Constraint Analysis Military Aircraft

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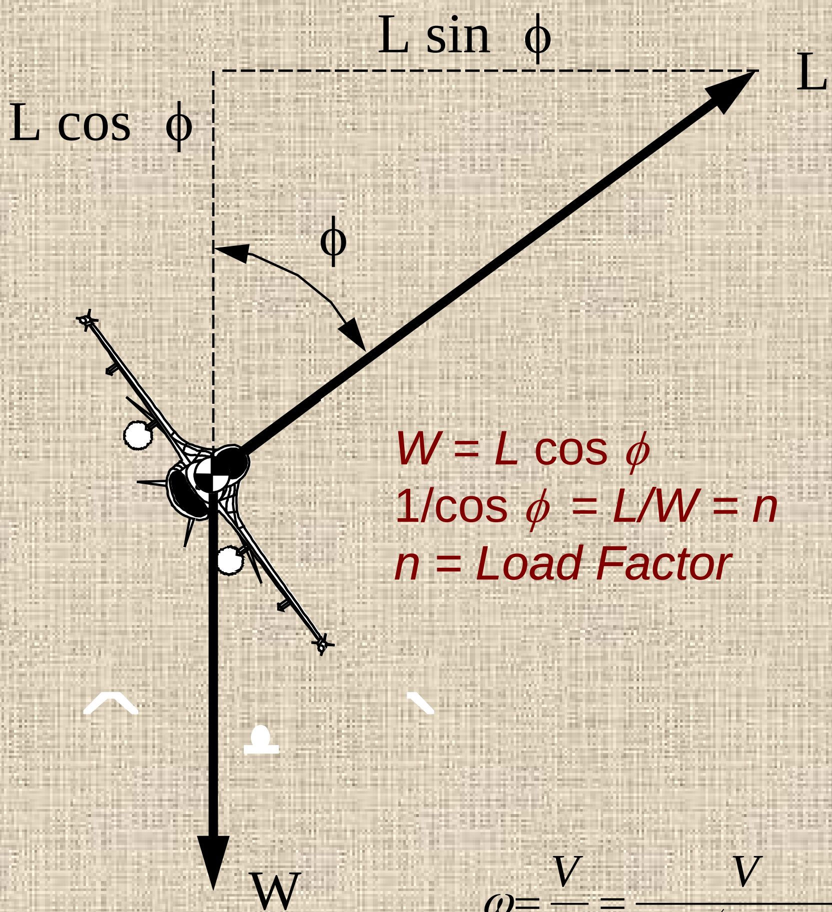
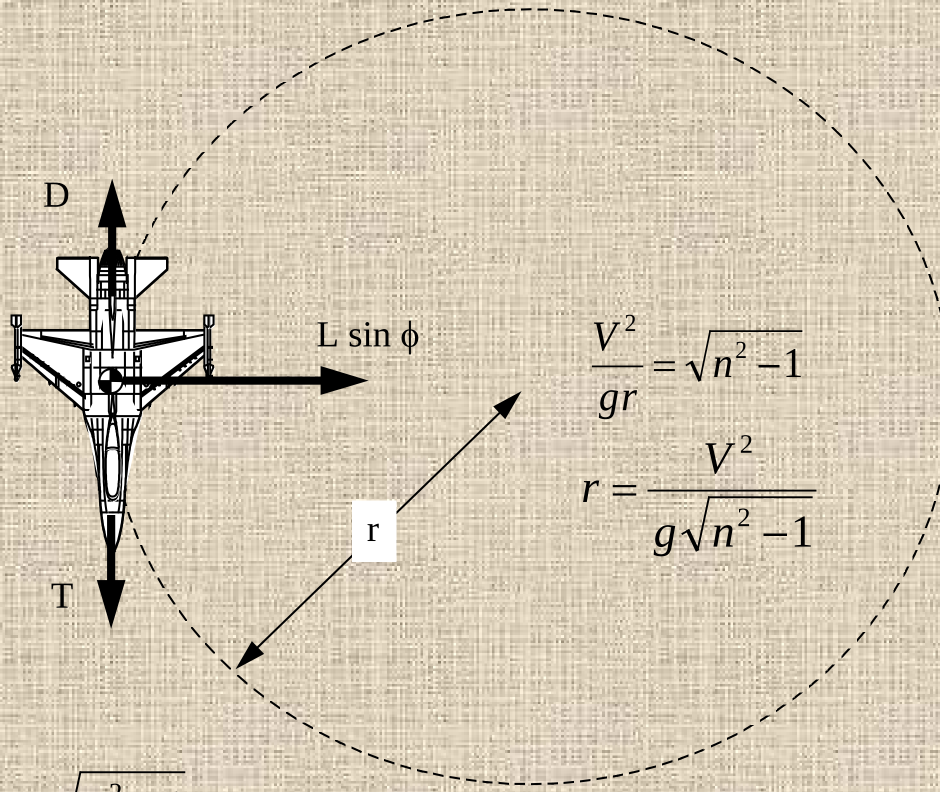
Key Operating Requirements

