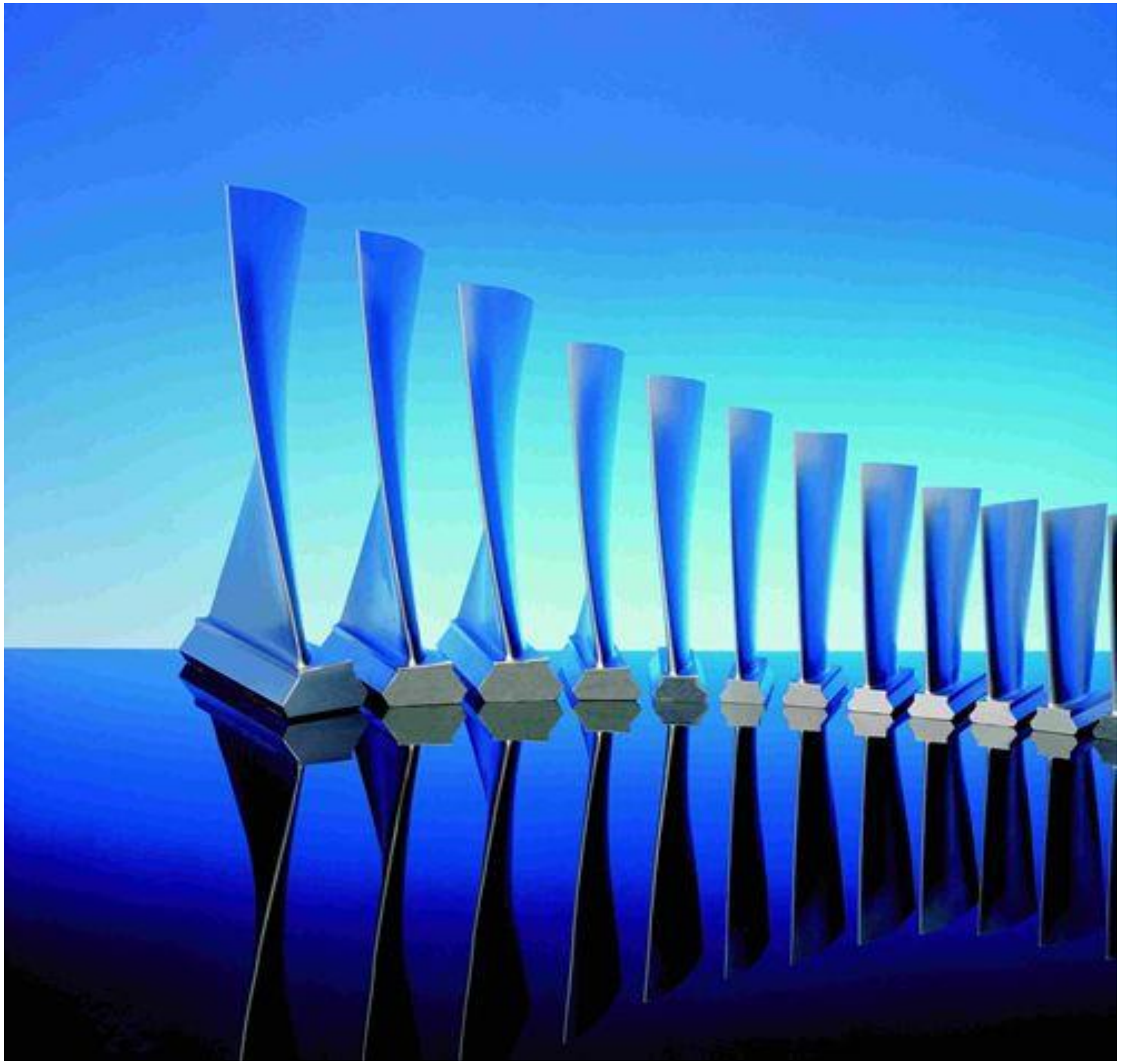
- Recap: Lecture 9: 18th August 2015, 1530-1630 hrs.
  - 3-D flow in axial flow compressors
    - 3-D blade shapes
    - 3-D flows:
      - secondary flows
      - tip clearance and leakage flows
      - passage vortices
      - scraping vortex
      - endwall flows

