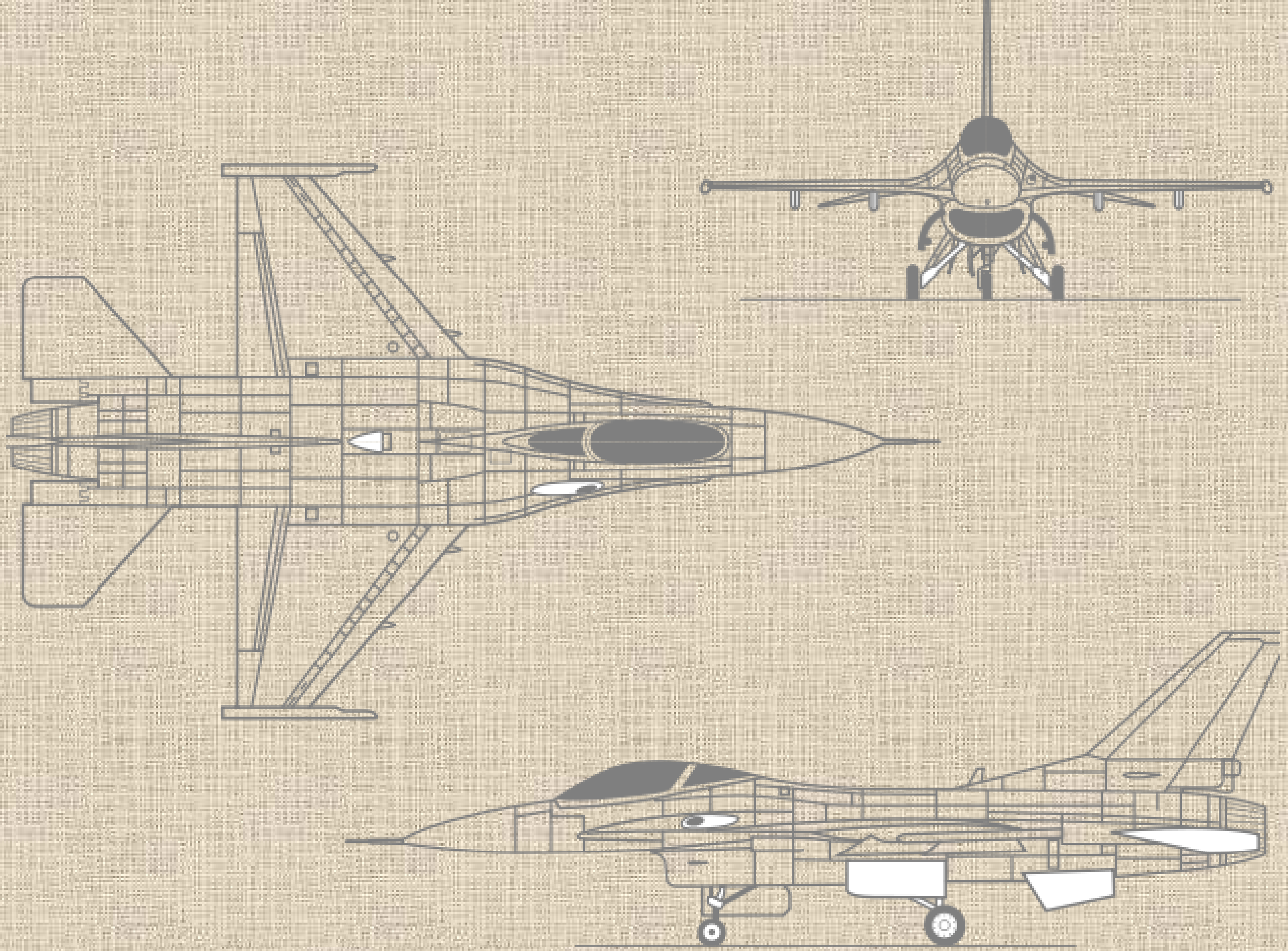
Single Seat Jet Engined Fighter Aircraft