- Turning Performance
  - Instantaneous & Sustained Turn Rates Rate of Climb
- Level Flight Acceleration
- Specific Excess Power
- Stalling Speed
- Take-off and Landing Distances

Turning Performance in Level Flight



W = L cos ϕ
1/cos ϕ = L/W = n
n = Load Factor



$$\frac{V^2}{gr} = \sqrt{n^2 - 1}$$

$$r = \frac{V^2}{g\sqrt{n^2 - 1}}$$

$$\omega = \frac{V}{r} = \frac{V}{V^2 \big/ g\sqrt{n^2 - 1}} = \frac{g\sqrt{n^2 - 1}}{V}$$

Source: Daniel P Raymer, *Aircraft Design, A Conceptual Approach*, AIAA Publications

Constraint on Instantaneous Turn Rate

$$\dot{\Psi} = \frac{g\sqrt{n^2 - 1}}{V} \qquad n = \sqrt{\left(\frac{\dot{\Psi}V}{g}\right)^2 + 1} \qquad n = \frac{qC_L}{W/S}$$

- Where n = load factor = L/W
- Two types of turn rates
  - Sustained
    - o Enough Thrust to maintain V and H in turn
    - o T = D
  - Instantaneous
    - o Highest turn rate possible, V or H reduce in turn

