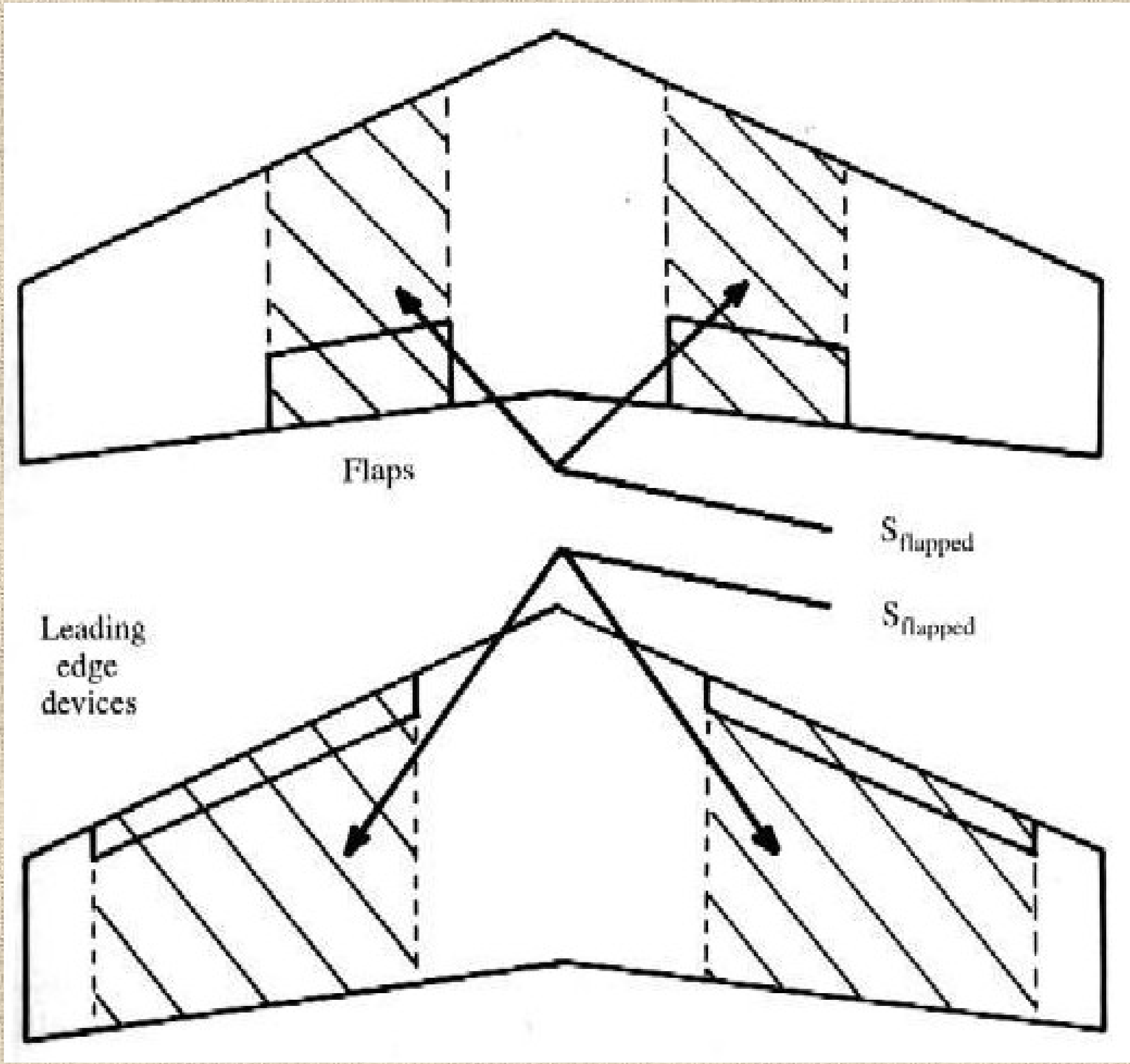
Source: Daniel P Raymer, *Aircraft Design, A Conceptual Approach*, AIAA Publications

Estimation of Wing $C_{L,max}$

- General Cases
  - Wings with low $\Lambda_{0.25c}$, AR > 5, $\lambda \approx 0.5$, large flaps
    - Wing $C_{Lmax} \approx 0.9$ Airfoil C_{Lmax}
  - Wings with partial span flaps