TUTORIAL ON CONSTRAINED ANALYSIS OF MILITARY AIRCRAFT



F-16C Fighting Falcon

Requirements

Parameter	Value	Operating Condition
Stalling Speed	300 kmph	1-g @ Sea Level @ W_{TO}
Sustained Turn Rate (subsonic)	9g	1500 m, M = 0.9
Sustained Turn Rate (supersonic)	4g	9000 m, M = 1.2
Instantaneous Turn Rate	18 deg/s	6000 m, M=0.9, $W_{man,}$ max. C_L=1.0
Maximum Mach Number	2.0	@ W_{man} @ 20 km AMSL
Specific Excess Power	150 m/s	1500 m, M=0.9
Maximum Rate of Climb	160 m/s	@W_{TO} and V = 500 kt
FAR 25 Takeoff Field Distance	900 m	1 km AMSL, C_L @ TO= 1.27
FAR 25 Landing Field Length	900 m	1 km AMSL, C_L @ Landing = 1.43
Absolute Ceiling	17 km	With 2 Missiles, 250 kg each
Service Ceiling	15 km	With 2 Missiles, 250 kg each

W_{TO} = 16875 kg

W_{man} = Weight with 50% fuel + Missiles + Gun

= 9862.5 kg

Aerodynamic Data

Drag Polar

- Parabolic Drag Assumption:
  - $C_D = C_{Do} + k_1 C_L^2$
- Subsonic Flight
  - $C_D = 0.0243 + 0.121\, C_L^2$
- Supersonic Level Flight
  - $C_D = 0.0368 + 0.321\, C_L^2$
- Supersonic Turn
  - $C_D = 0.0412 + 0.169\, C_L^2$

Lift Data

- Max. C_L in level flight = 1.00
- Max. C_L at Takeoff = 1.27
- Max. C_L at Landing = 1.43

Using Master Equation

CONSTRAINT ANALYSIS

The master equation

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{q}{\beta}\left[\frac{C_{D_o}}{\left(W_{TO}/S\right)} + k_1\left(\frac{n\beta}{q}\right)^2\left(\frac{W_{TO}}{S}\right)\right] + \frac{1}{V}\frac{dh}{dt} + \frac{1}{g}\frac{dV}{dt}\right\}$$

Stalling Speed

- Data
  - V_{stall} = 300 kmph
  - Altitude = Sea Level
  - Atmosphere = ISA
  - Max Takeoff Weight
- V_{stall} = 162 knots
- = 83.3 m/s
- $C_{L,max}$ clean = 1.0
  - $\beta = 1.0$
  - $\alpha = 1.0$
  - $\rho = 1.2256\ kg/m^3$
- Calculate W/S
  - $W/S \leq 0.5\ \rho V_{stall}^2\ C_{L,max}$
  - **$W/S \leq 433.3\ kg/m^2$**

Subsonic Combat Turn

- Data:
  - h = 1500 m
  - n = 9.0
  - M = 0.9
- Note
  - $C_D = C_{D0} + k_1 C_L^2$
    - k_1, C_{Do} obtained from basic aerodynamic model
- Assumptions
  - $C_{Do} = 0.0243$
  - $k_1 = 0.121$
  - $\beta = 0.8$
  - $\alpha = 1.408$
- Calculate ρ, a , V, q
- Answers
  - $\rho = 1.058$ kg/m^3
  - a = 334.5 m/s
  - V = 301 m/s
  - q = 4888.5 kg/m^2

Subsonic Combat Turn contd.

- For sustained turn, dh/dt = 0, dV/dt = 0
- Substituting in master equation

$$\frac{T_{SL}}{W_{TO}} = \left\{ \frac{q}{\alpha} \left[\frac{C_{D_o}}{\left(W_{TO} / S \right)} + k_1 \left(\frac{n\beta}{q} \right)^2 \left(\frac{W_{TO}}{S} \right) \right] \right\}$$

$(T/W)_{to} = A/(W_{to}/S) + B^*(W_{to}/S)$,Calculate A & B,

A=84.379; B=9.114*10^{-4}

Using this, get various W/S and T/W combinations:

W_{to}/S (kg/m^2)	100	150	200	250	300	350	400	450	500	550
T_{sl}/W_{to}	0.93	0.70	0.60	0.57	0.55	0.56	0.58	0.60	0.62	0.65

Specific Excess Power

- Data:
  - h = 1500 m
  - n = 1.0
  - M = 0.9
  - $P_s \geq 150$ m/s
- Note
  - $C_D = C_{D0} + k_1 C_L{}^2$
    - o k_1, C_{Do} obtained from basic aerodynamic model
- Assumptions
  - $C_{Do} = 0.0243$
  - $k_1 = 0.121$
  - $\beta = 0.8$
  - $\alpha = 1.408$
- Calculate ρ, *a* , V, *q*
- Answers
  - ρ = 1.058 kg/m^3
  - a = 334.5 m/s
  - V = 301 m/s
  - q = 4888.5 kg/m^2

Specific Excess Power contd.