- **Constraint on Instantaneous turn rate leads to an upper limit on W/S**

Constraint on Sustained Turn Rate

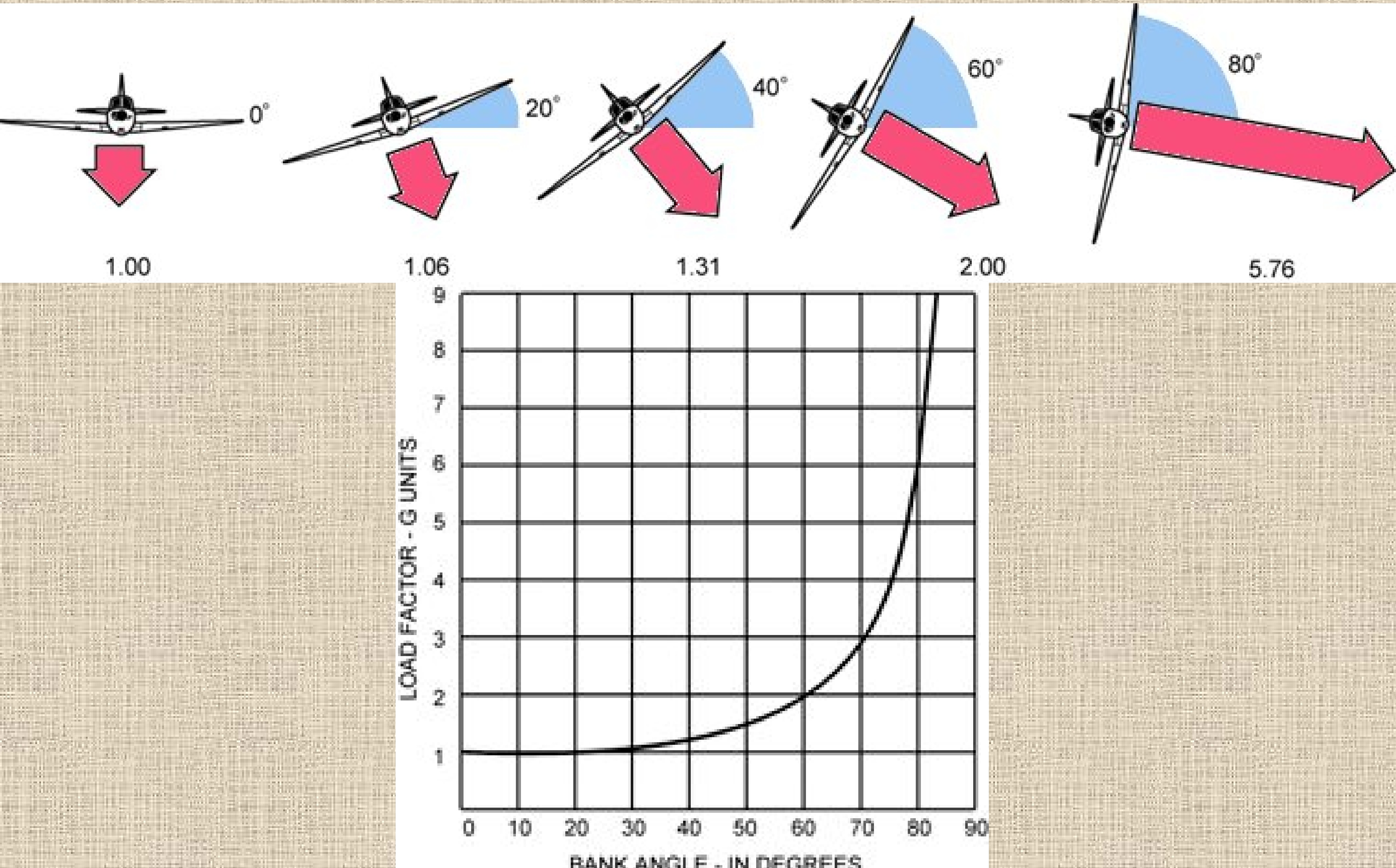
- In sustained turn $n = \left(T/W\right)\left(L/D\right)$
- For maximizing n, max L/D $W/S = \frac{q}{n}\sqrt{\pi A e C_{D_0}}$
- But this may give ridiculously low W/S !
- Using T = D $T = qSC_{D_0} + qS\left(\frac{C_L{}^2}{\pi A e}\right) = qSC_{D_0} + \frac{n^2 W^2}{qS\pi A e}$

$$\frac{T}{W} = \frac{qC_{D_0}}{W/S} + \frac{W}{S}\left(\frac{n^2}{q\pi A e}\right) \qquad \frac{W}{S} = \frac{\left(T/W\right) \pm \sqrt{\left(T/W\right)^2 - \left(4n^2 C_{D_0}/\pi A e\right)}}{2n^2/q\pi A e}$$

  - Note: W/S & T/W here are at combat conditions

- Important Observation $\frac{T}{W} \geq 2n\sqrt{\frac{C_{D_0}}{\pi A e}}$

Load Factor v/s Bank Angle



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Master Equation for Constraint Analysis

Energy Height Principles

- Energy Height = Specific Energy

$$H_e = \frac{P.E. + K.E.}{W} = \frac{mgh + \frac{1}{2} mV^2}{W} = h + \frac{V^2}{2g}$$