- Note:
  - Lecture on Tuesday, 25th August 2015 will start at 1545 hrs.
  - "Crib" session for Quiz # 1 between 1515-1545 hrs. on 25th August 2015

Radial equilibrium analysis

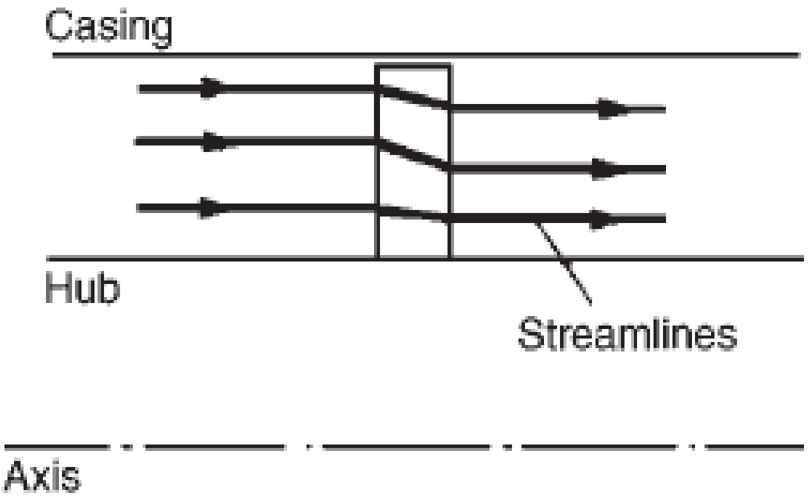
- Account for radial variations in
  - Blade speed
  - Axial velocity
  - Tangential velocity
  - Static pressure
- Large variations in these parameters can occur as the flow passes through a rotor.

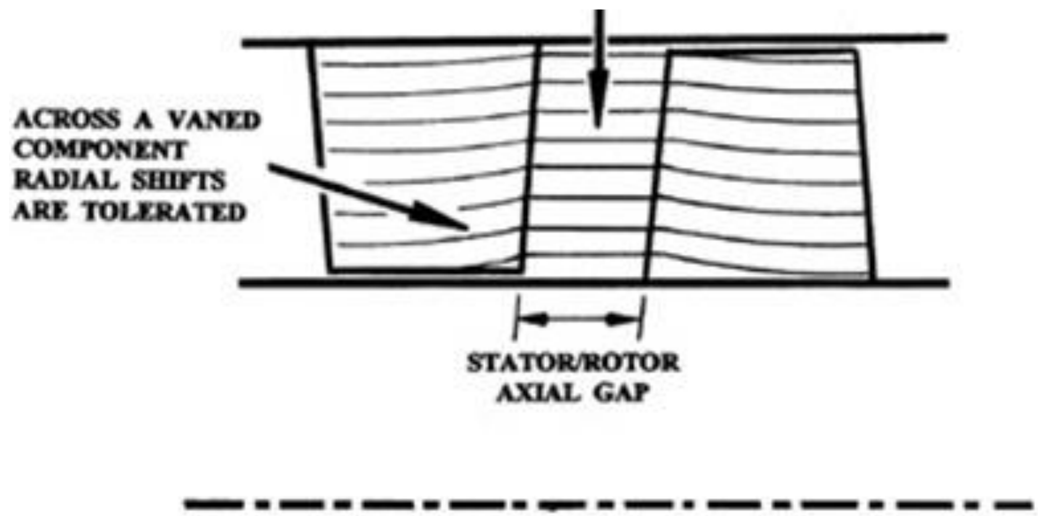


Radial equilibrium analysis

- The *radial equilibrium method* is widely used for three-dimensional design calculations in axial compressors and turbines.
- Is based upon the assumption that any radial flow which may occur is completed within a blade row, the flow outside the row then being in radial equilibrium.