- We know that : $P_s = \left\{ \frac{1}{V}\frac{dh}{dt} + \frac{1}{g}\frac{dV}{dt} \right\}$
- Substituting in master equation

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{ \frac{q}{\beta}\left[\frac{C_{D_o}}{\left(W_{TO}/S \right)} + k_1\left(\frac{n\beta}{q} \right)^2 \left(\frac{W_{TO}}{S} \right) \right] + \frac{P_s}{V} \right\}$$

$(T/W)_{to} = A/(W_{to}/S) + B^*(W_{to}/S) + C$, Calculate A, B, C

Answers: **A=84.379; B= 1.125 $*10^{-5}$ C = 0.2832**

Using this, get various W/S and T/W combinations :

W_{to}/S (kg/m^2)	100	150	200	250	300	350	400	450	500	550
T_{sl}/W_{to}	1.13	0.85	0.71	0.63	0.57	0.53	0.50	0.48	0.46	0.44

Supersonic Combat Turn

- Data:
  - $h = 9000$ m
  - $n = 4.0$
  - $M = 1.2$
- Note
  - $C_D = C_{D0} + k_1 C_L^2$
    - k_1, C_{Do} obtained from basic aerodynamic model

- Assumptions
  - $C_{Do} = 0.0412$
  - $k_1 = 0.169$
  - $\beta = 0.8$
  - $\alpha = 0.700$
- Calculate ρ, *a* , V, *q*
- Answers
  - $\rho = 0.4663$ kg/m^3
  - $a = 303.77$ m/s
  - $V = 364.52$ m/s
  - $q = 3159.14$ kg/m^2

Supersonic Combat Turn contd.

- Substituting in master equation

For sustained turn, dh/dt = 0, dV/dt = 0

$$\frac{T_{SL}}{W_{TO}} = \left\{ \frac{q}{\alpha} \left[\frac{C_{D_o}}{\left(W_{TO}/S \right)} + k_1 \left(\frac{n\beta}{q} \right)^2 \left(\frac{W_{TO}}{S} \right) \right] \right\}$$

$(T/W)_{to} = A/(W_{to}/S) + B*(W_{to}/S)$, Calculate A and B

Answers: **A=185.85, B=7.822*10^{-4}**

Using this, get various W/S and T/W combinations:

W_{to}/S (kg/m^2)	100	150	200	250	300	350	400	450	500	550
T_{sl}/W_{to}	1.94	1.36	1.09	0.94	0.85	0.80	0.78	0.77	0.76	0.77

Instantaneous Turn

- Data:
  - h = 6000 m
  - Ψ = 18 deg/sec
  - M = 0.9
  - W_{To} = 16875 kg
  - W_{man}
    - 50% internal fuel
    - 2*AIM-120 missiles
    - 200*20 mm bullets
    - = 9862.5 kg

