Constraint Analysis Transport Aircraft

Prof. Rajkumar S. Pant
Aerospace Engineering Department
IIT Bombay

Phases in Aircraft Design

- CONCEPTUAL (*Method*)
  - Establish concept feasibility
  - Identify the requirements that drive the design
  - Carry out initial sizing & layout
  - Estimate component masses, performance, and cost
- PRELIMINARY (*Numbers*)
  - Freeze the configuration
  - Ensure design practicality
  - Develop mechanical & structural concepts
  - Develop test and analytical base
- DETAIL (*Nuts & Bolts*)
  - Design various components
  - Develop tooling and fabrication process
  - Test major items
  - Finalize weight and performance estimates

Raymer's Big Six Parameters

- Wing Related
  1. t/c — Thickness Ratio
  2. λ — Taper Ratio
  3. Λ — Sweep
  4. AR — Aspect Ratio
- Aircraft Related
  5. W/S — Wing Loading
  6. T/W (or P/W) — Thrust (Power) Loading

Importance of W/S and T/W

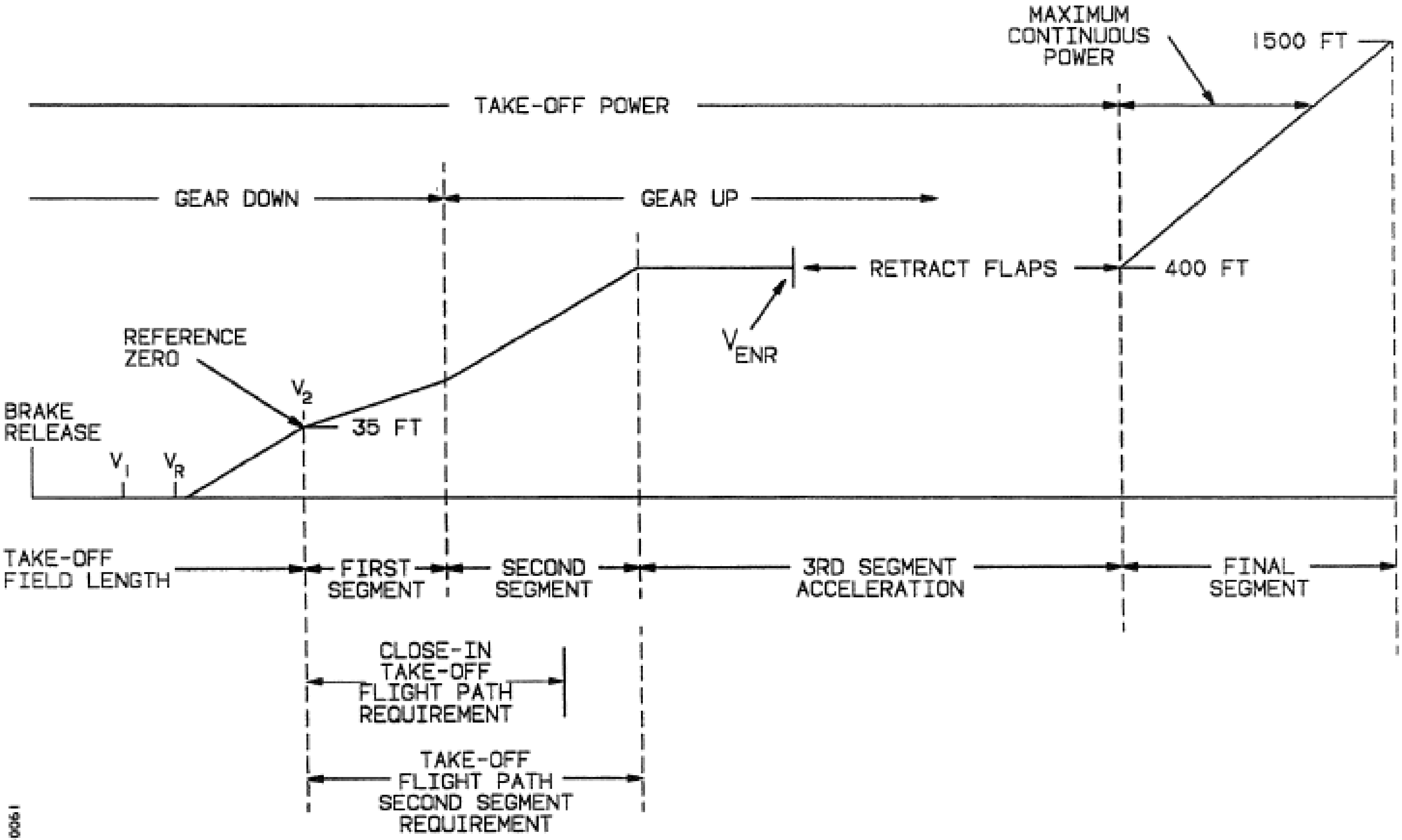
Appear in many performance equations

- W/S and T/W
  - Take-off Distance
  - Sustained Turn rate
  - Range and Endurance
  - Climb performance
- W/S alone
  - Stalling speed
  - Landing Distance
  - Ceiling
  - Instantaneous Turn
- T/W alone
  - Missed Approach Gradient
  - Climb Gradient