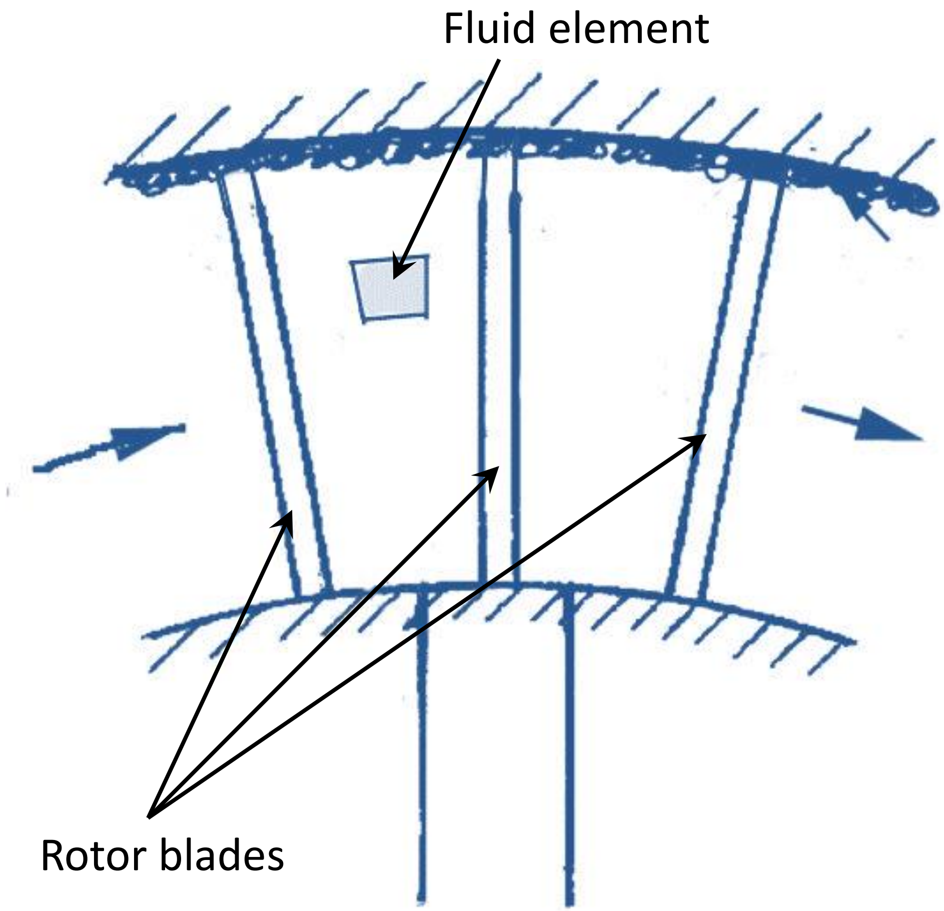
Radial equilibrium analysis





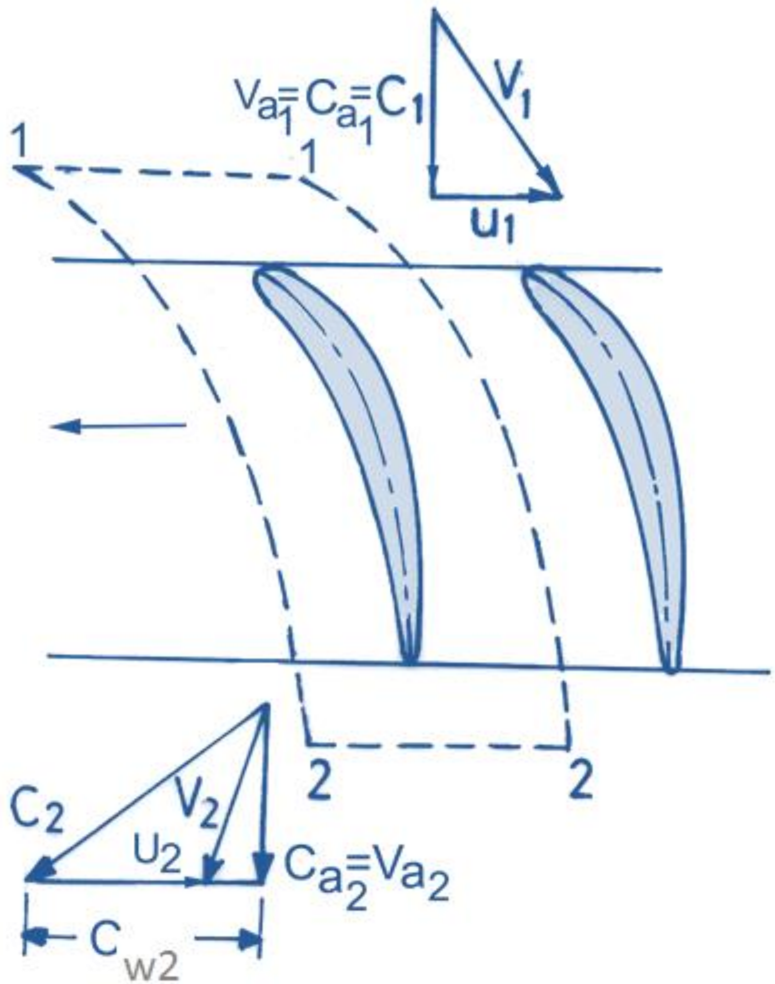
Radial equilibrium flow through a rotor blade row