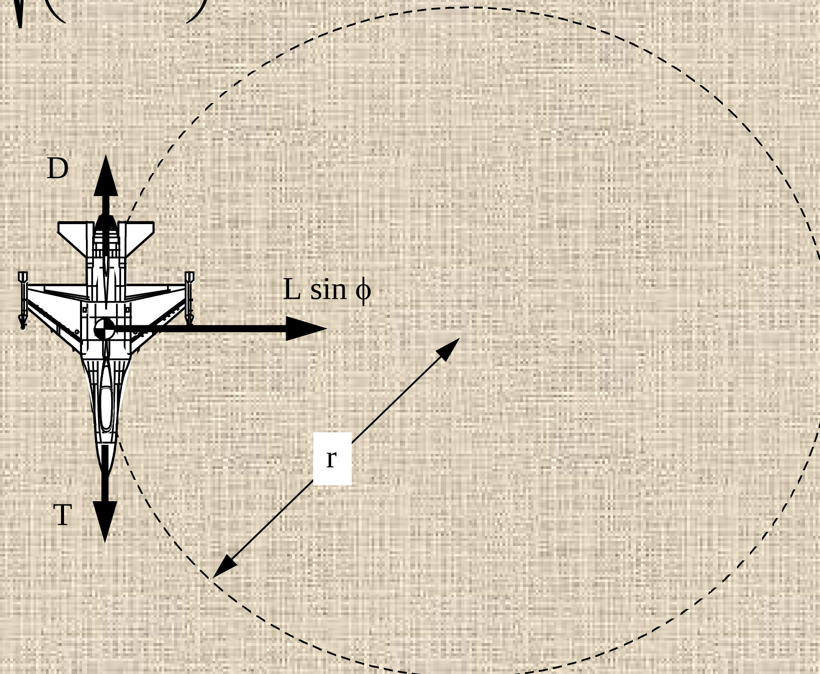
- Assumptions
  - Max $C_{L,turn}$ = 1.0

- Calculate β,ρ,a,V,q,n,W/S
  - β = 0.5845
  - ρ = 0.6597 kg/m^3
  - *a* = 316.4 m/s
  - V = 284.77 m/s
  - *q* = 2727.42 kg/m^2

Constraint on Instantaneous Turn Rate

$$\omega = \frac{V}{r} = \frac{V}{V^2 \Big/ g\sqrt{n^2-1}} = \frac{g\sqrt{n^2-1}}{V} \qquad \dot{\Psi} = \frac{g\sqrt{n^2-1}}{V} \qquad n = \sqrt{\left(\frac{\dot{\Psi}V}{g}\right)^2 + 1} \qquad n = \frac{qC_L}{W \big/ S}$$

- Here, $d\Psi/dt$ = 18 deg / sec
- V = 284.77 m/s
- Calculate n
- n = 9.176
- Calculate W_{TO}/S
- $W_{TO}/S \leq 508.54\ kg/m^2$



Maximum Mach Number

- Data:
  - SEP = 0, n = 1
  - h = 20000 m
  - M = 2.00
  - W_{To} = 16875 kg
  - W_{man}
    - o 50% internal fuel
    - o 2*AIM-120 missiles
    - o 200*20 mm bullets
    - o = 9862.5 kg
  - $\beta = 0.58447$
  - Also, $\alpha = \rho/\rho_0(1+0.7M)$

- Assumptions
  - $C_{Do} = 0.0368$
  - $K_1 = 0.321$
- Calculate ρ, α, a, V, q
  - ρ = 0.0880 kg/m^3
  - $\alpha = 0.1723$
  - a = 295.05 m/s
  - V = 590.10 m/s
  - q = 1562.33 kg/m^2

Max. Mach Number contd.

- In this case, $P_s = 0$, hence $P_s / V = \frac{P_s}{V} = \left\{ \frac{1}{V}\frac{dh}{dt} + \frac{1}{g}\frac{dV}{dt} \right\}$
- Substituting in master equation

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{ \frac{q}{\beta}\left[\frac{C_{D_o}}{\left(W_{TO}/S \right)} + k_1\left(\frac{n\beta}{q} \right)^2 \left(\frac{W_{TO}}{S} \right) \right] \right\}$$

$(T/W)_{to} = A/(W_{to}/S) + B^*(W_{to}/S)$, Calculate A&B

Answers **$A = 333.519$, $B = 4.0716 \times 10^{-4}$**

Using this, get various W/S and T/W combinations

W_{to}/S (kg/m^2)	100	150	200	250	300	350	400	450	500	550
T_{sl}/W_{to}	3.38	2.28	1.75	1.44	1.23	1.10	1.00	0.92	0.87	0.83

Max. Climb Rate

- Data
  - ROC = 160 m/s
  - Unaccelerated
    - $dV/dt = 0$
  - V_{fwd} = 500 knots
  - Altitude = Sea Level
  - Atmosphere = ISA+20
  - Max Takeoff Weight

- Assumptions
  - $C_{Do} = 0.0243$
  - $k_1 = 0.121$
  - $\beta = 1.0$
  - $\alpha = 1.0$
  - $n = 1.0$

- Calculate ρ, V_{climb}, q
  - $\rho = 1.1457\ kg/m^3$
  - $V_{fwd} = 257.36$ m/s
  - $V_{climb} = 303.04$ m/s
  - $q = 5364.13\ kg/m^2$

Maximum Climb Rate contd.