Takeoff & Landing Climb Gradients

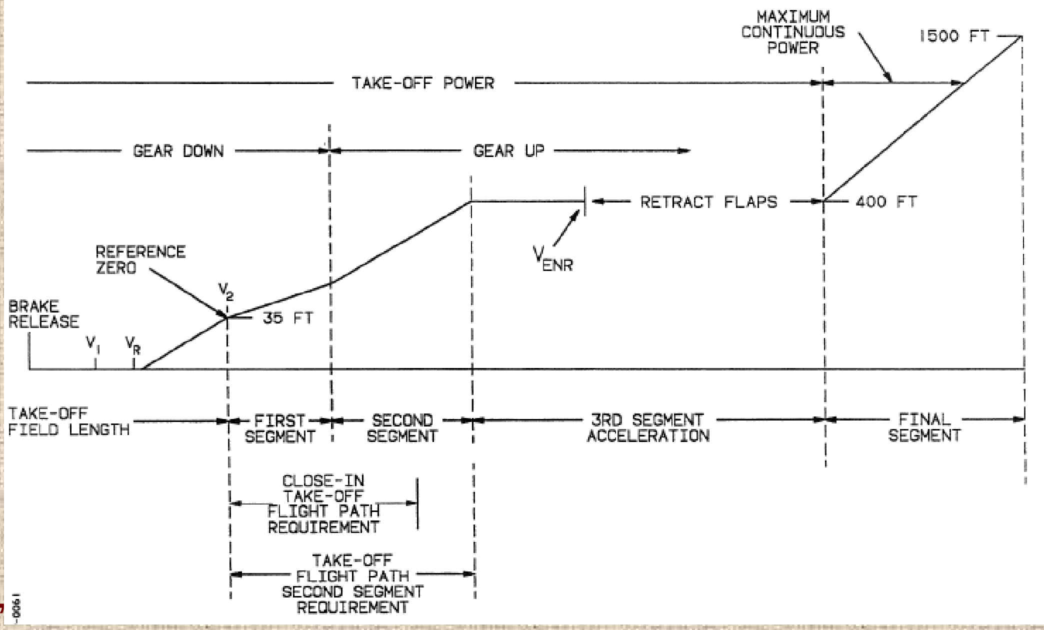
A brief explanation

A Closer Look at Take Off



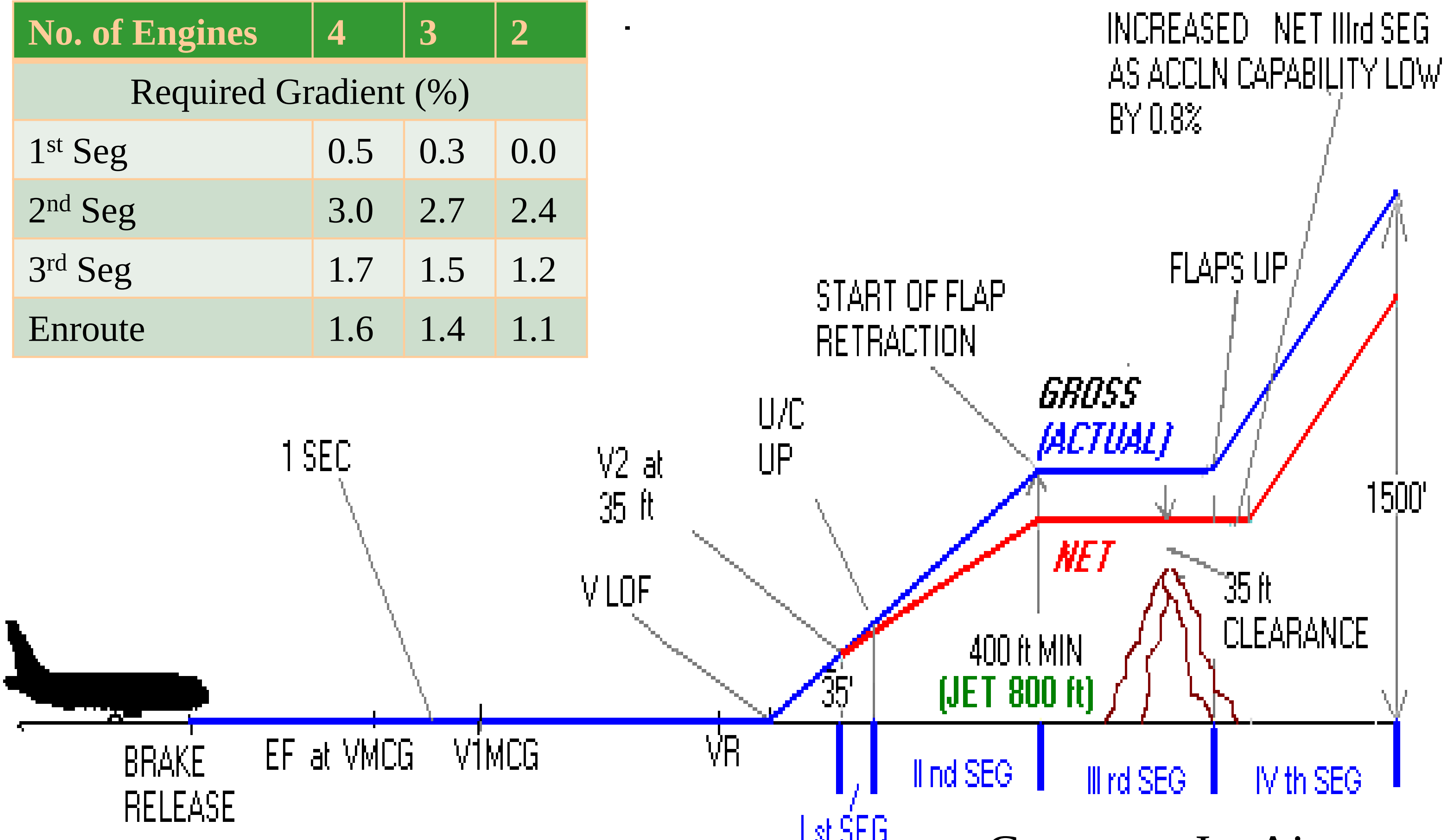
The Four Climb Segments

- **First Segment:**
  - From the end of the takeoff distance to the point the landing gear is fully retracted. (Speed = V_2)
- **Second Segment:**
  - The point where the landing gear is retracted to an altitude of at least 400’ (obstacle dependent). (Speed = V_2)
- **Third (Transition) Segment:**
  - The horizontal distance required to accelerate at a constant altitude to facilitate flap/slat retraction and acceleration to final climb speed.
- **Final Segment:**
  - End of third segment to at least 1500’ (obstacle dependent) with flaps/slats retracted, max. continuous power, and final climb speed.



Typical Takeoff Segments

No. of Engines	4	3	2
Required Gradient (%)			
1^{st} Seg	0.5	0.3	0.0
2^{nd} Seg	3.0	2.7	2.4
3^{rd} Seg	1.7	1.5	1.2
Enroute	1.6	1.4	1.1



Courtesy: Jet Airways

Detailed Definition of Climb Segments

The Takeoff Distance (TOD) ends at a 35 ft 'screen height' above the runway (15ft for Wet runway, 50 ft if a turning maneuver is involved), and at V2 speed. At the end of the TOD, the Gear is assumed to be DOWN.

The 1st Segment begins at the end of the TOD, and ends when the Gear is UP. Speed remains at V2.

The 2nd Segment begins at the end of the 1st Segment with the Gear UP, Power / Thrust at Takeoff, and the aircraft in the Takeoff configuration. The 2nd segment ends at 400 ft above Aerodrome Elevation, or higher if required for obstacle clearance. The 2nd segment is flown at V2, or limited to V2 plus an allowed margin.

The 3rd Segment begins at the end of the 2nd Segment climb with the Gear UP, Power / Thrust at Takeoff, and the aircraft in the Takeoff configuration. The 3rd segment is a level accelerating segment, where Flaps / Slats are retracted, and the aircraft accelerated to the Final Takeoff (Clean) speed, and Power / Thrust then reduced to Maximum Continuous.

The 4th Segment begins at the end of the 3rd Segment level acceleration segment with the aircraft in the Clean configuration, and Maximum Continuous Power / Thrust set. The 4th segment ends at 1500 ft, or higher if required for obstacle clearance.

Variation (1) - It is possible that the gear is retracted by the time that the aircraft has reached 'screen height' at the end of the Takeoff, in which case no 1st segment exists. This is not a typical case, but does exist.