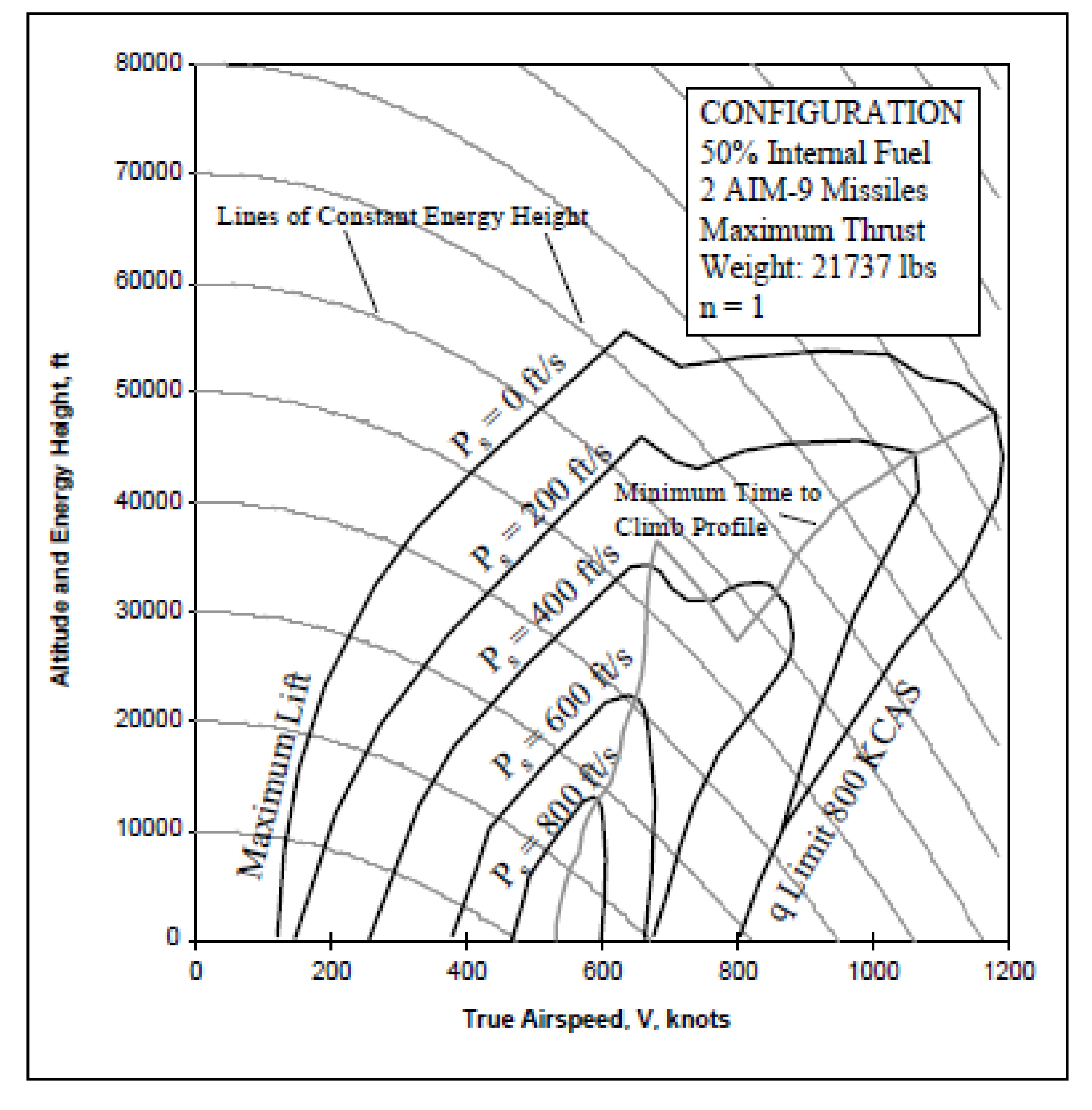
- Excess Power = Rate of change of energy

$$P_{avail} - P_{required} = V(T - D) = \frac{d(P.E. + K.E.)}{dt}$$

- Specific Excess Power = Excess Power /W

$$P_s \equiv \frac{P_{avail} - P_{required}}{W} = \frac{V(T - D)}{W} = \frac{d}{dt}\left(\frac{P.E. + K.E.}{W}\right) = \frac{dH_e}{dt} = \frac{d}{dt}\left(h + \frac{V^2}{2g}\right)$$