- We know that : $ROC = \left\{ \dfrac{dh}{dt} \right\}$
- Substituting in master equation

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{ \frac{q}{\beta}\left[\frac{C_{D_o}}{\left(W_{TO}/S \right)} + k_1 \left(\frac{n\beta}{q} \right)^2 \left(\frac{W_{TO}}{S} \right) \right] + \frac{1}{V}\frac{dh}{dt} \right\}$$

$(T/W)_{to} = A/(W_{to}/S) + B*(W_{to}/S) + C$, Calculate A, B, C

Answers: **A = 108.62; B= 1.8798 $*10^{-5}$ C = 0.6217**

Using this, get various W/S and T/W combinations:

W_{to}/S (kg/m^2)	100	150	200	250	300	350	400	450	500	550
T_{sl}/W_{to}	1.71	1.35	1.17	1.06	0.99	0.94	0.90	0.87	0.85	0.83

Takeoff Ground Roll

- Data:
  - $h = 1000m$
  - $S_{To} \leq 900$ m
- Note
  - C_{Lmax} @ TO
    - o obtained from basic aerodynamic model
- Assumptions
  - C_{Lmax} @ TO = 1.27
  - $\beta = 1.0$
  - $\alpha = 1.0$
- Calculate T, ρ
  - T = 281.66 K
  - ρ = 1.1122 kg/m^3

Take Off Ground Roll Constraint

Substituting in the equation $$s_{TO} = \frac{1.44 W_{TO}^{\;2}}{\rho S C_{L_{\max}} T}$$

We get $$\frac{T}{W_{TO}} = \frac{1.44}{\rho C_{L_{\max}} s_{TO}} \cdot \frac{W_{TO}}{S}$$

Thus, $T_{sl}/W_{to} = A*\{W_{to}/S\}$, Calculate A

Answer **$A=1.1328*10^{-3}$**

Using this, we get various W/S and T/W combinations as

W_{to}/S (kg/m^2)	100	150	200	250	300	350	400	450	500	550
T_{sl}/W_{to}	0.11	0.17	0.23	0.28	0.34	0.40	0.45	0.51	0.57	0.62

Landing Ground Roll

- Data:
  - $h = 1000$ m
  - $S_{land} \leq 900$ m
- Note
  - C_{Lmax} @ TO
    - o obtained from basic aerodynamic model
- Assumptions
  - $C_{L,land} = 1.43$
  - $\beta = 1.0$
  - $\alpha = 1$
  - $\mu_{roll} = 0.5$
  - $D_{land} = L_{land} = 0$
- ρ is already known !
  - $\rho = 1.1122\ \mathrm{kg/m^3}$

Landing Constraint

Substituting in the equation

$$s_{Land} = \frac{1.69 \cdot (\beta \cdot W_{TO})^2}{\rho S C_{L_{land}} \left[D_{land} + \mu_{roll} (\beta \cdot W_{TO} - L_{land} \right]}$$