Variation (2) - It is possible that the time limit for Takeoff thrust may be reached before the 3rd segment is complete, although this is becoming rare with increasing availability of a 10 minute limit. It is then necessary to re-evaluate whether the aircraft has the performance capability to accomplish the acceleration with MCT, and re-assess the length of the 3rd segment. Manufacturer's data is rarely available for this, necessitating an alternative steeper 2nd segment climb, higher than dictated by obstacles, to reach the 3rd segment in a shorter time, leaving sufficient Takeoff Power / Thrust availability to accomplish the 3rd segment.

Source: http://www.pprune.org/archive/index.php/t-95292.html

Second Segment Climb Gradient

- **FAR-25 requirement:**
  - *Sufficient thrust must be installed in the aircraft so that in the event of an engine failure, the following minimum gradient may be sustained, with flaps in the take-off position, but with the landing gear retracted*

γ_{SSCG}	No.of Engines
3.0 %	Four
2.7 %	Three
2.4 %	Two

Missed Approach Gradient

- *The situation in which the aircraft is on final approach to a landing but does not land for one of several reasons; instead, power is applied and the aircraft climbs, usually to circle the airport and initiate another landing approach.*
- **FAR-25 Regulation:** *Sufficient thrust for the aircraft to climb at a specified gradient (γ_{MA}) with one engine inoperative and at maximum landing weight.*

γ_{MA} = 2.7 % for four-engine aircraft
= 2.4 % for three-engine aircraft
= 2.1 % for two-engine aircraft

Where, $\gamma_{MA} = (T/W - D/W)V$ during the climb

Two possible approaches

- Approach No 1
  - Estimate T/W from constraints on
    - Missed Approach Gradient or Take off Climb Gradient
  - Estimate W/S meeting other constraints
- Approach No 2
  - Estimate W/S from constraints on
    - Stalling speed, Landing Distance, Ceiling, ….
  - Estimate T/W meeting other constraints
- Approach No 1 generally preferred, **WHY ?**
  - Less variability in T/W from past data
  - Constraints on gradients easy to implement

Approach-1
Estimation of T/W from specified constraints

Estimation of T/W

- T/W directly affects aircraft performance
  - High T/W
    - ☺ Higher V_{cr}, acceleration, ROC, sustained turn rate
    - ☹ Higher Fuel Consumption, heavier aircraft
- T/W keeps changing during mission
  - Lower W due to fuel consumption
  - T (or P) changes due to changes in H, V and η_p
- Design T/W
  - ISA, SLS , at W_{gross} , max. throttle setting
  - Calculated T/W to be adjusted to Design T/W

Typical values of T/W and P/W

- Jet Engined a/c (T/W)
- Dimensionless
  - Civil

Transport (2 eng)	0.2
Transport (3 eng)	0.3
Transport (4 eng)	0.4

  - Military

Strategic Bomber	0.2
Tactical Bomber	0.3
Trainer	0.4
Fighter	0.6
Interceptor	0.9
Air Superiority	> 1.0

- Prop. aircraft (P/W)
- Units = Watts/g
  - Civil

Powered Sailplane	0.07
G.A. (1 eng)	0.12
Homebuilt	0.13
Agricultural	0.15
Flying Boat	0.16
G.A. (2 eng)	0.30
Twin Turboprop	0.33
Aerobatic	0.45

  - Military

Bomber	0.35
Cargo	0.40

T/W or P/W as a function of M_{max} or V_{max}

$T/W_0 = a\, M_{max}^C$	a	C
Jet trainer	0.488	0.728
Jet fighter (dogfighter)	0.648	0.594
Jet fighter (other)	0.514	0.141
Military cargo/bomber	0.244	0.341
Jet transport	0.267	0.363

$P/W_0 = a\, V_{max}^C$: hp/lb or {Watt/g}	a	C
Sailplane—powered	0.043 {0.071}	0
Homebuilt—metal/wood	0.005 {0.006}	0.57
Homebuilt—composite	0.004 {0.005}	0.57
General aviation—single engine	0.025 {0.036}	0.22
General aviation—twin engine	0.036 {0.048}	0.32
Agricultural aircraft	0.009 {0.010}	0.50
Twin turboprop	0.013 {0.016}	0.50
Flying boat	0.030 {0.043}	0.23

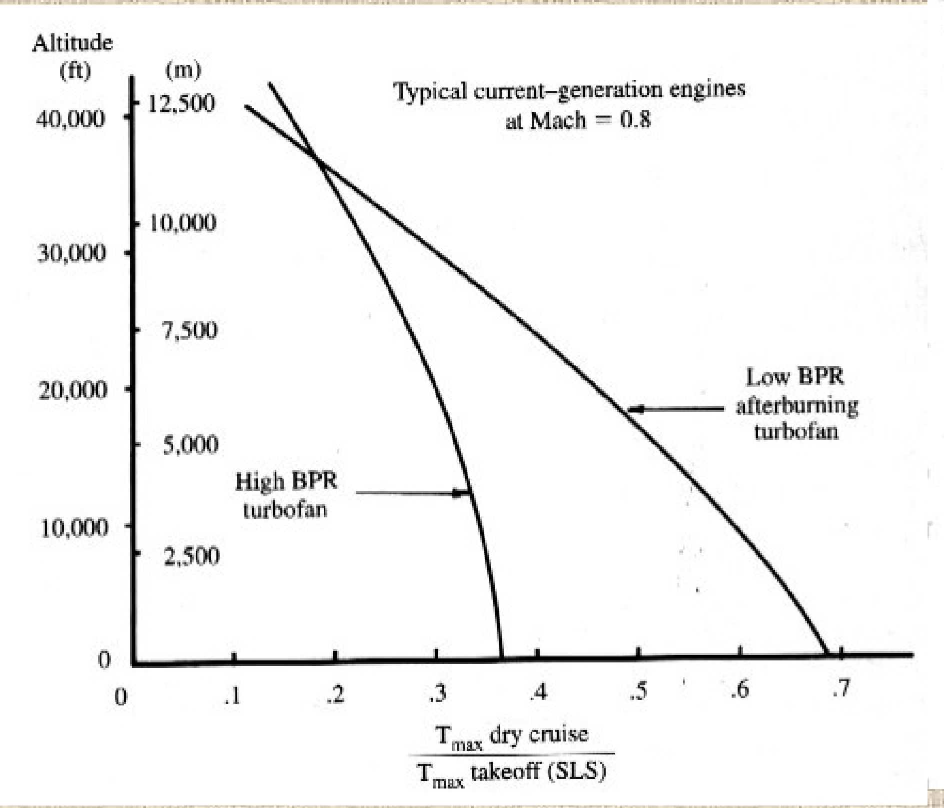
Source: Daniel P Raymer, *Aircraft Design, A Conceptual Approach*, AIAA Publications