Ps diagram at n=1



Setting Up the Master Equation

- A master equation will be used to represent the relation between T/W & W/S
- From excess power requirements, we get

$$P_s \equiv \frac{P_{avail} - P_{required}}{W} = \frac{V(T-D)}{W} = \frac{d}{dt}\left(\frac{P.E.+K.E.}{W}\right) = \frac{dH_e}{dt} = \frac{d}{dt}\left(h + \frac{V^2}{2g}\right)$$

$$\frac{T}{W} - \frac{D}{W} = \frac{1}{V}\frac{dh}{dt} + \frac{1}{g}\frac{dV}{dt}$$

Thrust Lapse Ratio

- a = Thrust lapse Ratio, depends on σ and M (or V)
- $T = \alpha\, T_{SL}$
- a depends on powerplant type, as follows:

Type	Thrust Model
Piston Engine/Propeller	$T_A = SHP_{SL} \dfrac{\rho}{\rho_{SL}} \dfrac{\eta_P}{V_\infty}$
Turboprop	$T_A = ESHP_{SL} \left(\dfrac{\rho}{\rho_{SL}}\right) \dfrac{\eta_P}{V_\infty}$
High Bypass-Ratio Turbofan (Use $M = 0.1$ thrust for all $M < 0.1$)	$T_A = \left(\dfrac{01}{M_\infty}\right) T_{SL} \left(\dfrac{\rho}{\rho_{SL}}\right)$
Turbojet and Low-Bypass-Ratio Turbofan Dry (No Afterburner)	$T_A = T_{SL} \left(\dfrac{\rho}{\rho_{SL}}\right)$
Wet (With Afterburner Operating)	$T_A = T_{SL} \left(\dfrac{\rho}{\rho_{SL}}\right)(1 + 0.7\, M_\infty)$

Other Factors

- Weight $W = \beta W_{TO}$
  - where β = the weight fraction for a given constraint

- Drag $D = C_D qS = (C_{D_o} + k_1 C_L^2) qS$

- Lift Coefficient $C_L = \dfrac{L}{qS} = \dfrac{nW}{qS}$

Building up the master equation

$$\frac{T}{W} - \frac{D}{W} = \frac{1}{V}\frac{dh}{dt} + \frac{1}{g}\frac{dV}{dt}$$

$$D = C_D q S = (C_{D_o} + k_1 C_L^2) q S$$

$$C_L = \frac{L}{qS} = \frac{nW}{qS}$$

$$W = \beta W_{TO}$$

$$T = \alpha T_{SL}$$

SUBSTITUTE

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{q}{\beta}\left[\frac{C_{D_o}}{\left(W_{TO}/S\right)} + k_1\left(\frac{n\beta}{q}\right)^2\left(\frac{W_{TO}}{S}\right)\right] + \frac{1}{V}\frac{dh}{dt} + \frac{1}{g}\frac{dV}{dt}\right\}$$