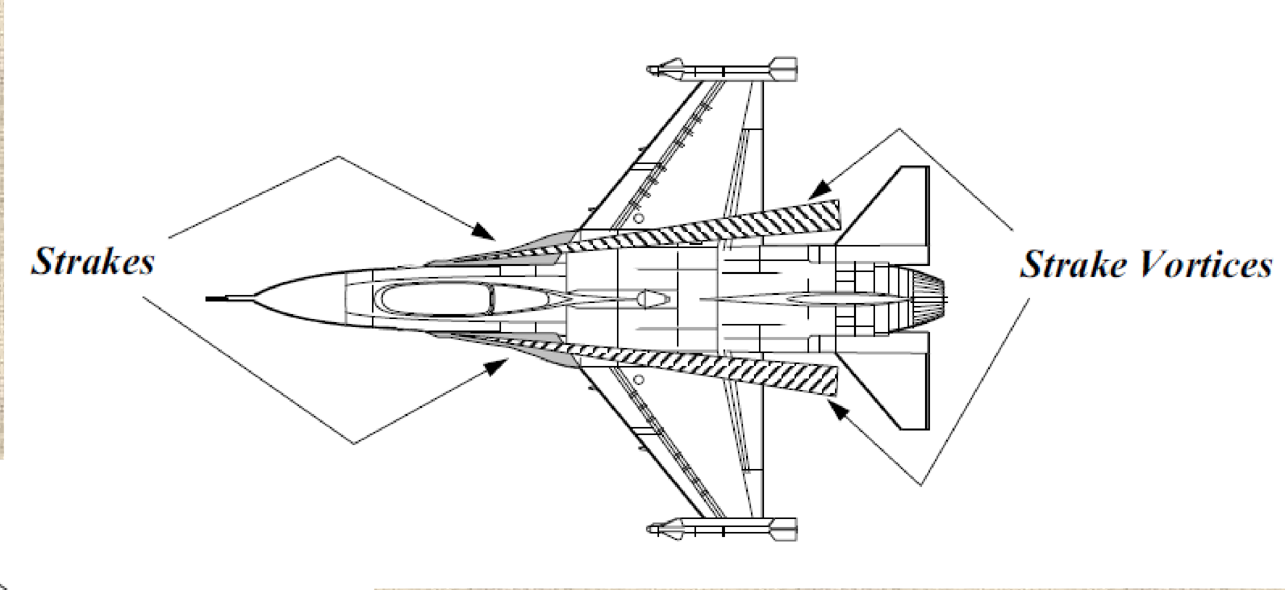
$$C_{L_{\max}} \cong 0.9\left\{\left(C_{L_{\max}}\right)_{flapped}\frac{S_{flapped}}{S_{ref}}+\left(C_{L_{\max}}\right)_{unflapped}\frac{S_{unflapped}}{S_{ref}}\right\}$$

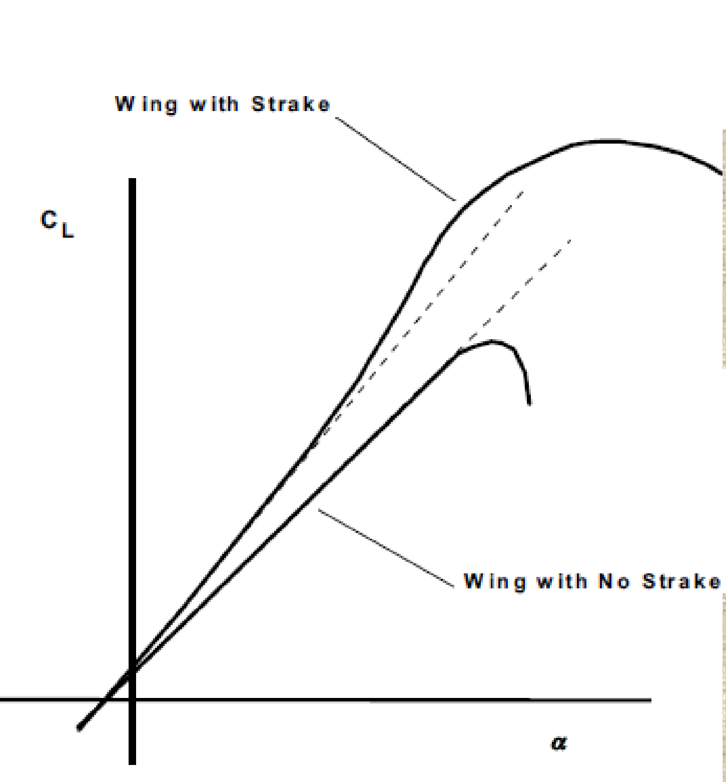
Flapped & Unflapped Area



Source: Daniel P Raymer, *Aircraft Design, A Conceptual Approach,* AIAA Publications