Radial equilibrium analysis



Let us assume that a small element inside the rotating blade passage represents the fluid flow inside the rotor, such that the analysis of the status of this element may wholly represent the status of the whole flow inside the rotor passage

Radial equilibrium analysis



It may be recalled that this element is also executing a path through the curved diffusing passage between the rotor blades.

Simple three dimensional flow analysis: *Initial assumptions*

1) Radial movement of the flow is governed by the radial equilibrium of forces.

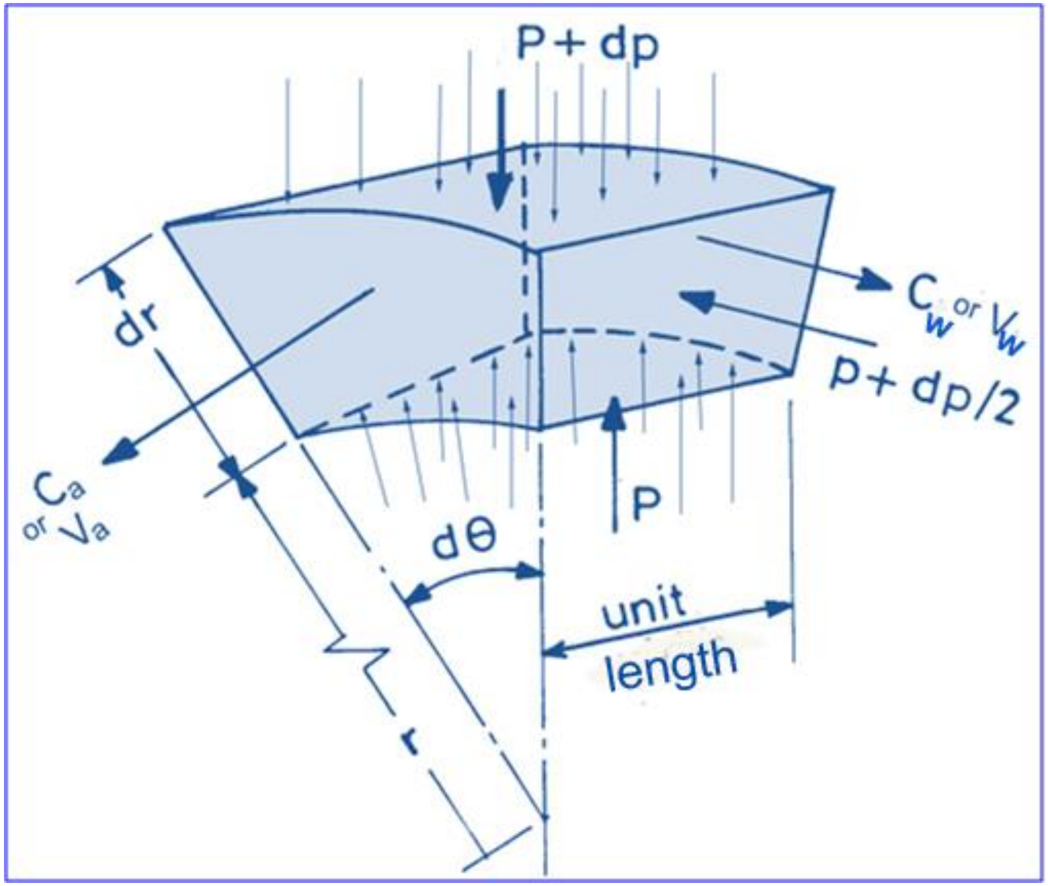
2) Radial movements occur within the blade passage only and not outside it.