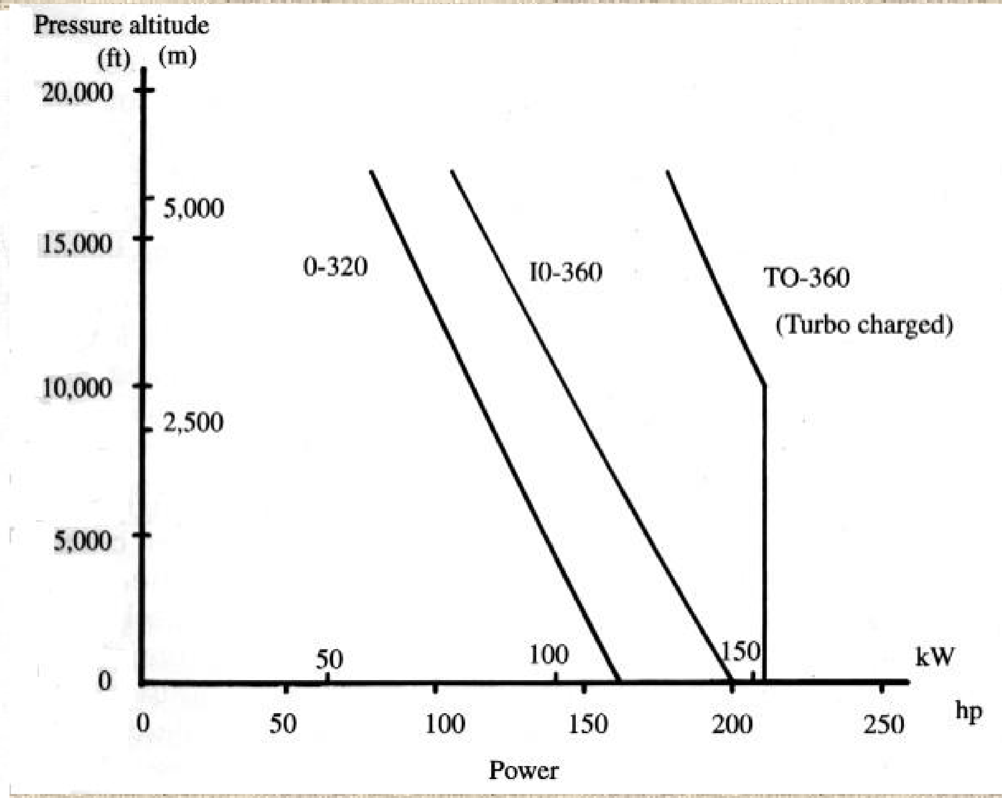
T/W from
Cruise Considerations

Thrust Matching from Cruise Considerations

- $\left(\frac{T}{W}\right)_{cr}$ can be estimated from $\left(\frac{L}{D}\right)_{cr}$
- $\left(\frac{T}{W}\right)_{cr} = \dfrac{a}{\left(\frac{L}{D}\right)_{cr}}$

$$\left(\frac{T}{W}\right)_{TO} = \left(\frac{W_{cr}}{W_{TO}}\right)\left(\frac{T_{TO}}{T_{cr}}\right)\left(\frac{T}{W}\right)_{cr}$$





Source: Daniel P Raymer, *Aircraft Design, A Conceptual Approach*, AIAA Publications

T/W from
Climb Gradient Considerations

FAR 25 requirements

Segment	Engine	Thrust	LG	Flaps	Speed	Weight	Altitude
1^{st}	OEI	Takeoff	Down	Takeoff	V_{lo}	Takeoff	35 ft
2^{nd}	OEI	Takeoff	Up	Takeoff	$1.2V_s$	Takeoff	35 – 400 ft
3^{rd}	OEI	Max. Cont.	Up	Up	$1.25\ V_s$	End of Takeoff	400 ft
4^{th} Enroute	OEI (2) 2EI (3/4)	Max. Cont.	Up	Up	Any	End of Seg 3	Clear Obstacles
Approach	OEI	Takeoff	Up	< Landing	$1.5\ V_s$	Landing	
Landing	AEO	8 sec after idle to Takeoff	Down	Landing	$1.3\ V_s$		

T/W from Constraint on Climb Gradient

- Climb Gradient = Excess Thrust / Weight
- If desired Climb Gradient is specified then

$$\left(\frac{T}{W}\right)_{cl} = \frac{1}{\left(L/D\right)_{cl}} + \frac{V_{ver}}{V}$$

- The two major climb gradient constraints are
  - Missed Approach
  - 2[nd] Stage Climb

Constraint on Missed Approach Gradient

$$\frac{T}{W} = \frac{N}{N-1} \cdot \left\{ \frac{1}{L/D} + \gamma_{MA} \right\}$$

Where, N = number of engines

L/D = aircraft L/D during missed approach

Flaps in approach configuration

Landing Gear Down

γ_{MA} = required Missed approach gradient

Constraint on γ_{MA} puts a lower limit on T/W

Constraint on 2nd stage Climb Gradient

$$\frac{T}{W} = \frac{N}{N-1} \cdot \left\{ \frac{1}{L/D} + \gamma_{SSCG} \right\}$$

Where, N = number of engines

L/D = aircraft L/D in 2nd Stage climb configuration

Flaps in Takeoff configuration

Landing gear up

γ_{SSCG} = Required Missed approach gradient

Constraint on γ_{SSCG} puts a lower limit on T/W

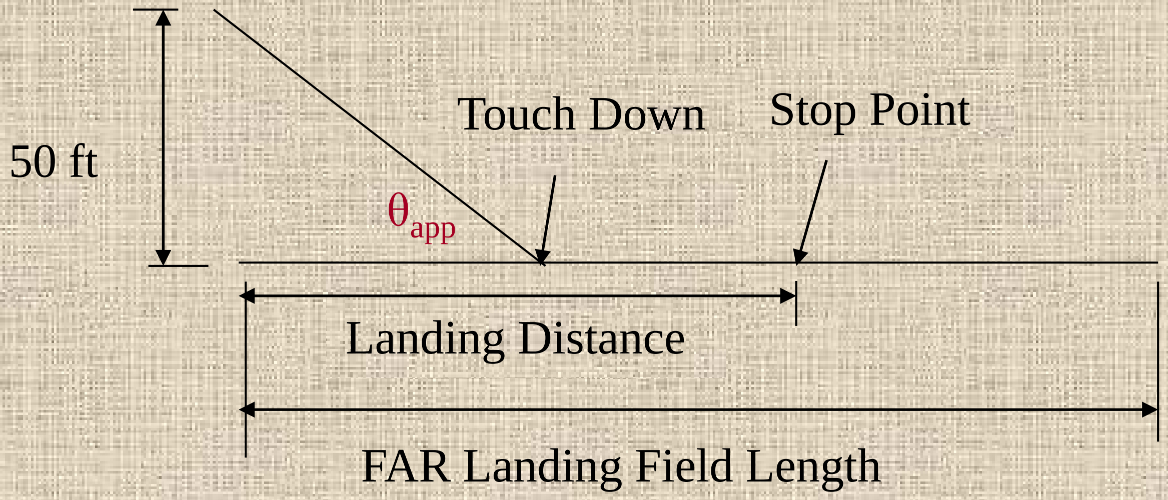
Approach-2
Estimation of W/S from specified constraints