Effect of LeX and Strakes



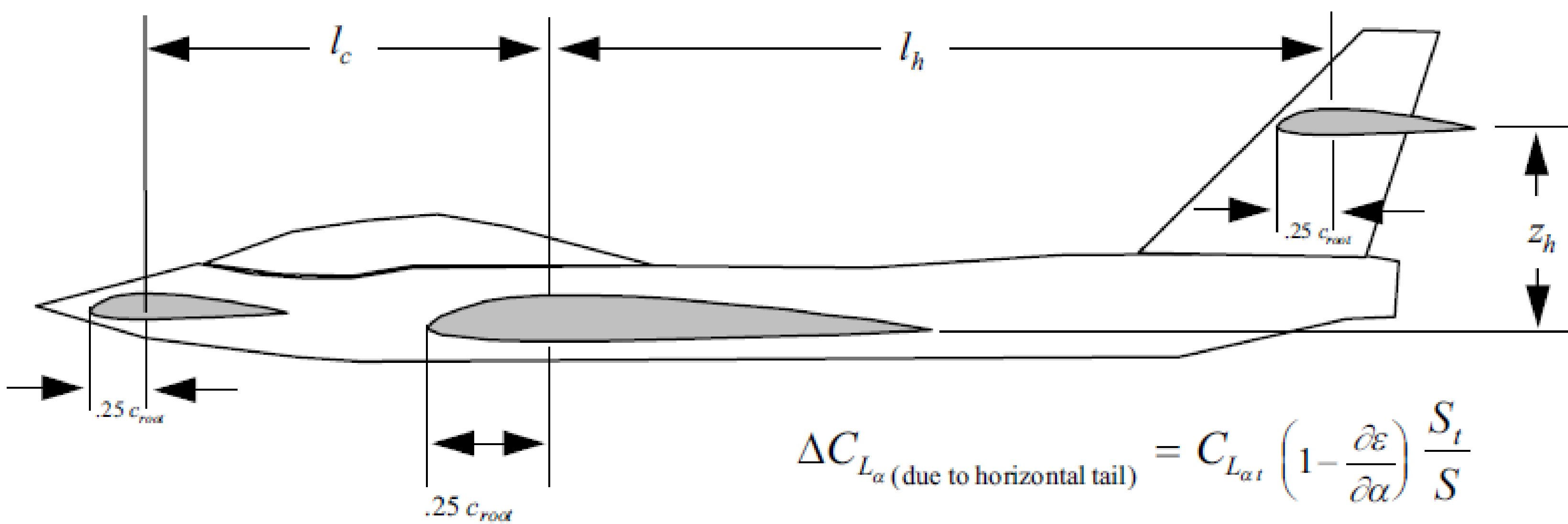


For low AoAs, in which strakes are not effective

$$C_{L_\alpha\,(\text{with strake})} = C_{L_\alpha\,(\text{without strake})}\,\frac{S + S_{strake}}{S}$$

Effect of Horizontal Tail & Canard

$$\frac{\partial \varepsilon}{\partial \alpha} = \frac{21^{\circ} C_{L_\alpha}}{AR^{0.725}} \left(\frac{c_{avg}}{l_h}\right)\left(\frac{10-3\lambda}{7}\right)\left(1-\frac{z_h}{b}\right)$$

