A Brief Introduction to

V-N Diagram

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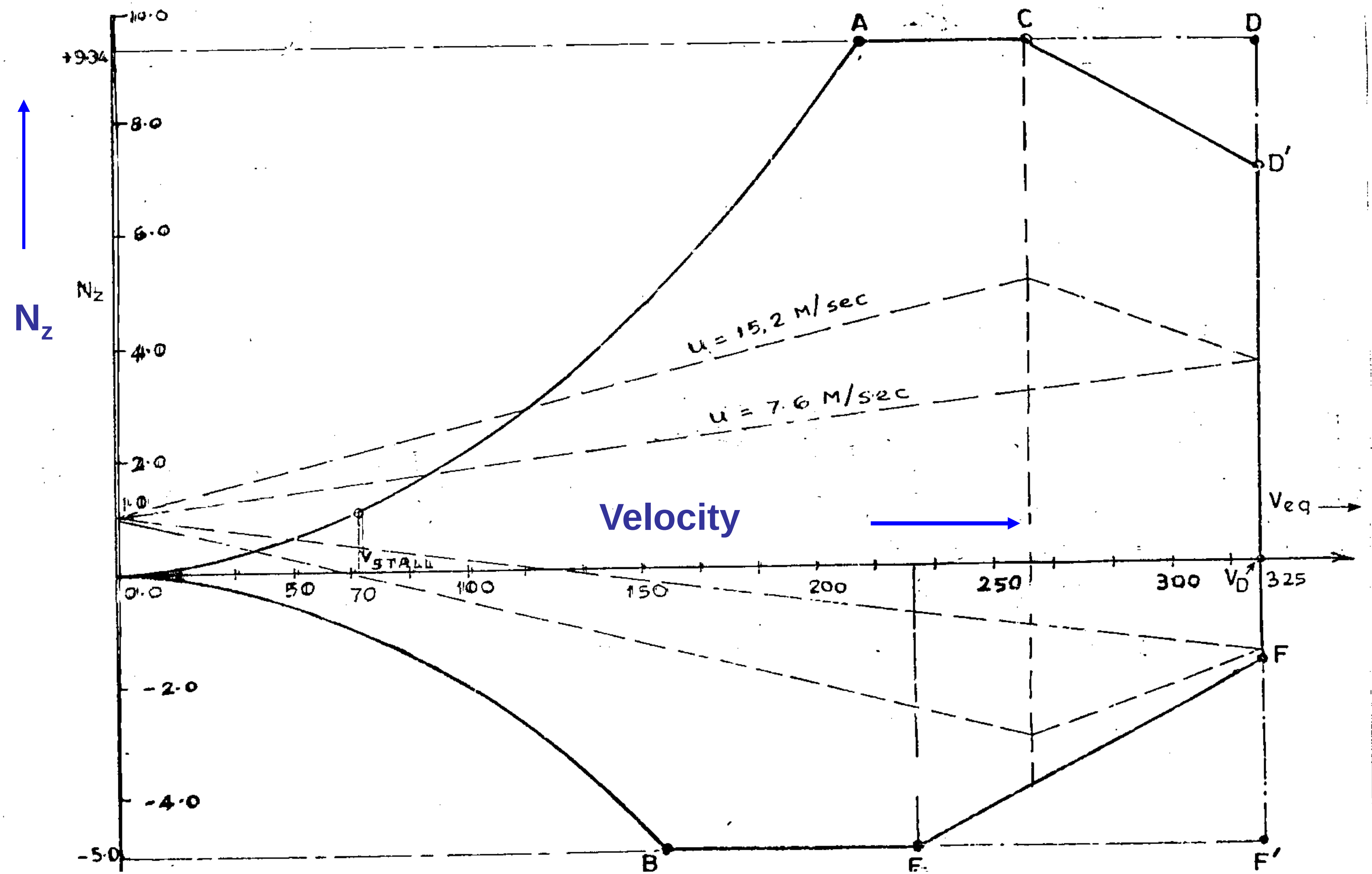

V-N Diagram of HF- 24 (MARUT) A/C (as per AP-970)