Stalling Speed Constraint

- W/S directly affected by V_{stall} $\quad W/S = \frac{1}{2}\rho V^2_{stall} C_{L_{\max}}$
- Airworthiness requirements for operational safety
  - $V_{TO} \geq 1.1\ V_{stall}$, $V_{CL} \geq 1.2\ V_{stall}$ {Military & Civil Aircraft}
  - $V_{app} \geq 1.2\ V_{stall}$,$V_{TD} \geq 1.1\ V_{stall}$ { Military Aircraft}
  - $V_{app} \geq 1.3\ V_{stall}$,$V_{TD} \geq 1.15\ V_{stall}$ { Civil Aircraft}
- Airworthiness regulations
  - $V_{stall} \leq 61$ kt for single engined FAR-23 certified aircraft
- Determination of $C_{L,max}$ is important
- Constraint on V_{stall} puts upper limit on W/S

Landing Distance Constraint

- Raymer's approx. formula $$S_{landing}(ft) = 80\left(\frac{W}{S}(\frac{lb}{ft^2})\right)\left(\frac{1}{\sigma C_{L_{max}}}\right) + S_a(ft)$$
- S_a = approach distance, depends on θ_{app} and H_{obs}
  - o = 1000 ft for transport a/c with 3 deg. glideslope
  - o = 600 ft for G. A. a/c with power off approach
  - o = 450 ft for STOL a/c with 7 deg. glideslope
- FAR-25 LFL = 1.67 * Landing Distance



Constraint on LFL puts an upper limit on W/S

Constraint on Ceiling

- At Ceiling, Excess power = 0, i.e.,T = D ➔ G = 0
- For Optimum Cruise of Prop. a/c, $$qSC_{D_0} = qS\frac{C_L^{\ 2}}{\pi Ae}$$
- Thus, $\left(W/S\right) = q\sqrt{\pi AeC_{D_0}}$
  - e.g. if h = 30 km, M = 0.8, A = 8, C_{D0} = 0.015, e = 0.85
    - o ρ = 0.01786 kg/m^3, a = 295 m/s, **ESTIMATE V, q and W/S**
    - o V = 236.1 m/s, q = 497.7 N/m^2 and W/S = 281.5 N/m^{2,} **highly impractical !**
- In such cases, W/S estimated from level flight $$W/S = qC_L$$
  - C_L = 0.95 to 1.0 for high altitude a/c
- **Constraint on Ceiling puts a lower limit on W/S**

Constraints that depend on both W/S and T/W

Constraint from Climb Requirements

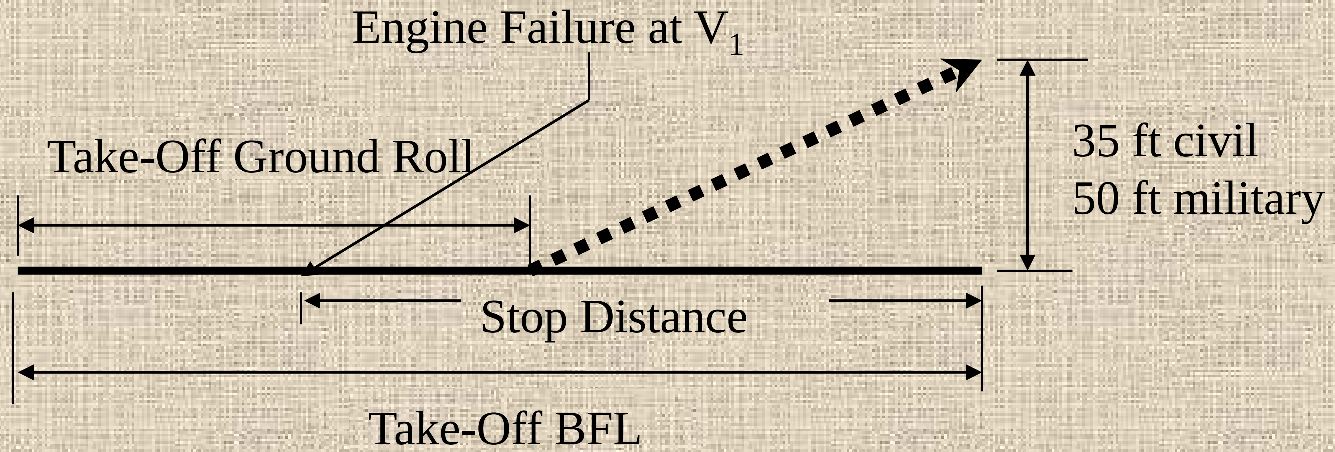
- Rate of Climb = vertical velocity
- Climb Gradient $G = (T - D)/W$
- Thus $\dfrac{D}{W} = \dfrac{T}{W} - G$ $\quad \dfrac{D}{W} = \dfrac{qSC_{D_0} + qS\left(C_L^{\,2}/\pi Ae\right)}{W} = \dfrac{qC_{D_0}}{W/S} + \dfrac{W}{S}\dfrac{1}{\pi Ae}$

$$\frac{W}{S} = \frac{\left[\left(T/W\right) - G\right] \pm \sqrt{\left[\left(T/W\right) - G\right]^2 - \left(4C_{D_0}/\pi Ae\right)}}{2/q\pi Ae}$$

- Interesting Observation $\dfrac{T}{W} \geq G + 2\sqrt{\dfrac{C_{D_0}}{\pi Ae}}$