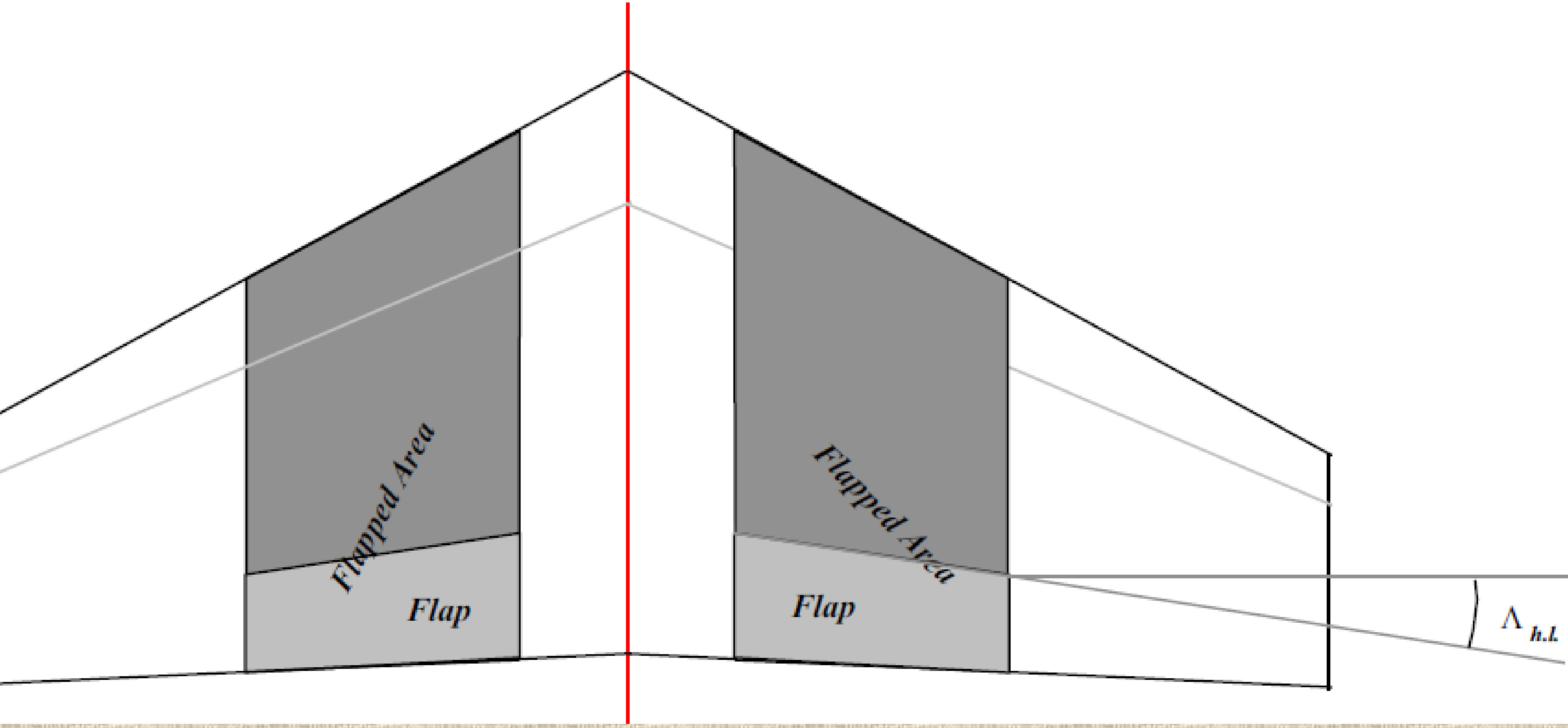


$$\Delta C_{L_{\alpha\,(\text{due to horizontal tail})}} = C_{L_{\alpha t}} \left(1-\frac{\partial \varepsilon}{\partial \alpha}\right)\frac{S_t}{S}$$

Effect of High Lift Devices

- Most Flaps increase $\alpha_{L=0}$ but don't change $C_{L\alpha}$
  - Equivalent to increase in α_a
- For full span flaps, $\Delta\alpha_{a3\text{-}D} \approx \Delta\alpha_{a2\text{-}D}$
  - $\Delta\alpha_{a2\text{-}D}$ = increment in absolute AoA for airfoil
  - $\Delta\alpha_{a3\text{-}D}$ = increment in absolute AoA for 3-D wing
- For Partial Span Flaps, $\Delta\alpha_a = \Delta\alpha_{a2D}\,(S_f/S)\,\mathrm{Cos}\,\Lambda_{h.l.}$
  - S_f/S = Ratio of Flapped Area to Wing Ref. Area
  - $\Lambda_{h.l.}$ = Sweep of Flap Hinge Line
  - $C_{Lmax,\ flapped} = C_{Lmax,\ no\ flaps} + C_{L\alpha} \cdot \Delta\alpha_a$
- Note: $\Delta\alpha_{a\text{-}2D} = 10^0$ @ Takeoff, 15^0 @ Landing