V-N diagram is a graph of a/c velocity and the load factor

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Aircraft Load Factors

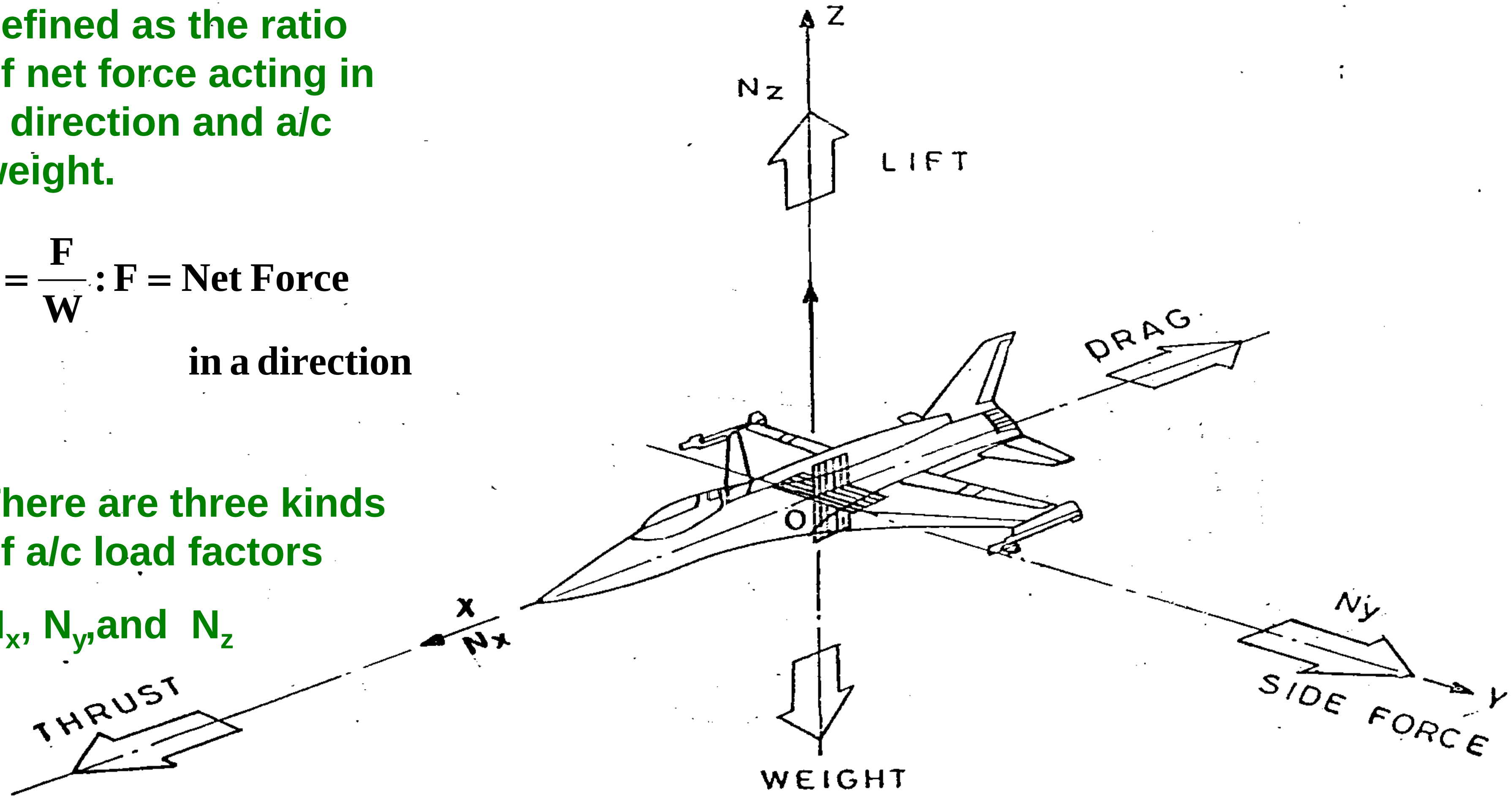
Load factor is defined as the ratio of net force acting in a direction and a/c weight.

$$N = \frac{F}{W} \; : F = \text{Net Force}$$

in a direction

There are three kinds of a/c load factors

N_x, N_y, and N_z



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Some General Points

V-N diagram is applicable only for symmetrical maneuvers in the vertical planes. Why?

Because **N_z has the highest numerical value** and in symmetrical maneuvers in vertical plane **N_x & N_y remain constant.**

V-N diagram is drawn only for N_z. Why?

Because the numerical values of N_x, N_y are small and can't lead to structural damage to a/c if they are too high.

It can be seen that $N_z \alpha V^2$ and (AOA) How?

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Factors that governs the upper limit of N_z

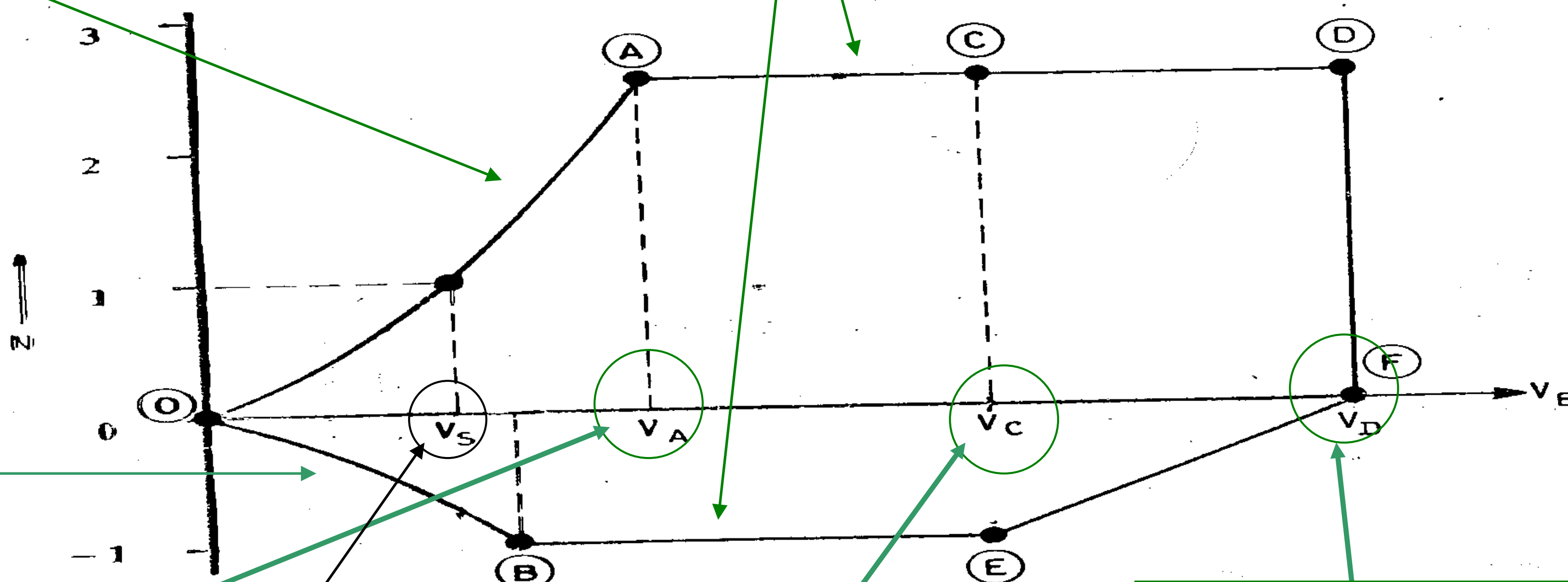
- Structural strength of a/c
  - high N_z means designing the aircraft structure to bear higher loads
- Safety and Comfort of Passengers and Pilot