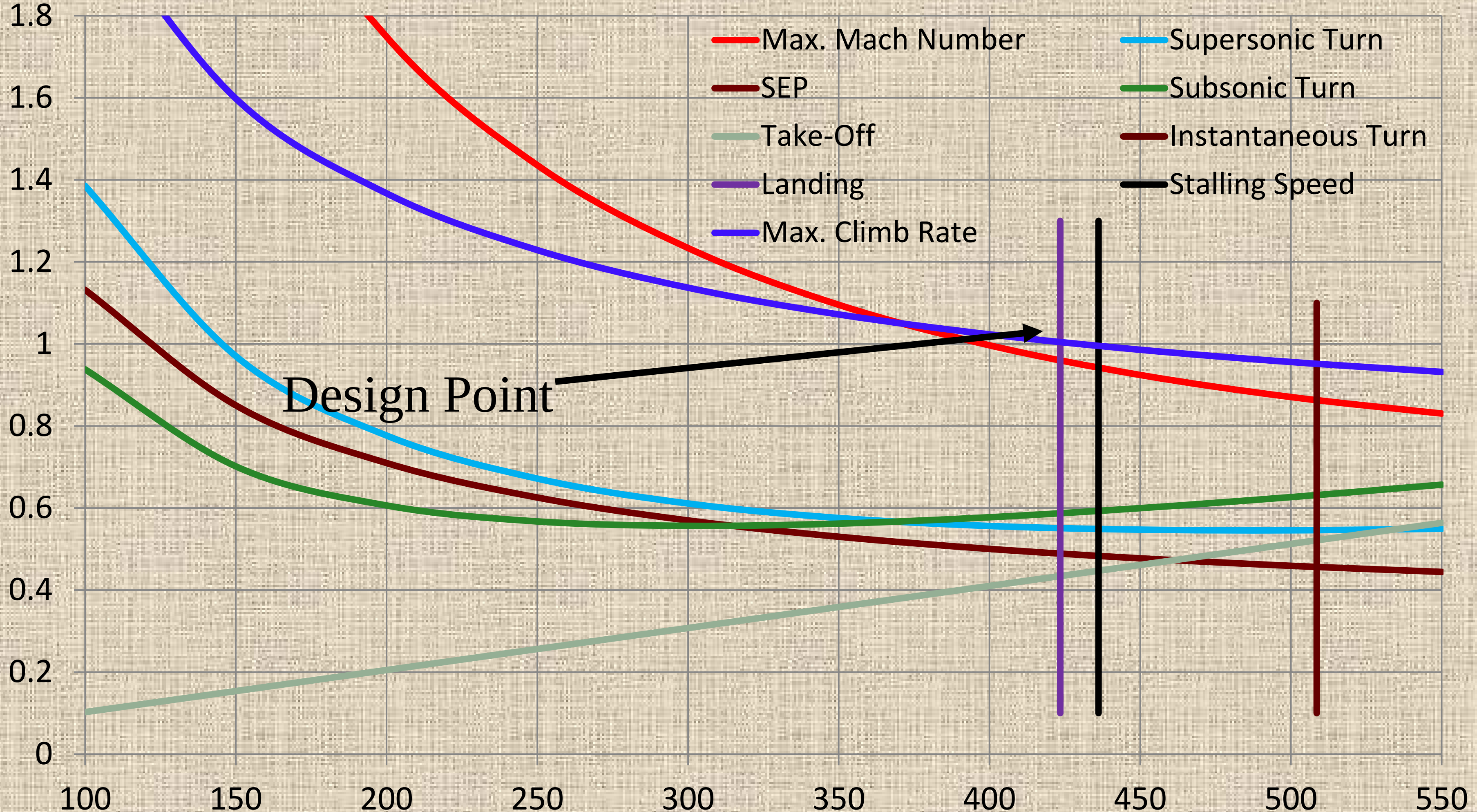
With $D_{land} = L_{land} = 0$, we get

$$\frac{W_{TO}}{S} = \frac{s_{Land} \cdot \rho \cdot C_{L_{land}} \cdot \mu_{roll}}{1.69 \cdot \beta}$$