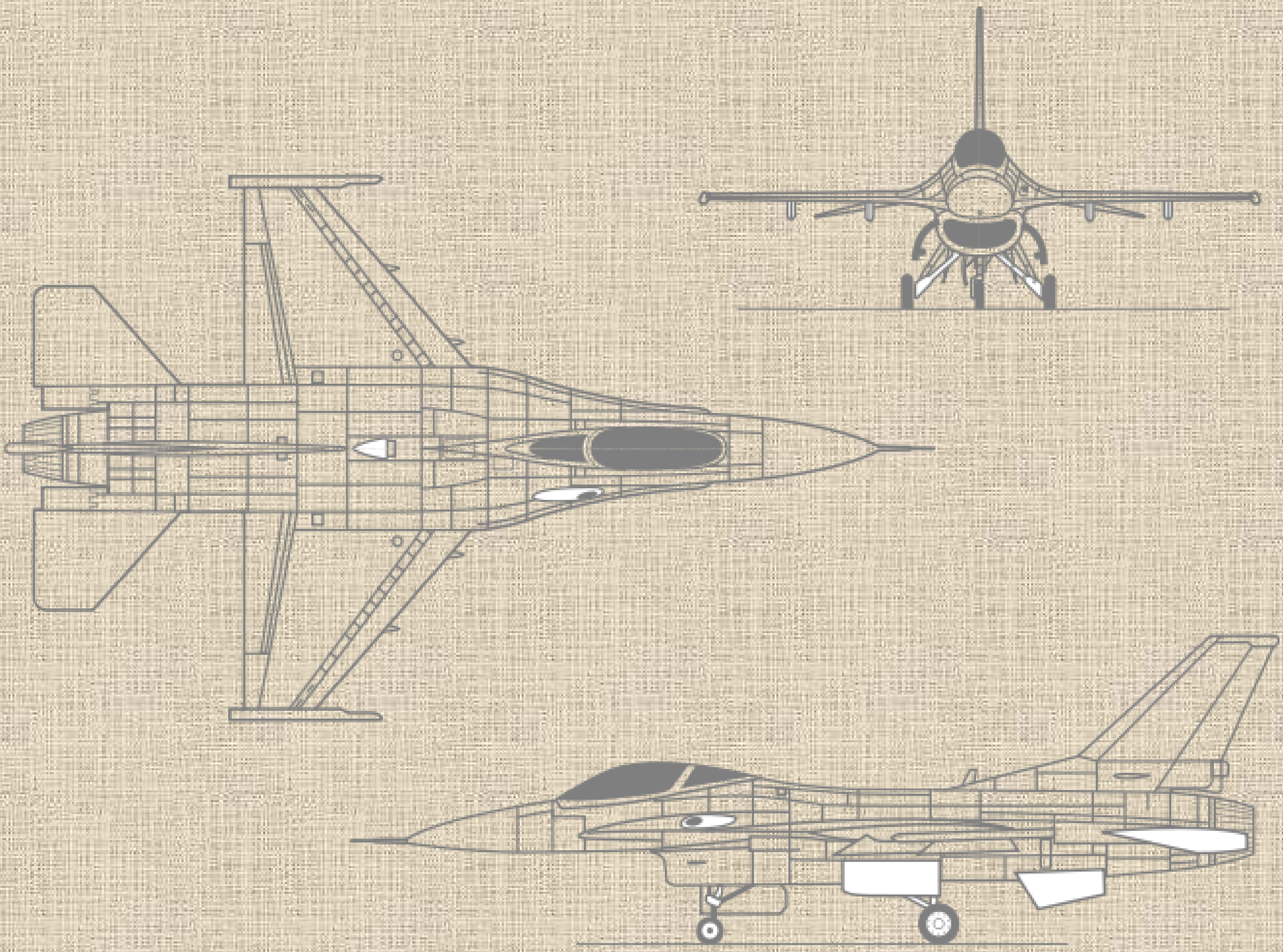
Definition: Flapped Area
Hinge Sweep Line