See this TABLE

Next

Lines AD and BE are externally imposed limits

Parabolic curve refers to stalling angle of attack



Corner speed

V-N diagram as per FAR-23

Design diving spee
(refers to max
.dynamic pressure

$V_D=1.2*V_c$

Design cruising speed

$$F\left(\frac{W}{S}\right)$$

One-g stall speed

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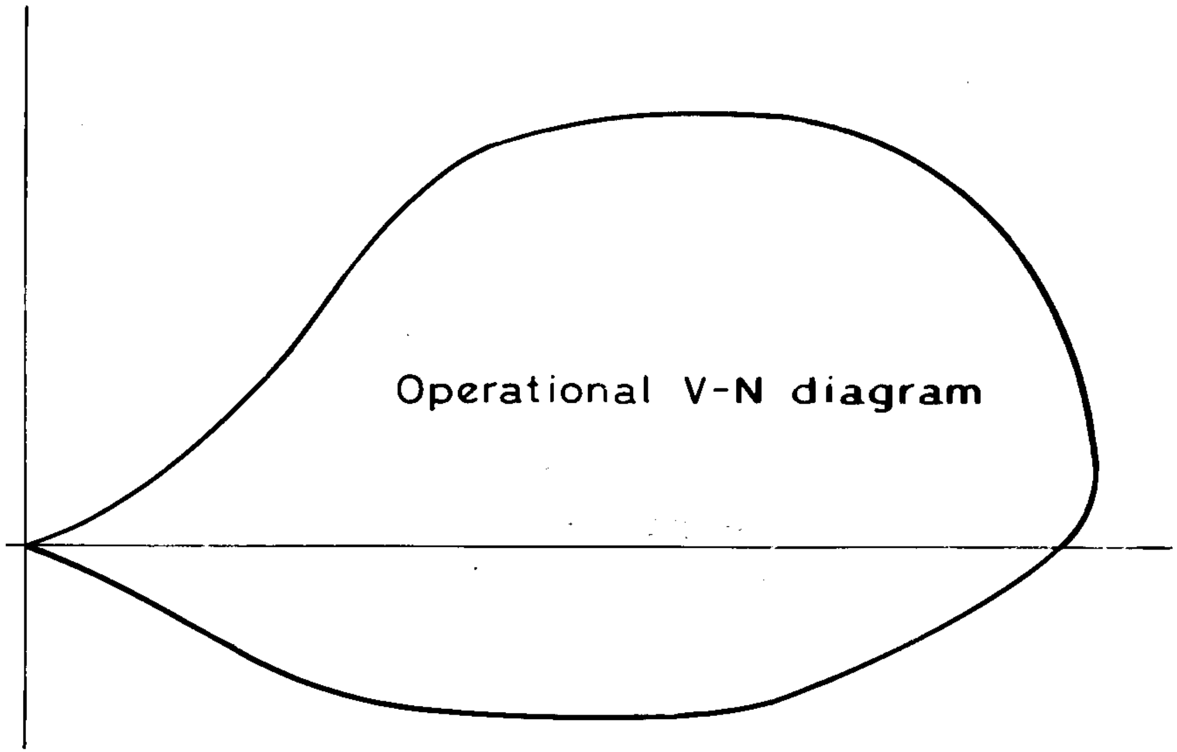

Certain Areas are not operationally possible leading to this "**Operational " V-N Diagram**

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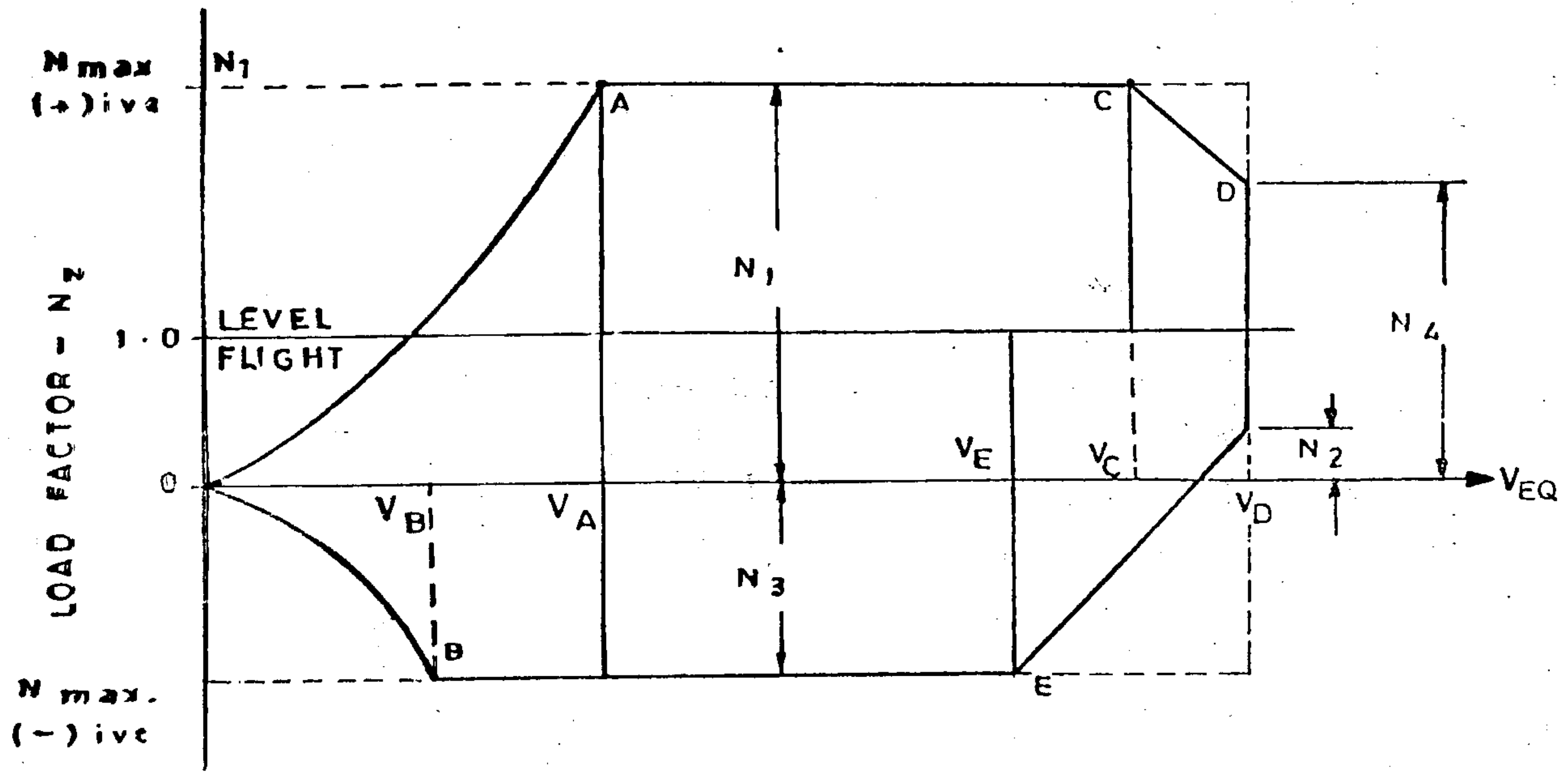