Calculate W/S

$W_{to}/S \leq 423.48\ \mathrm{kg/m^2}$

Constraint Diagram



Design Point W/S = 423.5 kg/m² ; T/W = 1.00

Specifications of F-16C

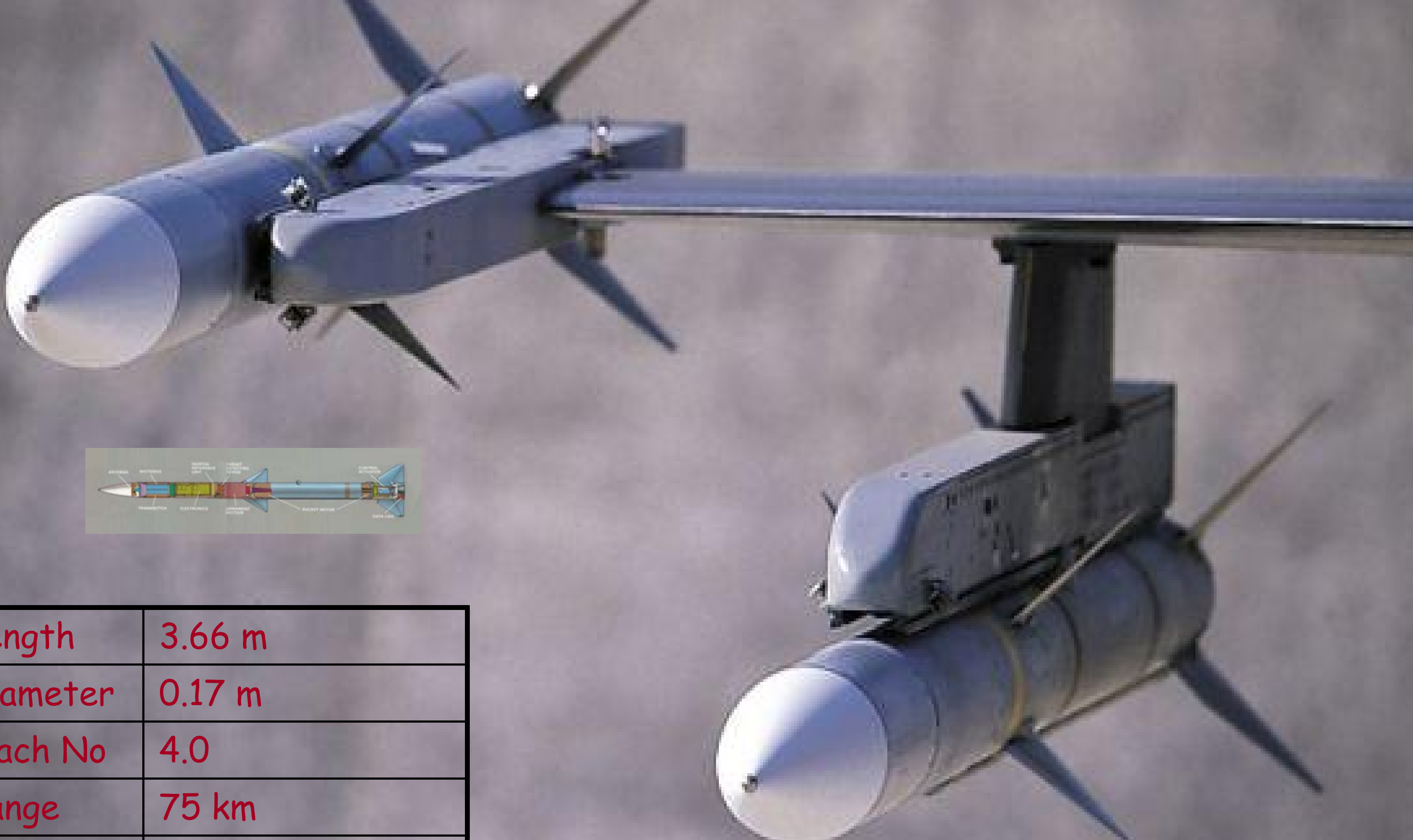
- **Role:**
  - single-engined, multi-role tactical aircraft
- **Crew:** 1
- **Length:** 14.8 m
- **Wingspan:** 9.8 m
- **Height:** 4.8 m
- **Wing area:** 27.87 m² (300 ft²)
- **Airfoil:** NACA 64A204 root and tip
- **Empty weight:** 8272 kg
- **Loaded weight:** 12003 kg
- **Max takeoff weight:** 16875 kg
- **Powerplant:**
  - 1× Pratt & Whitney F100-PW-220 afterburning turbofan
  - **Thrust :** 64.9 kN (dry) 105.7 kN (wet)
- **Alternate:**
  - 1× General Electric F110-GE-100 afterburning turbofan
  - **Thrust :** 76.3 kN (dry) 128.9 kN (wet)

Performance

- **V_{Max}:** > Mach 2 (2,124 km/h) at altitude
- **Combat radius:** 295 nm (550 km)
  - hi-lo-hi mission with 6x 450 kg bombs
- **Ferry range:** > 2,800 nm (4,800 km)
- **Service ceiling:** > 55,000 ft (15 km)
- **Rate of climb:** 50,000 ft/min (260 m/s)
- **Wing loading: 431 kg/m²**
- **Thrust/weight: with F100 = 0.98**
- with F110 = 1.095