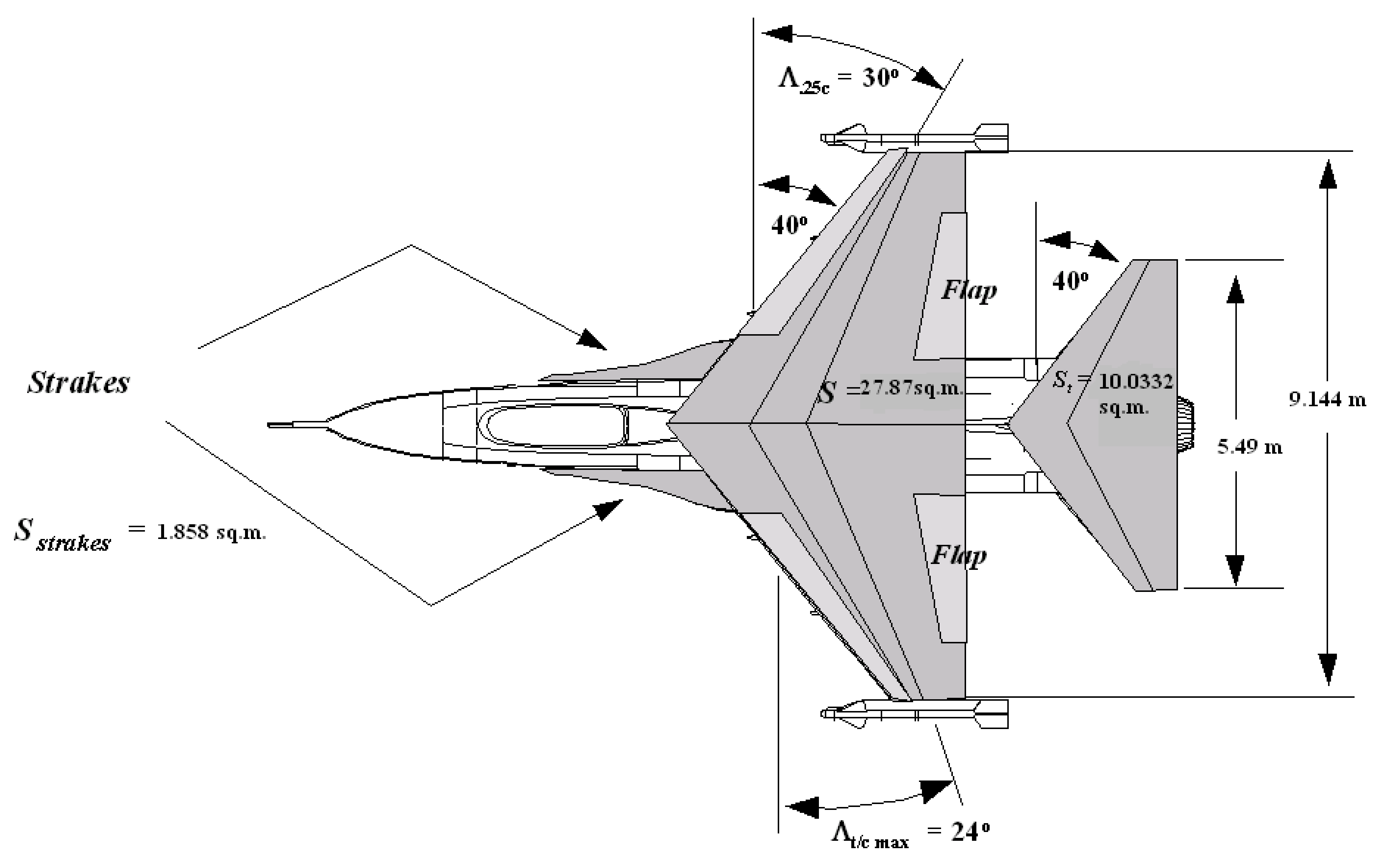
Example

LIFT COEFFICIENT ESTIMATION

Lift Coefficient Estimation of F-16



F-16 Aircraft Geometry



Useful Data for F-16

- NACA 64_A-204 airfoil, $c_{l\alpha}$ = 0.1 per degree
- Sweep of Max. Thickness line = Λ_{tmax} = 24°
- Max. Absolute AoA = 14 deg
- Distance from the quarter chord of the main wing's mean chord to the same point on horizontal tail, l_h = 4.48 m
- Wing taper ratio, λ = 1.07 m/5.03 m = 0.21
- Height of center line of HT from Wing = 0.3048 m