Constraint on Take-Off Field Length

- Definition of Takeoff Balanced Field length
- V_1 = Decision Speed

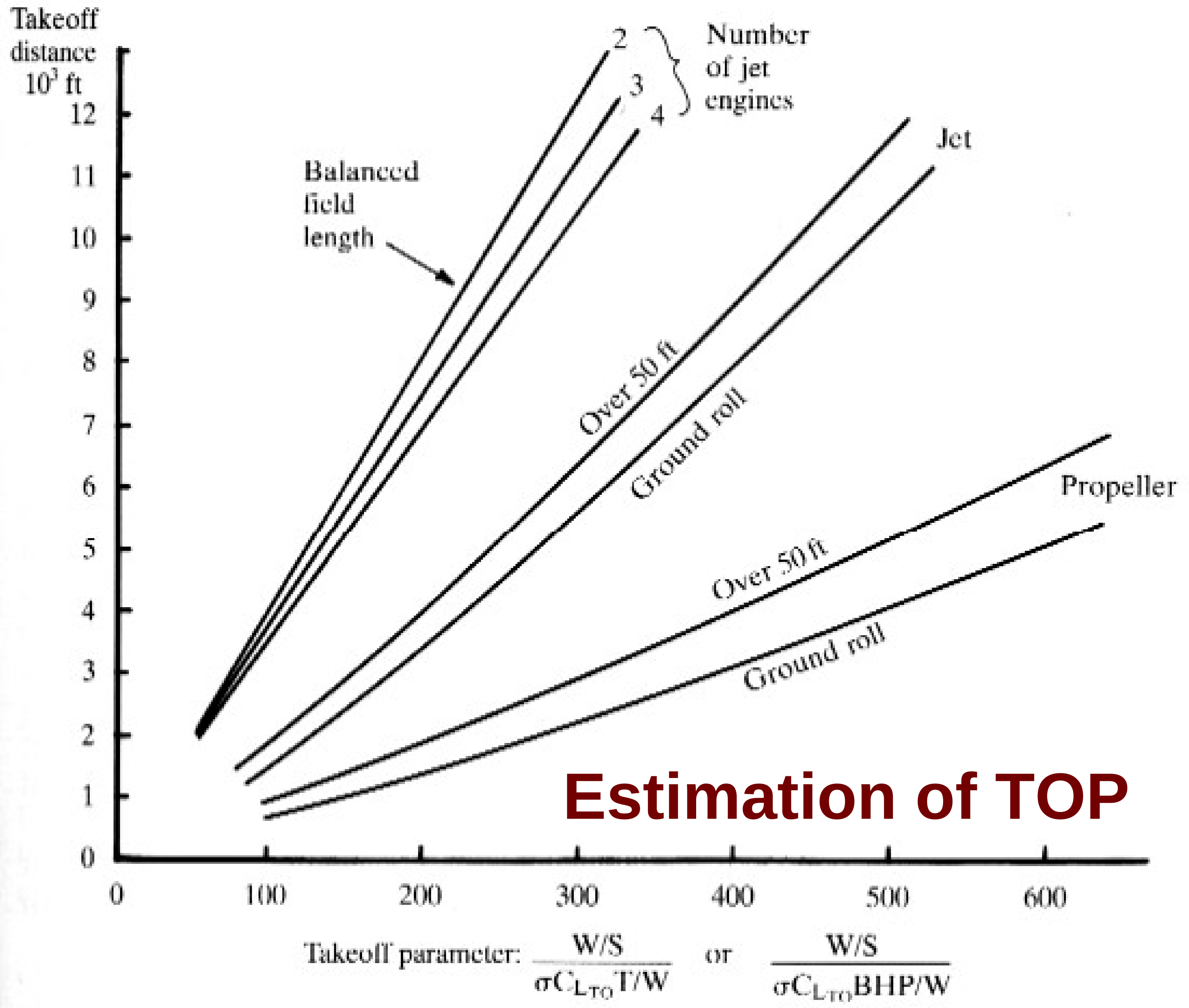


Note: FAR-25 Obstacle height = 50 ft for military aircraft

TO Ground Roll Constraint

- Take Off Ground Roll = $\frac{{V_{LO}}^2}{2a}$, where a = acceleration
- Lift Off Velocity V_{LO} = $\sqrt{\frac{2W}{\rho\, SC_{L,LO}}}$ and $C_{L,LO} = \frac{C_{L,max}}{1.21}$
- F = m. a and hence T = (W/g).a, hence $\frac{T}{W} = \frac{a}{g}$
- Thus, TOGR $= \frac{k\, {}^{W}\!/_{S}}{\sigma\, C_{L,LO}\, {}^{T}\!/_{W}}$, where $\sigma = \frac{\rho}{\rho_0}$ and $k = \frac{1}{g\, \rho_0}$
- Let us define Takeoff Parameter = $TOP = \frac{TOGR}{k}$
- Hence, $\frac{W}{S} = \mathrm{TOP}\ \sigma\ C_{L,TO}\ \frac{T}{W}$ or $\mathrm{TOP}\ \sigma\ C_{L,TO}\ \frac{hp}{W}$

Estimation of TOP



Source: Daniel P Raymer, *Aircraft Design, A Conceptual Approach,* AIAA Publications

Optimum W/S from Cruise and Loiter

Propeller Aircraft

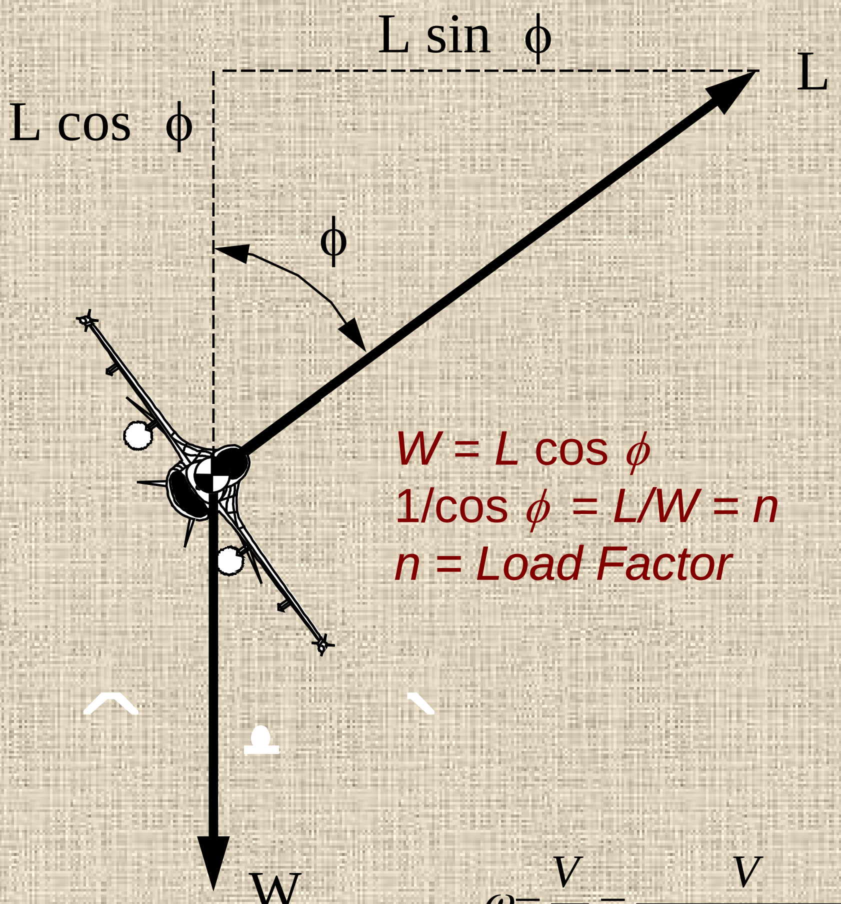
- Max. Range → $C_{Di} = C_{Do}$
- L/D = L/D max
- $\frac{W}{S} = q\sqrt{\pi \, A \, e \, C_{D0}}$
- Max. Loiter→ $C_{Di} = 3.C_{Do}$
- L/D = 0.866 L/D_{max}
- $\frac{W}{S} = q\sqrt{3\pi \, A \, e \, C_{D0}}$