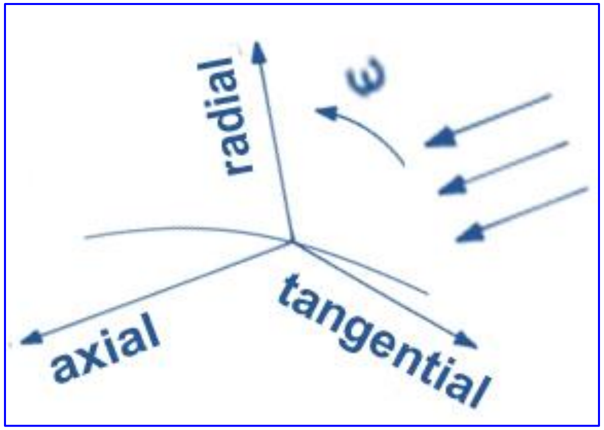
3) Flow analysis involves balancing the radial force exerted by the blade rotation.

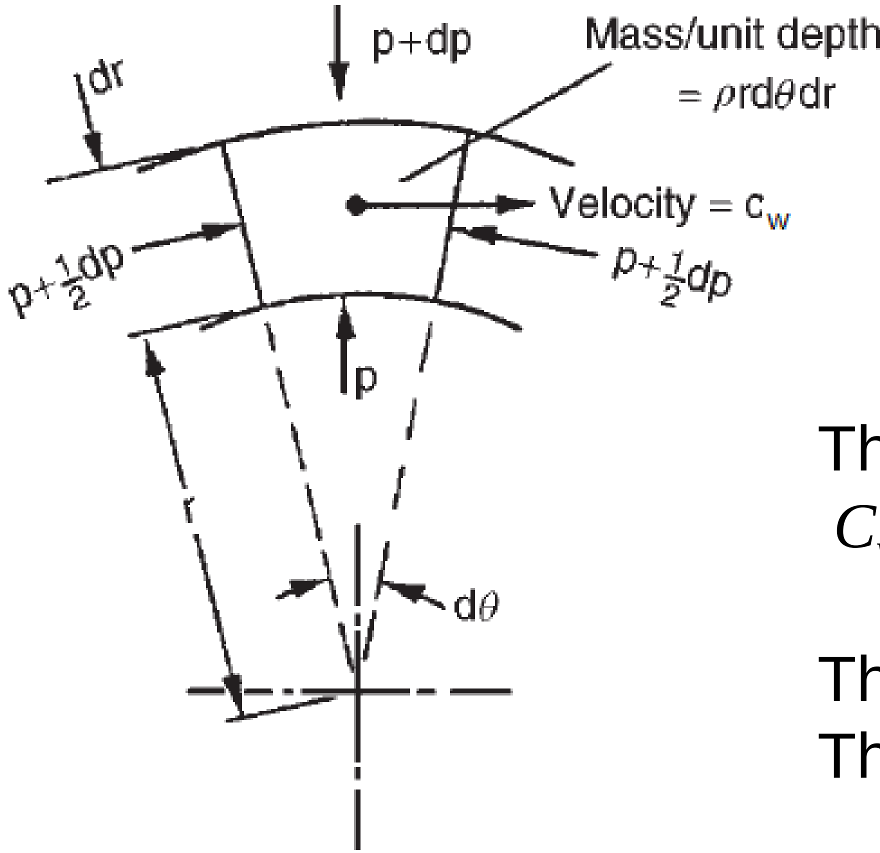
4) Gravitational forces can be neglected.

5) Radial velocities are considered negligibly small (when compared with other velocity components)

Consider this *fluid element* of unit axial length subtended by an angle $d\theta$, of thickness dr, along which the pressure variation is from P to $P+dP$.







The centrifugal force = $(\rho r dr d\theta)\omega^2 r$
$C_w = r\boldsymbol{\omega}$

The centrifugal force = $\rho\, C_w{}^2 dr d\theta$
The pressure force = $r\; dP\; d\theta$

$\rho\, C_w{}^2 dr d\theta = r\; dP\; d\theta$
$\boldsymbol{(dP/dr)(1/\rho) = (1/r)\, C_w{}^2}$

A fluid element in radial equilibrium (C_r = 0)

Resolving all the aerodynamic forces, acting on this element,

We get,

$$(P+dP)(r+dr).d\theta.1 - P.r.d\theta.1 -$$

$$2(P+dP/2).dr.(d\theta/2).1$$

$$= \rho.r.dr.d\theta.C_w^2 / r$$

LHS represents the sum total of all static forces acting on the element and RHS represents the force due to the centripetal action.

