- Recap: Lecture 9: 21st August 2015, 1530-1655 hrs.
  - Radial Equilibrium
    - Simple radial equilibrium equation
    - Vortex laws: free vortex law

Simple three dimensional flow analysis: *Initial assumptions*

1) Radial movement of the flow is governed by the radial equilibrium of forces.

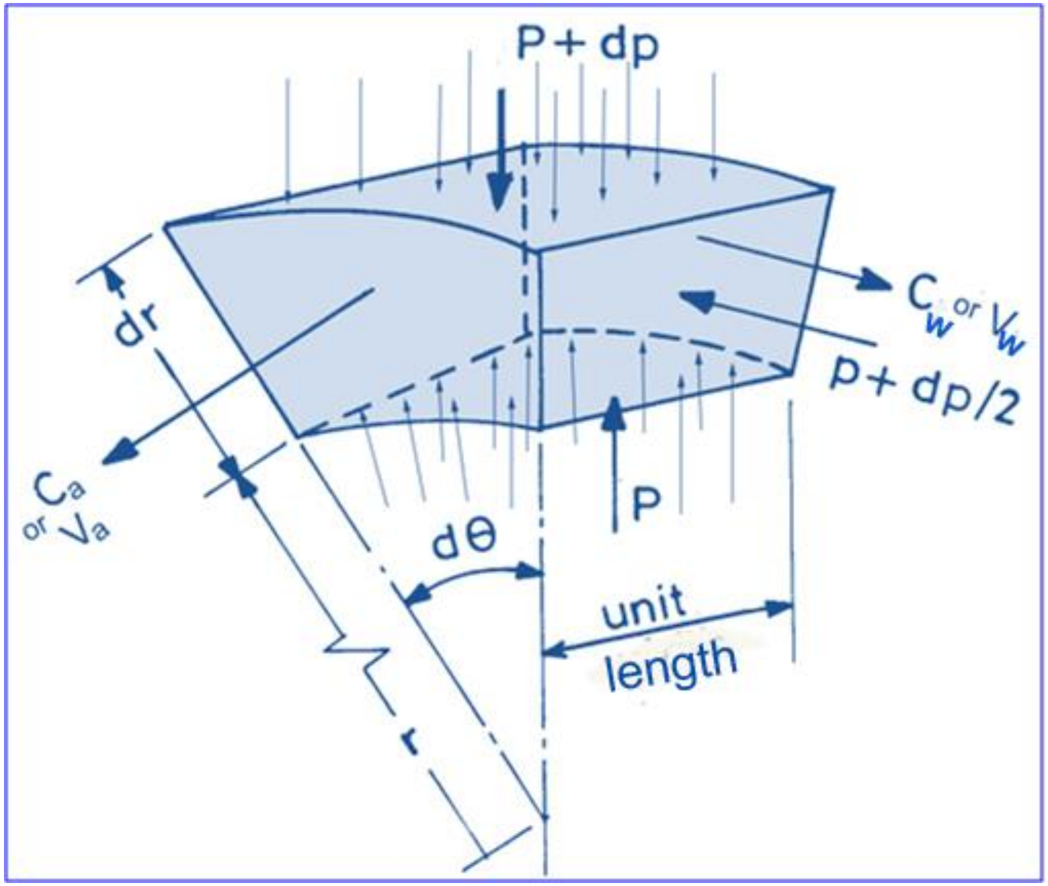
2) Radial movements occur within the blade passage only and not outside it.

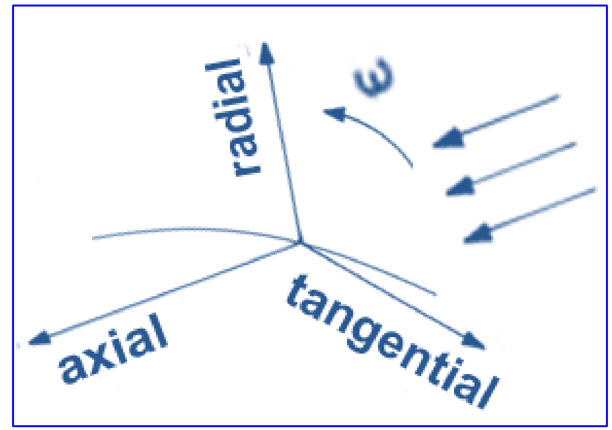
3) Flow analysis involves balancing the radial force exerted by the blade rotation.

4) Gravitational forces can be neglected.

5) Radial velocities are considered negligibly small (when compared with other velocity components)

Consider this *fluid element* of unit axial length subtended by an angle $d\theta$, of thickness dr, along which the pressure variation is from P to $P+dP$.