Jet Aircraft

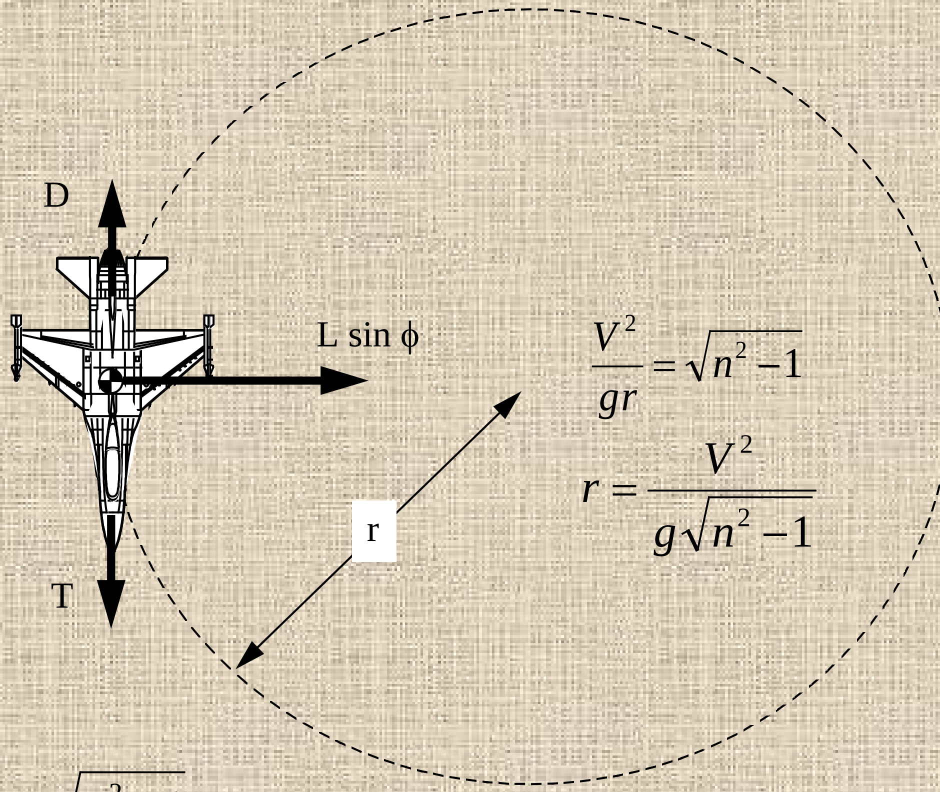
- Max. Range→ $C_{Di} = C_{Do}/3$
- L/D = 0.866 L/D_{max}
- $\frac{W}{S} = q\sqrt{\pi \, A \, e \, \frac{C_{D,o}}{3}}$
- Max. Loiter → $C_{Di} = C_{Do}$
- L/D = L/D max
- $\frac{W}{S} = q\sqrt{\pi \, A \, e \, C_{D,o}}$

These optimal values do not usually impose constraints

Turning Performance in Level Flight



W = L cos ϕ
1/cos ϕ = L/W = n
n = Load Factor



$$\frac{V^2}{gr} = \sqrt{n^2 - 1}$$

$$r = \frac{V^2}{g\sqrt{n^2-1}}$$

$$\omega = \frac{V}{r} = \frac{V}{V^2 \big/ g\sqrt{n^2-1}} = \frac{g\sqrt{n^2-1}}{V}$$

Source: Daniel P Raymer, *Aircraft Design, A Conceptual Approach*, AIAA Publications

Constraint on Instantaneous Turn Rate

$$\dot{\Psi} = \frac{g\sqrt{n^2 - 1}}{V} \qquad n = \sqrt{\left(\frac{\dot{\Psi} V}{g}\right)^2 + 1} \qquad n = \frac{qC_L}{W/S}$$

- Where n = load factor = L/W
- Two types of turn rates
  - Sustained
    - o Enough Thrust to maintain V and H in turn
    - o T = D
  - Instantaneous
    - o Highest turn rate possible, V or H reduce in turn