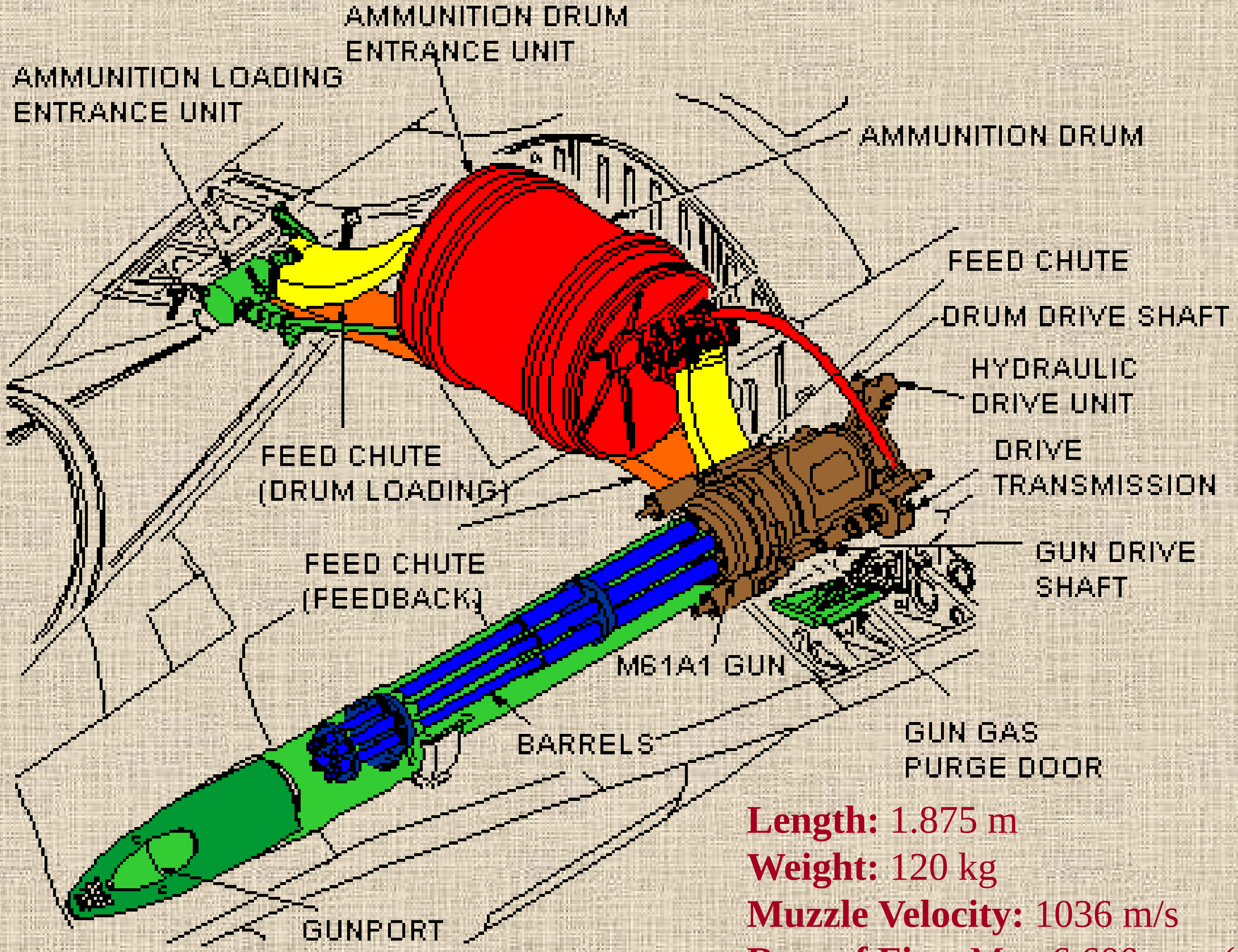
Armament on F-16 aircraft

AIM-120 A missile



Length	3.66 m
Diameter	0.17 m
Mach No	4.0
Range	75 km
Weight	152 kg (warhead = 23 kg)

M61A1 20mm Automatic Gun



Length: 1.875 m

Weight: 120 kg

Muzzle Velocity: 1036 m/s

Rate of Fire: Max 6,600 rpm (set to 4000-6000)

Bullet weight: = 0.25 kg