V-N Diagram (AP 970)

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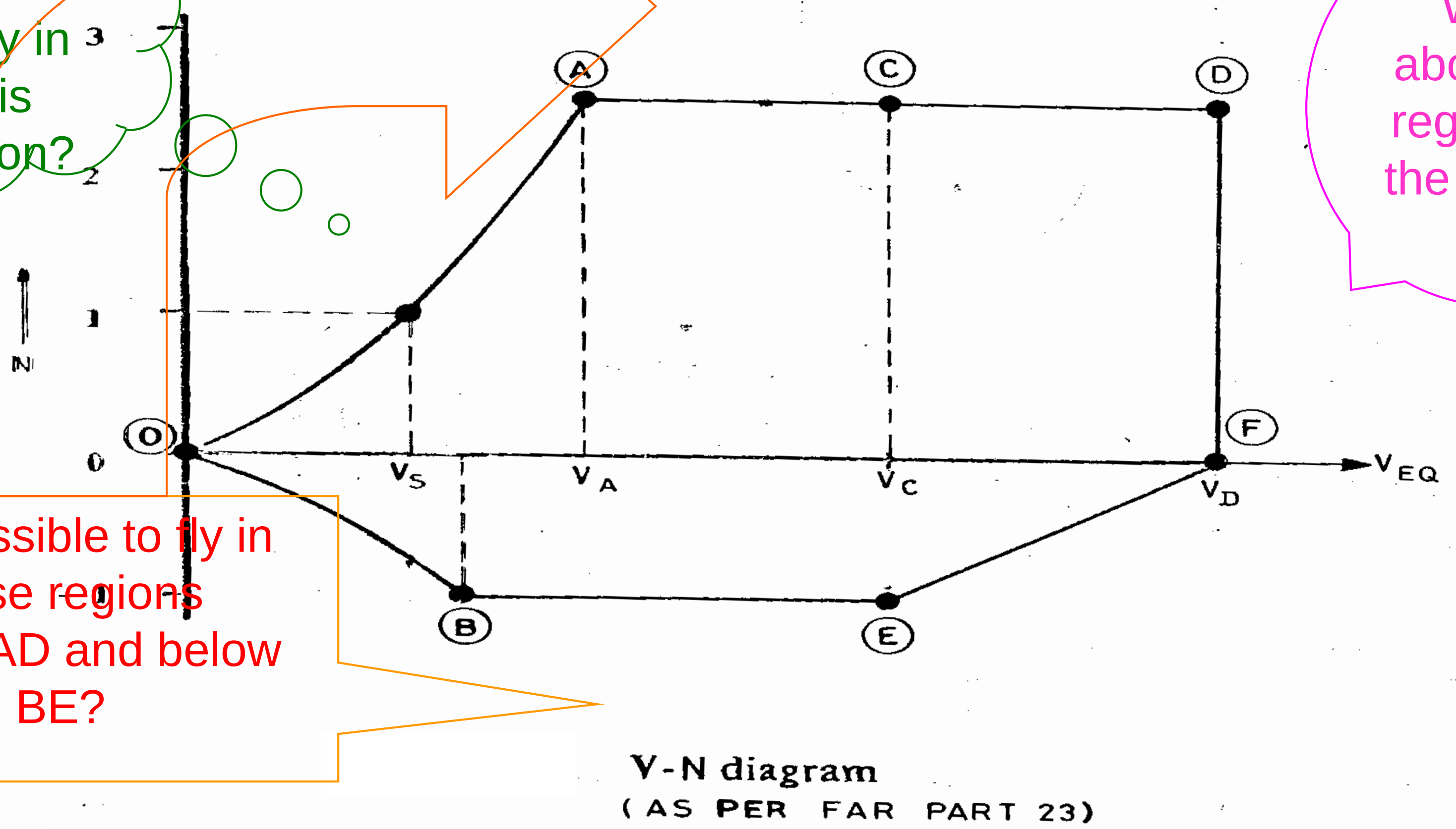
Many airworthiness requirements suggest a cut in upper part of the V-N diag. as well From pt C to line DF because flight is not possible in these regions due to **limitations of power plant**

What happens when pilot exceeds the limits of load factor?



Is it possible to fly in this region?

What about this region? to the right of DF

Is it possible to fly in these regions Above AD and below BE?

Yes, How? ..

Next

Gusts

Next

Effect of Gusts

Gusts are vertical draughts of air, they could be upwards or downwards

They impose additional vertical load factors in an aircraft.

The direction of relative wind is changed by Δα

Next

$$\Delta N_z = \frac{a_0 * V_{Eq} * \rho * V_G * S}{2 * W}$$

Where V_G = Vertical Gust

a_0 = Slope of lift curve

V_{Eq}= Equivalent Velocity

- If the a/c was in level flight than this additional load factor will add to the existing load factor of 1 (level flight)
- The graph of load factor will start from (0,1)
- The airworthiness authorities have specified certain values of gust velocities to be considered in V-N diagram depending on the type of a/c and the altitude of flight.