Estimation of Wing Efficiency Factor

- NACA 64_A-204 airfoil, $c_{l\alpha} = 0.1$ per degree
- $\Lambda_{tmax} = 24^o$
- Calculate Wing and Tail aspect ratios

$$AR = \frac{b^2}{S} = \frac{9.144^2}{27.87} = 3 \qquad AR_t = \frac{b_t^{\,2}}{S_t} = \frac{5.49^2}{10.03} = 3$$

- Estimation of span efficiency factor e

$$e = \frac{2}{2 - AR + \sqrt{4 + AR^2(1 + \tan^2 \Lambda_{t_{\max}})}} = \frac{2}{2 - 3 + \sqrt{4 + 9(1 + \tan^2 24^o)}}$$

$= .703 = e_t$

Estimation of Lift Curve Slope

$$C_{L_\alpha} = \frac{c_{l_\alpha}}{1 + \dfrac{57.3\, c_{l_\alpha}}{\pi\, e\, AR}} = 0.0536\ /^{o} = C_{L_{\alpha t}}$$

$$C_{L_\alpha\,(\text{with strake})} = C_{L_\alpha\,(\text{without strake})} \frac{S + S_{strake}}{S}$$

$$= (0.0536\ /^{o})\ \frac{27.87 + 1.86}{27.87} = 0.0572\ /^{o}$$

Estimation of Lift Curve Slope contd.

$$\frac{\partial \varepsilon}{\partial \alpha} = \frac{21^{\circ} C_{L_\alpha}}{AR^{0.725}} \left(\frac{c_{avg}}{l_h} \right) \left(\frac{10 - 3\lambda}{7} \right) \left(1 - \frac{z_h}{b} \right)$$

$$= \frac{\left(21^{\circ}\right)\left(0.0572/^{\circ}\right)}{3^{0.725}} \left(\frac{3.048 \text{ m}}{4.48 \text{ m}} \right) \left(\frac{10 - 3\,(0.21)}{7} \right) \left(1 - \frac{0.3048 \text{ m}}{9.14 \text{ m}} \right)$$