Neglecting the second order terms (products of small terms e.g. ***dP.dr*** etc.) the equation reduces to

$$\frac{1}{\rho}\frac{dP}{dr} = \frac{1}{r}C_w^2$$

This is called the

Simple Radial Equilibrium Equation

Consider the following governing equations:

1) $h_0 = h + C^2/2 = c_pT + \frac{1}{2}(C_a^2 + C_w^2)$

$\longrightarrow$ Energy Equation (1)

2) $c_p.T = \dfrac{\gamma}{\gamma - 1}\dfrac{p}{\rho}$ $\longrightarrow$ From Equation of state (2)

3) $\dfrac{p}{\rho^\gamma} = c$ $\longrightarrow$ Isentropic Law (3)

Where, h_0 is total enthalpy, h is static enthalpy pressure p, density ρ, are the fluid properties and c_p and γ are the thermal properties of air at the operating condition

Substituting for c_p from Eqn (2) and then differentiating the Eqn (1) w.r.t. r, we get

$$\frac{dh_0}{dr} = C_a \frac{dC_a}{dr} + C_w \frac{dC_w}{dr} + \frac{\gamma}{\gamma - 1}\left[\frac{1}{\rho}\frac{dP}{dr} - \frac{P}{\rho^2}\frac{d\rho}{dr}\right]$$

Differentiating the Eqn (3) (isentropic law) we get

$$\frac{d\rho}{dr} = \frac{\rho}{\gamma P}\frac{dP}{dr}$$

Substituting this in the new energy equation we get (after neglecting products of smaller terms)

$$\frac{dh_0}{dr} = C_a \frac{dC_a}{dr} + C_w \frac{dC_w}{dr} + \frac{1}{\rho}\frac{dP}{dr}$$

Now invoking the simple *radial equilibrium equation* developed earlier in the energy equation

$$\frac{1}{\rho}\frac{dP}{dr} = \frac{1}{r}C_w^2$$

We get

$$\frac{dh_0}{dr} = C_a\frac{dC_a}{dr} + C_w\frac{dC_w}{dr} + \frac{C_w^2}{r}$$

• At entry to the compressor, except near the hub and the casing, enthalpy $h_0(r)$ = constant.

• Using the condition of uniform work distribution along the blade length (i.e. radially constant) we can say that

$$\frac{dh_0}{dr} = 0$$

Thus, the energy equation would be written as,

$$C_a \frac{dC_a}{dr} + C_w \frac{dC_w}{dr} + \frac{C_w^2}{r} = 0$$

Now, if C_a = constant at all radii, then the first term is zero and the above equation reduces to

$$C_w \frac{dC_w}{dr} = -\frac{C_w^{2}}{r}$$

Therefore, the equation becomes

$$\frac{dC_w}{C_w} = -\frac{dr}{r}$$

This, on integration, yields

$\boldsymbol{C_w . r}$ **= constant**.

This condition is commonly known as the **Free Vortex Law**

- The term *Free Vortex* essentially denotes that the strength of the vortex (*or lift per unit length*) created by each airfoil section used from the root to the tip of the blade remains constant

$$\text{Lift}\ ,\ L = \rho.V.\Gamma$$

where, ρ is the density,

V is the inlet velocity, and

Γ is the circulation

- It, therefore, means that at the trailing edge of the blade, the trailing vortex sheet has constant strength from the root to the tip of the blade.

- The simple Radial Equilibrium may be used to explain some of the basic characteristics of an axial compressor

- Radial equilibrium requires that in a medium (<1.0) to low ($\ll 1.0$) hub/tip radius ratio in a rotor blade, change of whirl component (ΔC_w or ΔV_w) must be very large near the hub (root) compared to that near the casing (tip)

- Radial equilibrium, thus, requires that flow turning at hub, $\Delta\beta$, must be much larger at hub than at the tip.
- Hence, the hub airfoil must be of much higher camber than that of the tip airfoil
- Whirl component downstream of the rotor (C_{w2} or V_{w2}) is higher than the upstream