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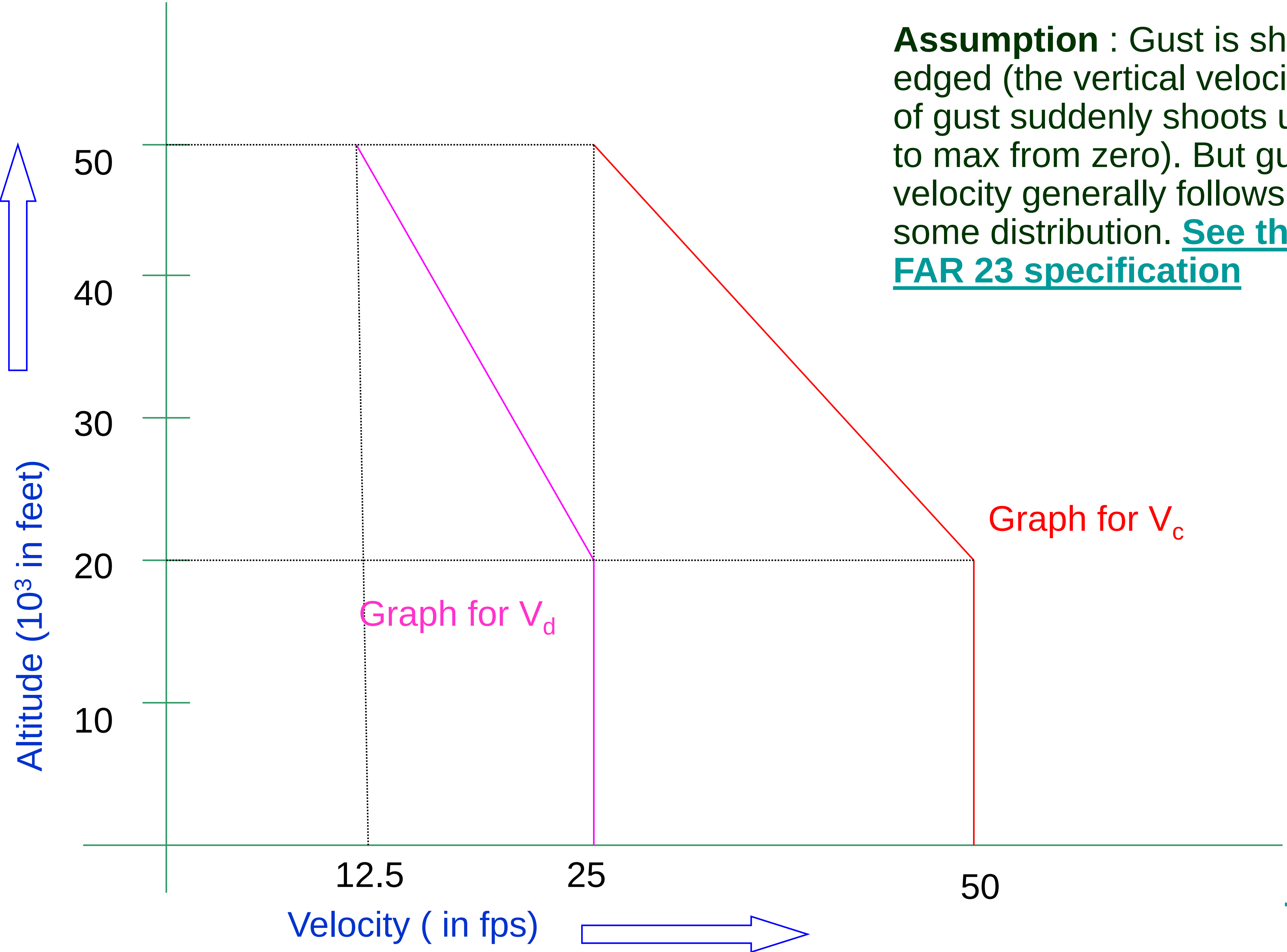
FAR 23 Standard for Gust Velocities



Assumption : Gust is sharp edged (the vertical velocity of gust suddenly shoots up to max from zero). But gust velocity generally follows some distribution. **See the FAR 23 specification**

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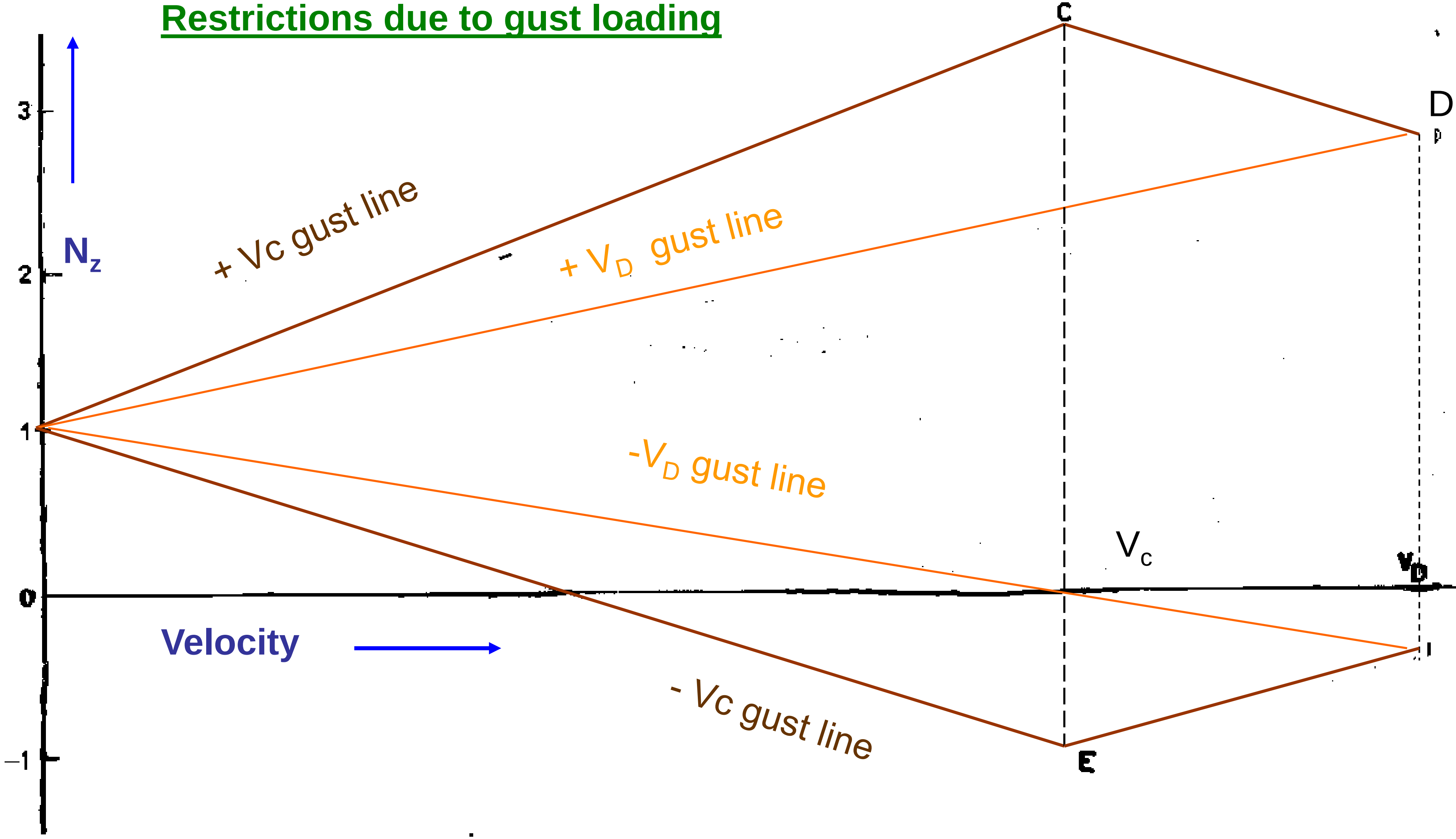
Restrictions due to gust loading