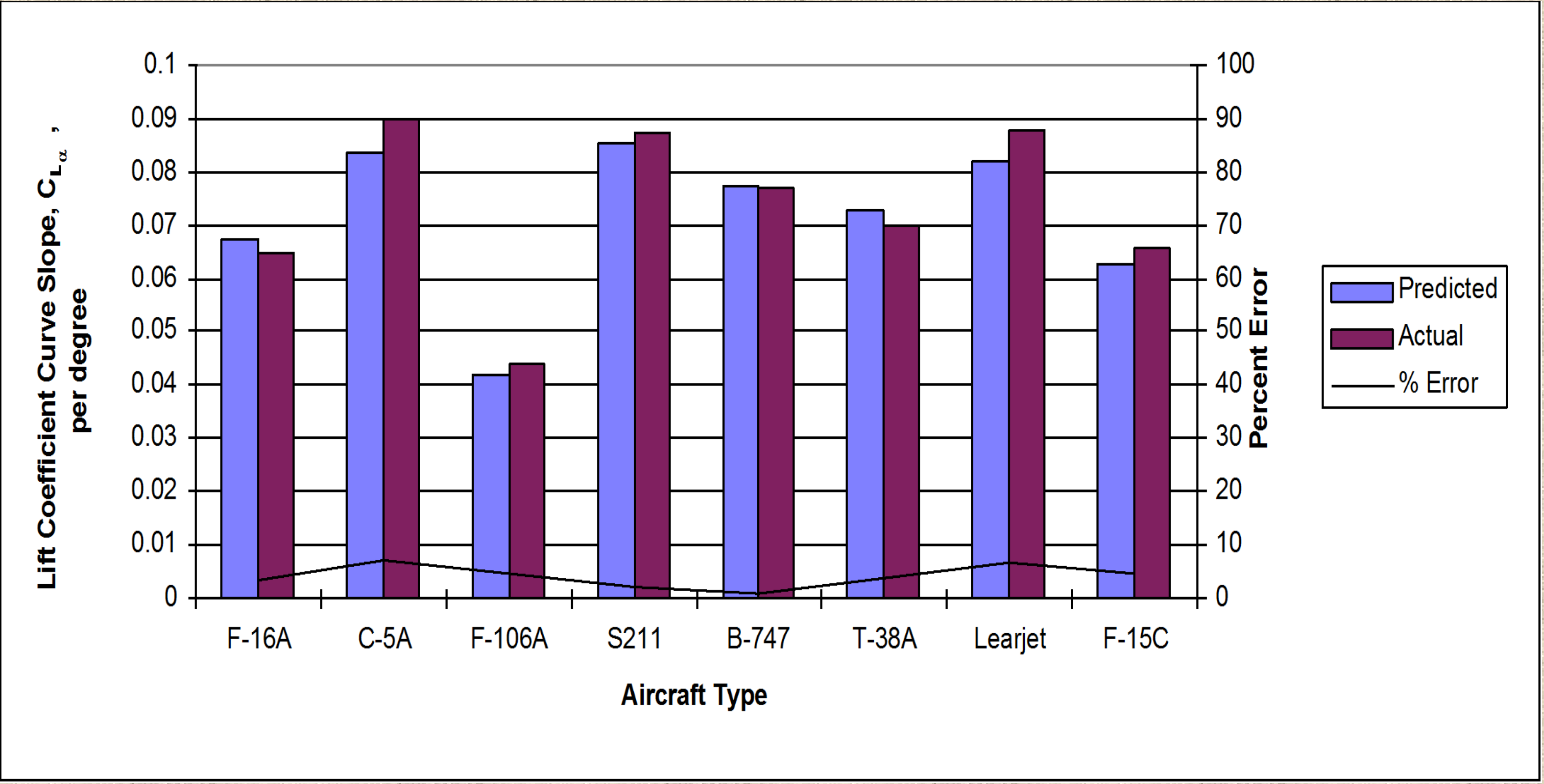
= 0.48

$$C_{L_\alpha (\text{whole aircraft})} = C_{L_\alpha (\text{with strake})} + C_{L_{\alpha\, t}} \left(1 - \frac{\partial \varepsilon}{\partial \alpha} \right) \frac{S_t}{S}$$

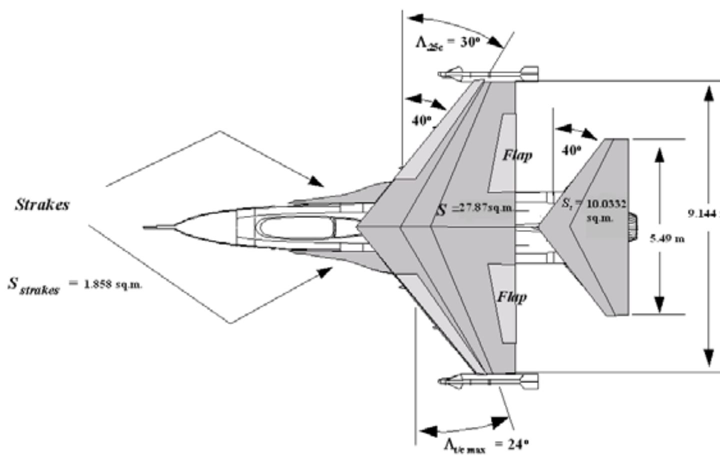
= 0.0572 /° + 0.0536 /° (1-.48) (10.03/27.87) = .067 /°

= 0.067 /°

Comparison of Lift Coefficient Slopes for various aircraft



Estimation of Max Lift Coeff.



$$\Delta\alpha_a = \Delta\alpha_{a_{2-D}} \frac{S_f}{S} \cos\Lambda_{h.l.} = 4.9^{\circ}$$

C_{Lmax} = 0.067/° (14° +4.9°) = 1.27 for takeoff

$$\Delta\alpha_a = \Delta\alpha_{a_{2-D}} \frac{S_f}{S} \cos\Lambda_{h.l.} = 7.36^{\circ}$$

C_{Lmax} = 0. 067 /° (14° +7.36°) = 1.43 for landing