Other Constraints

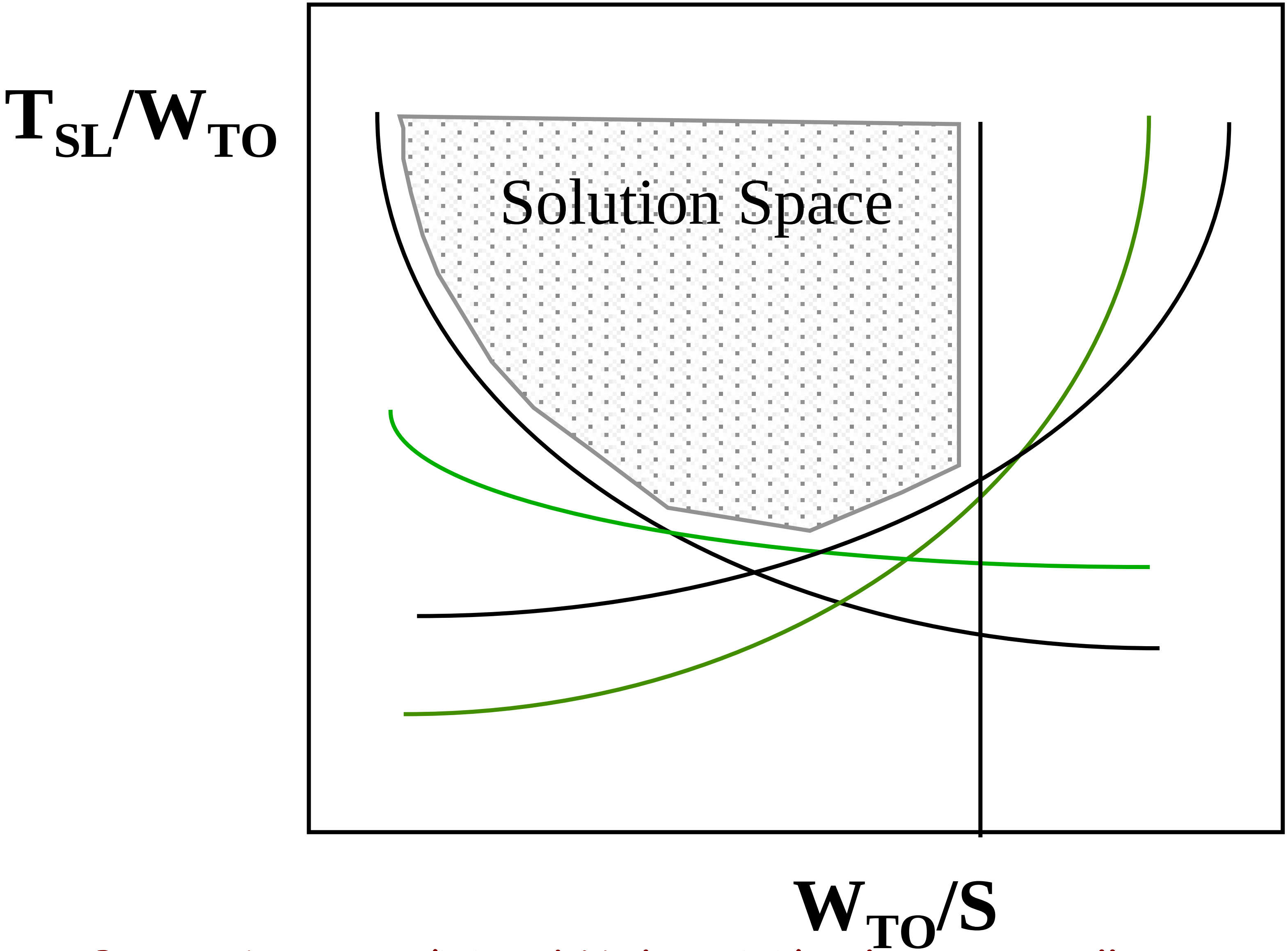
- The master equation takes all constraints except Take-off & Landing
- Take Off Constraint
  - Usually specified on take off distance S_{TO}
  - Assuming $V_{to} = 1.2*V_{stall}$

$$s_{TO} = \frac{1.44 W_{TO}^{2}}{\rho S C_{L_{\max}} g T}$$

- Landing Constraint
  - Usually specified at Landing Distance S_{Land}
  - Assuming $V_{land} = 1.3* V_{stall}$
  - μ_{roll} = Rolling Friction Coeff.
  - L_{land} = Lift at Landing
  - D_{land} = Drag at Landing

$$s_{Land} = \frac{1.69 \cdot \left(\beta \cdot W_{TO}\right)^2}{\rho S C_{L_{land}} g\left[D_{land} + \mu_{roll}\left(\beta \cdot W_{TO} - L_{land}\right]\right.}$$

Sample Constraint Diagram



Design Point = Lowest T/W and Highest W/S that meets all constraints