Limit Gust Line

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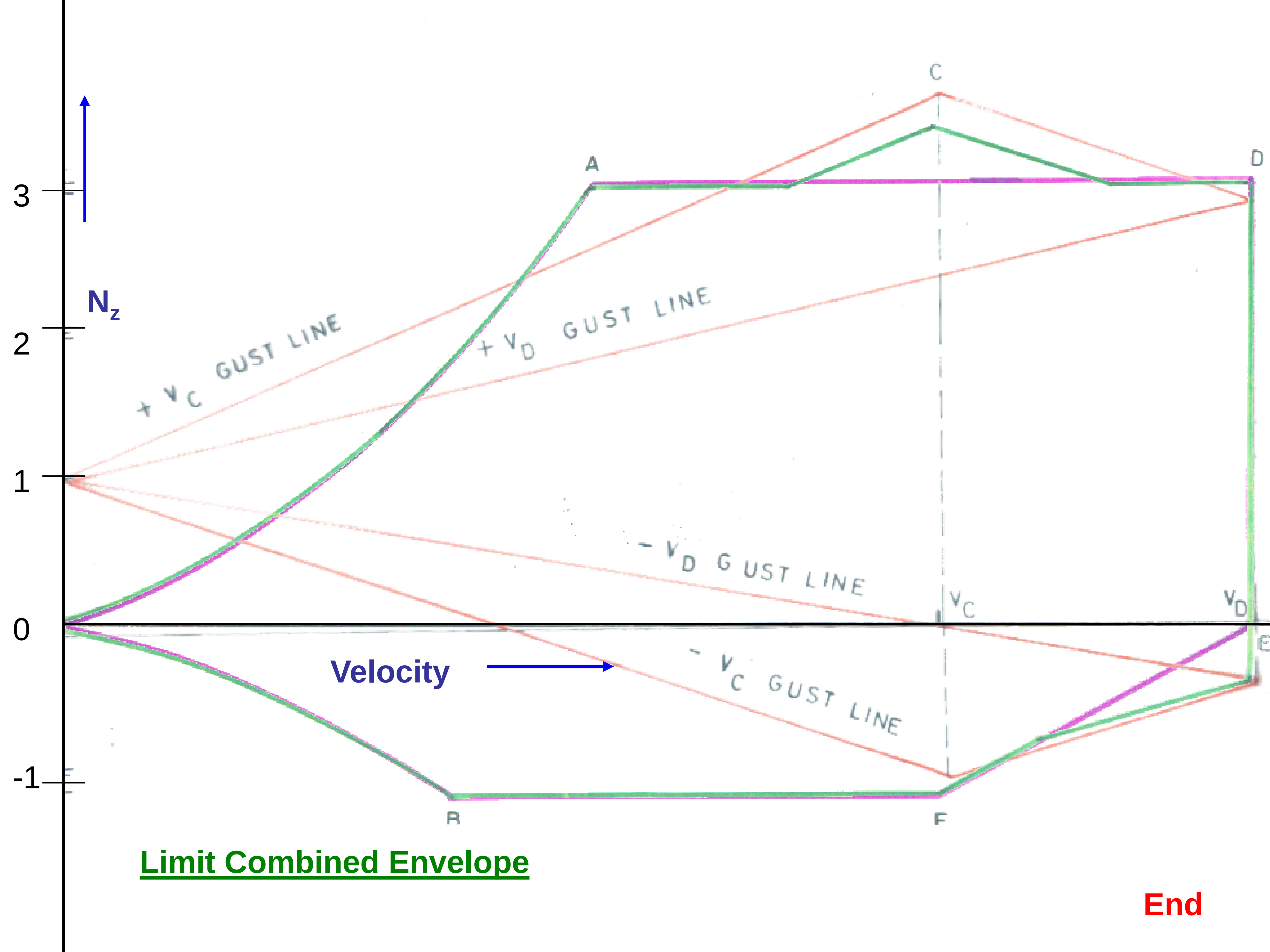

Limit Combined Envelope

End

The End

- Pilot can make the a/c fly in this region if he has enough engine control power
- But it could lead to structural damage as well as health problems to pilots and passengers.
- But during the **Dive-Pull out Manoeuvre** it is possible that pilot may exceed the N_{max} prescribed at the lowest point of the dive

that's why this manoeuvre is called "**checked manoeuvre**"

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Typical Limit Load Factors

Aircraft Type	N(positive)	N(Negative)
General Aviation-normal	2.5 to 3.8	-1 to -1.5
General Aviation-utility	4.4	-1.8
General Aviation-aerobatics	6	-3
Homebuilt	5	-2
Transport	3 to 4	-1 to -2
Strategic Bomber	3	-1
Tactical bomber	4	-2
Fighter	6.5 to 9	-3 to -6

Observe:- N(negative) is almost half of N(positive).