- **Constraint on Instantaneous turn rate leads to an upper limit on W/S**

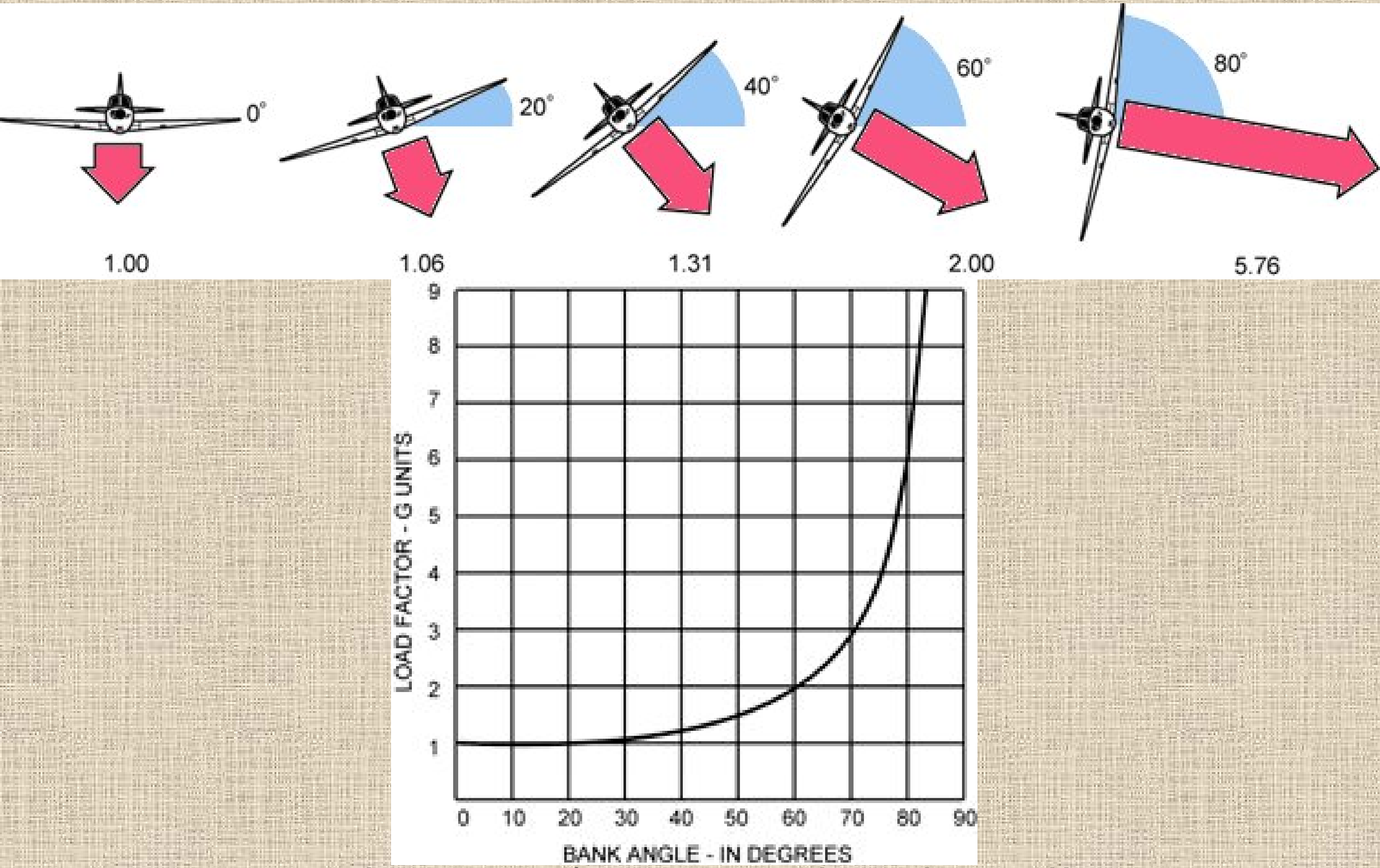
Constraint on Sustained Turn Rate

- In sustained turn $n = \left({}^{T}\!/_{W}\right)\left({}^{L}\!/_{D}\right)$
- For maximizing n, max L/D $\;{}^{W}\!/_{S} = \frac{q}{n}\sqrt{\pi AeC_{D_0}}$
- But this may give ridiculously low W/S !
- Using T = D $\;T = qSC_{D_0} + qS\left(\frac{C_L{}^2}{\pi Ae}\right) = qSC_{D_0} + \frac{n^2W^2}{qS\pi Ae}$

$$\frac{T}{W} = \frac{qC_{D_0}}{{}^{W}\!/_{S}} + \frac{W}{S}\left(\frac{n^2}{q\pi Ae}\right) \qquad \frac{W}{S} = \frac{\left({}^{T}\!/_{W}\right) \pm \sqrt{\left({}^{T}\!/_{W}\right)^2 - \left({}^{4n^2C_{D_0}}\!/_{\pi Ae}\right)}}{{}^{2n^2}\!/_{q\pi Ae}}$$

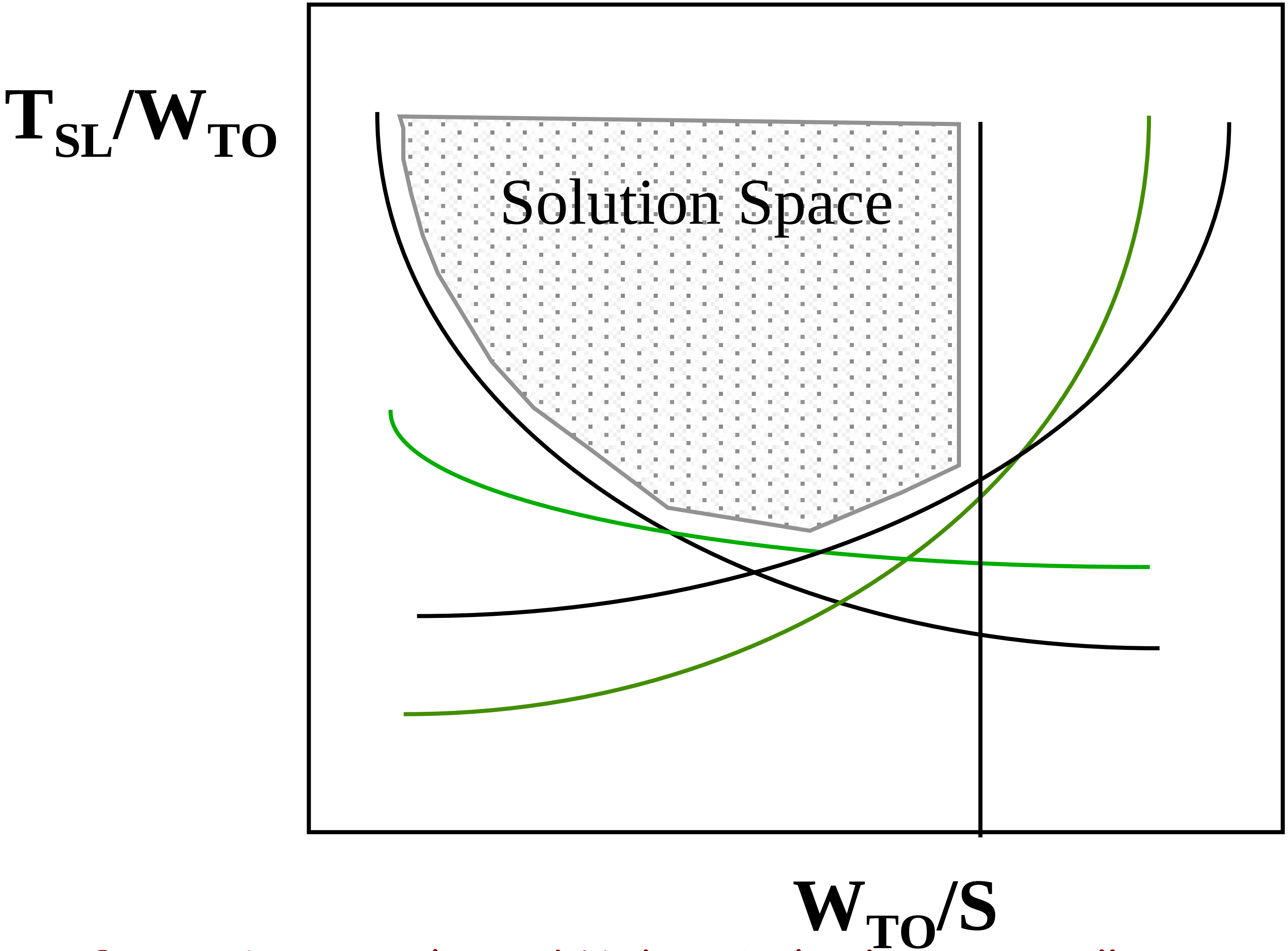
  - Note: W/S & T/W here are at combat conditions
- Important Observation $\;\frac{T}{W} \geq 2n\sqrt{\frac{C_{D_0}}{\pi Ae}}$

Load Factor v/s Bank Angle



Source: Daniel P Raymer, *Aircraft Design, A Conceptual Approach*, AIAA Publications

Sample Constraint Diagram



Design Point = Lowest T/W and Highest W/S that meets all constraints

Source: Daniel P Raymer, *Aircraft Design, A Conceptual Approach*, AIAA Publications