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Equivalent Airspeed is used in calculations instead of True airspeed as found by Pitot-Static tube

- The velocity (True Airspeed [TAS]) indicated by the Airspeed Indicator is proportional to dynamic pressure
- Taking into account the errors in calibrated instruments we get the calibrated airspeed [CAS].
- And after taking into considerations the compressibility effects we get Equivalent airspeed [EAS] (so it is that speed at which the a/c would be flying at sea level under same conditions of pressure and temp.)
- By using this equivalent speed the variable ‘ρ’ can be eliminated
- So $N_z \quad \alpha$ AOA

  $\alpha \quad V_{eq}^2$ ONLY

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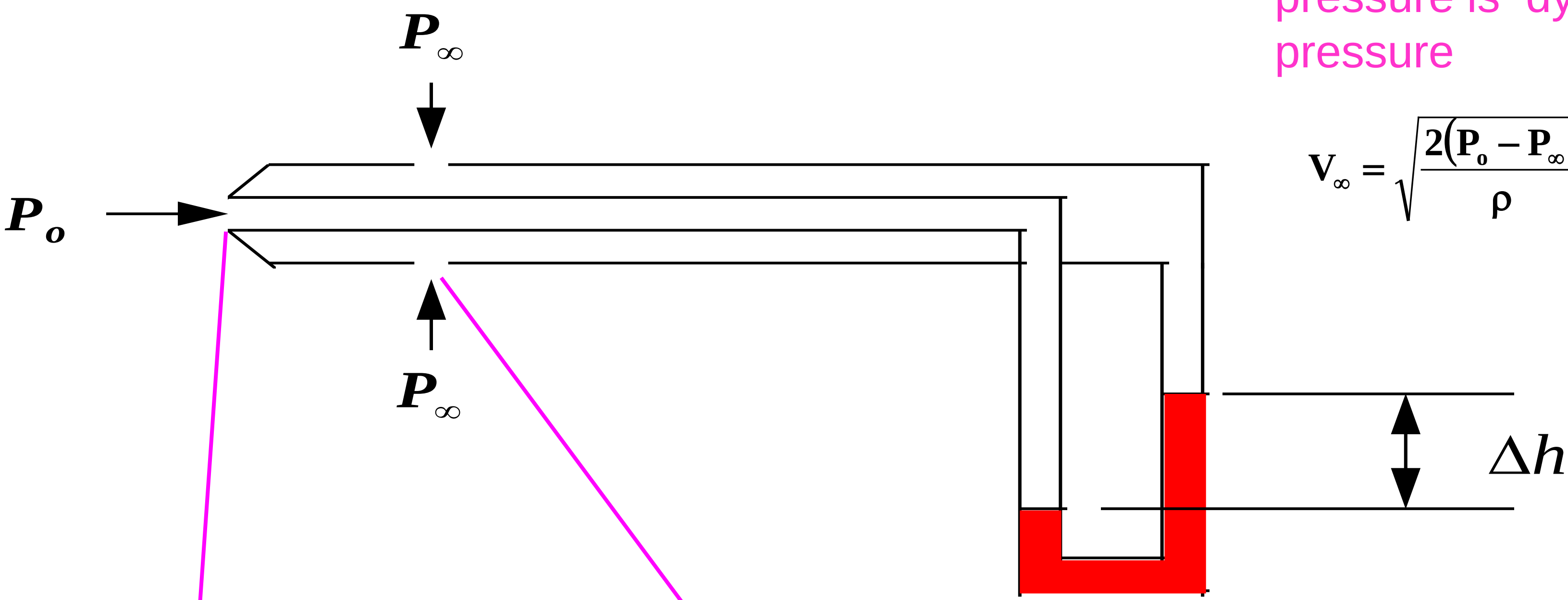
Determining flow speed by Pitot -Static tube

Only for Incompressible flow

Difference between total and static pressure is dynamic pressure

$$V_\infty = \sqrt{\frac{2(P_o - P_\infty)}{\rho}}$$



Pitot – Static Probe

Static pressure is sensed

Total pressure is sensed

Back to Question

- FAR 23 specifies a cosine distribution for the gust shape

$$V_G = \frac{V_{G_{\max}}}{2.0}\left(1-\cos\left(\frac{\pi\delta}{24C_{mean}}\right)\right)$$

where C_{mean} Mean Geometric Chord

δ= [Penetration in gust = 100 ft. or 12 chord lengths (whichever is less)]

- The Gust Alleviation Factor 'K' is specified as follows:-

$$k = \frac{0.88\mu}{5.3+\mu}$$ for subsonic a/c

$$k = \frac{\mu^{1.03}}{6.95+\mu^{1.03}}$$ for supersonic a/c

$$\mu = \frac{2(w/s)}{\rho g C_{mean} a_0}$$ a/c mass ratio

The factor k is multiplied to V_G to give us the effective sharp gust velocity

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Corner Speed

- Point A in the graph is important because it corresponds to highest N_z permissible, and also the max. lift coefficient of a/c.

Implications:-

1. It leads to smallest turn radius (tightest turn)
2. And Fastest turn rate

The speed corresponding to this a/c is called the Design Manoeuvre speed or Corner speed

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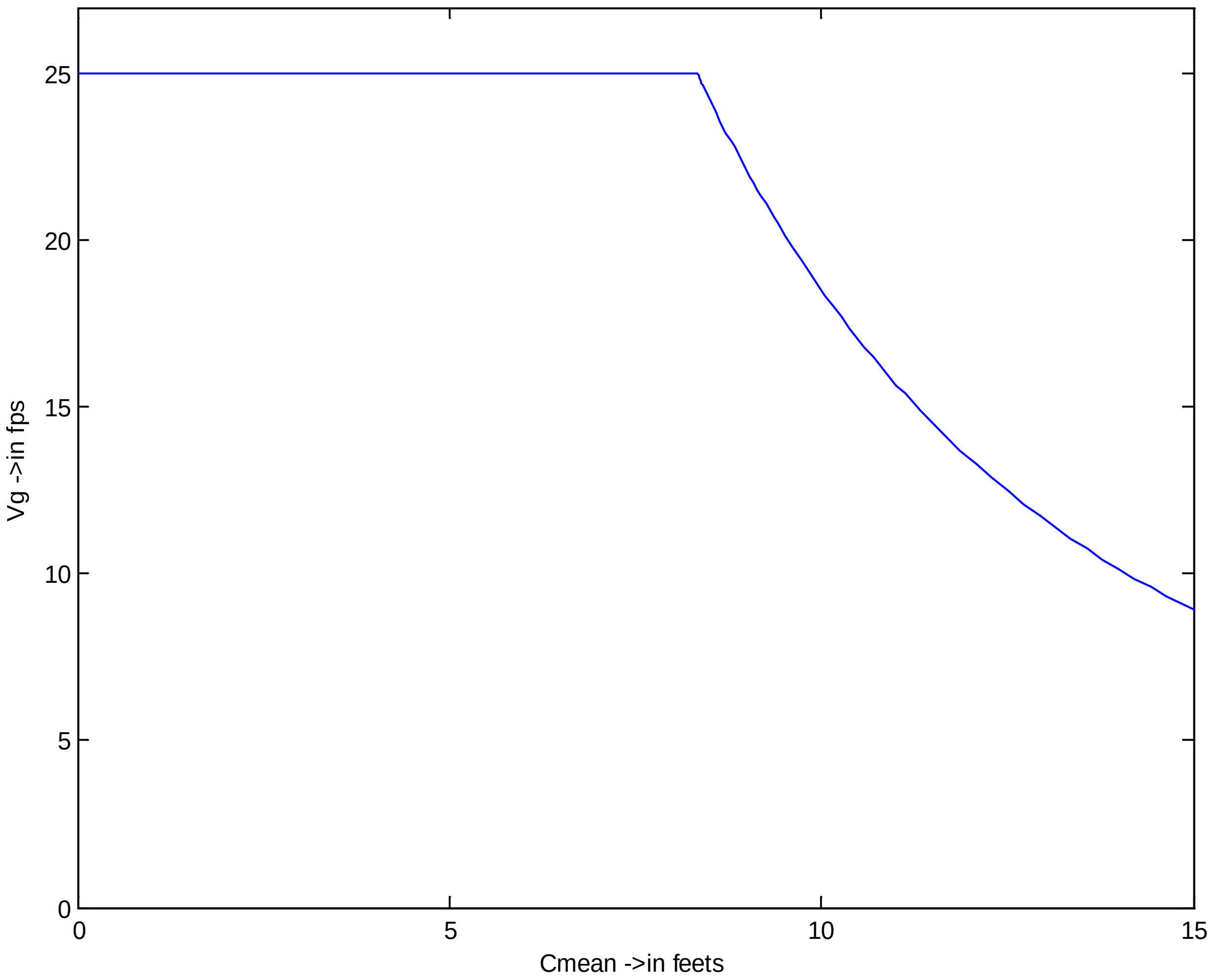
Cosine distribution as per FAR 23 specification

$$V_G = \frac{V_{G_{\max}}}{2.0}\left(1 - \cos\left(\frac{\pi\delta}{24C_{mean}}\right)\right)$$

This distribution is for V_c for altitude between 0-20000 ft.

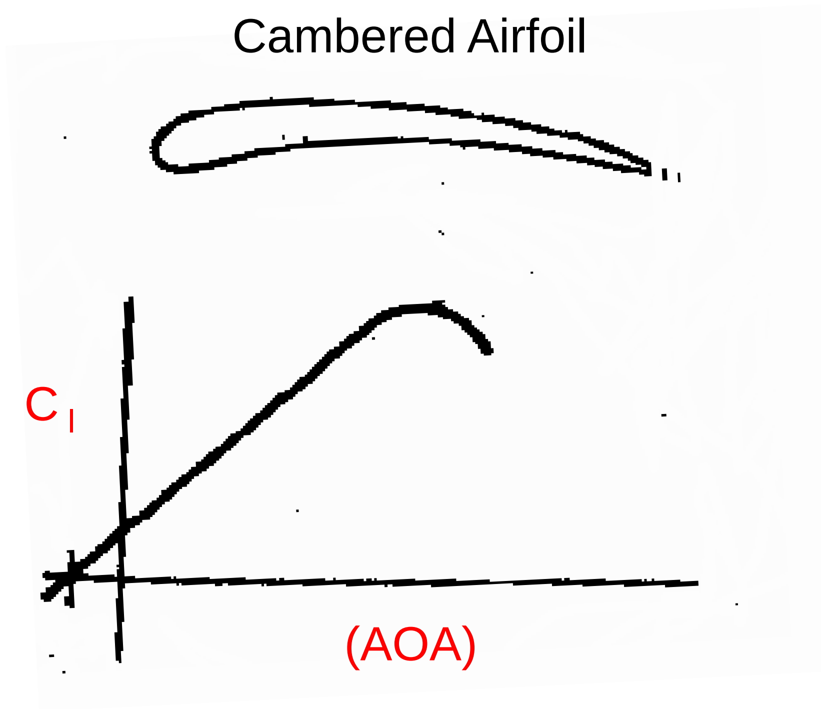


Back

$$N = \frac{L}{W}$$

$L=1/2\rho_{\infty}v^2_{\infty}SC_L$: Lift

$L=1/2\rho_{\infty}v^2_{\infty}S(AOA)a_0$



where

ρ_{∞} =density of air C_l = Lift Coefficient

v = a/c speed S = wing area

a_0 = Lift curve slope

Thus $N_z \; \alpha \; \rho V^2$

and $N_z \; \alpha \; (AOA)$

But this would imply that we need to draw a different V-N diagram for every possible altitude.

So how do we eliminate this problem?

Back to general points