Neglecting the second order terms (products of small terms e.g. ***dP.dr*** etc.) the equation reduces to

$$\frac{1}{\rho}\frac{dP}{dr} = \frac{1}{r}C_w^2$$

This is called the

Simple Radial Equilibrium Equation

Consider the following governing equations:

1) $h_0 = h + C^2/2 = c_p T + \frac{1}{2}(C_a^2 + C_w^2)$

$\longrightarrow$ Energy Equation (1)

2) $c_p.T = \frac{\gamma}{\gamma - 1}\frac{p}{\rho}$ $\longrightarrow$ From Equation of state (2)

3) $\frac{p}{\rho^\gamma} = c$ $\longrightarrow$ Isentropic Law (3)

Where, h_0 is total enthalpy, h is static enthalpy pressure p, density ρ, are the fluid properties and c_p and γ are the thermal properties of air at the operating condition

Therefore, the equation becomes

$$\frac{dC_w}{C_w} = -\frac{dr}{r}$$

This, on integration, yields

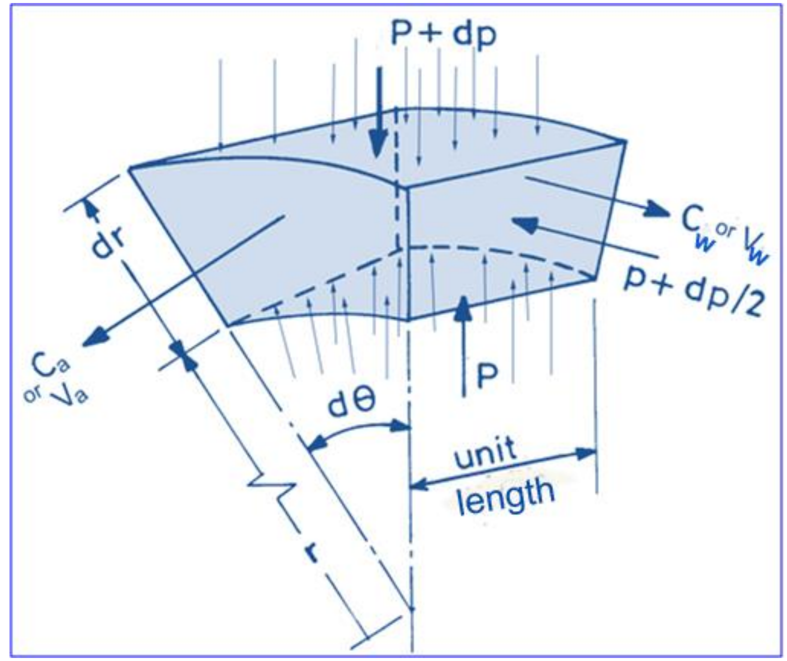
$$\boldsymbol{C_w .\, r = \text{constant}}.$$

This condition is commonly known as the **Free Vortex Law**

- The simple Radial Equilibrium may be used to explain some of the basic characteristics of an axial compressor
- Radial equilibrium requires that in a medium (<1.0) to low ($\ll 1.0$) hub/tip radius ratio in a rotor blade, change of whirl component (ΔC_w or ΔV_w) must be very large near the hub (root) compared to that near the casing (tip)

- Radial equilibrium, thus, requires that flow turning at hub, $\Delta\beta$, must be much larger at hub than at the tip.
- Hence, the hub airfoil must be of much higher camber than that of the tip airfoil
- Whirl component downstream of the rotor (C_{w2} or V_{w2}) is higher than the upstream

- The radial static pressure gradient *dP/dr* will be greater downstream of the rotor than upstream



- Static pressure rise across the blade root will be lesser than that across the rotor tip.

- Thus degree of reaction, R_x across the root will be much less compared to that at the tip.

- If one looks at a stage consisting of a rotor and a stator, the radial equilibrium would also impact the flow across the stator
- Stator blade rows reduce the whirl component
- Downstream of the stator, radial pressure gradient *dP/dr* will be much lower than upstream of the stator
- Static pressure rise, ΔP across the stator at hub would be higher than at the tip
    - This may lead to high blade loading and even flow separation at stator hub

- Consider a compressor where, $rC_{w1}=K_1$ before the rotor and $rC_{w2}=K_2$ after the rotor.
- The specific work done by the rotor is

$$w = U(C_{w2} - C_{w1}) = \Omega r\left({}^{K_2}/_{r} - {}^{K_1}/_{r}\right) = \text{const.}$$

- Since, the specific work is constant at all radii
- The flow angles,

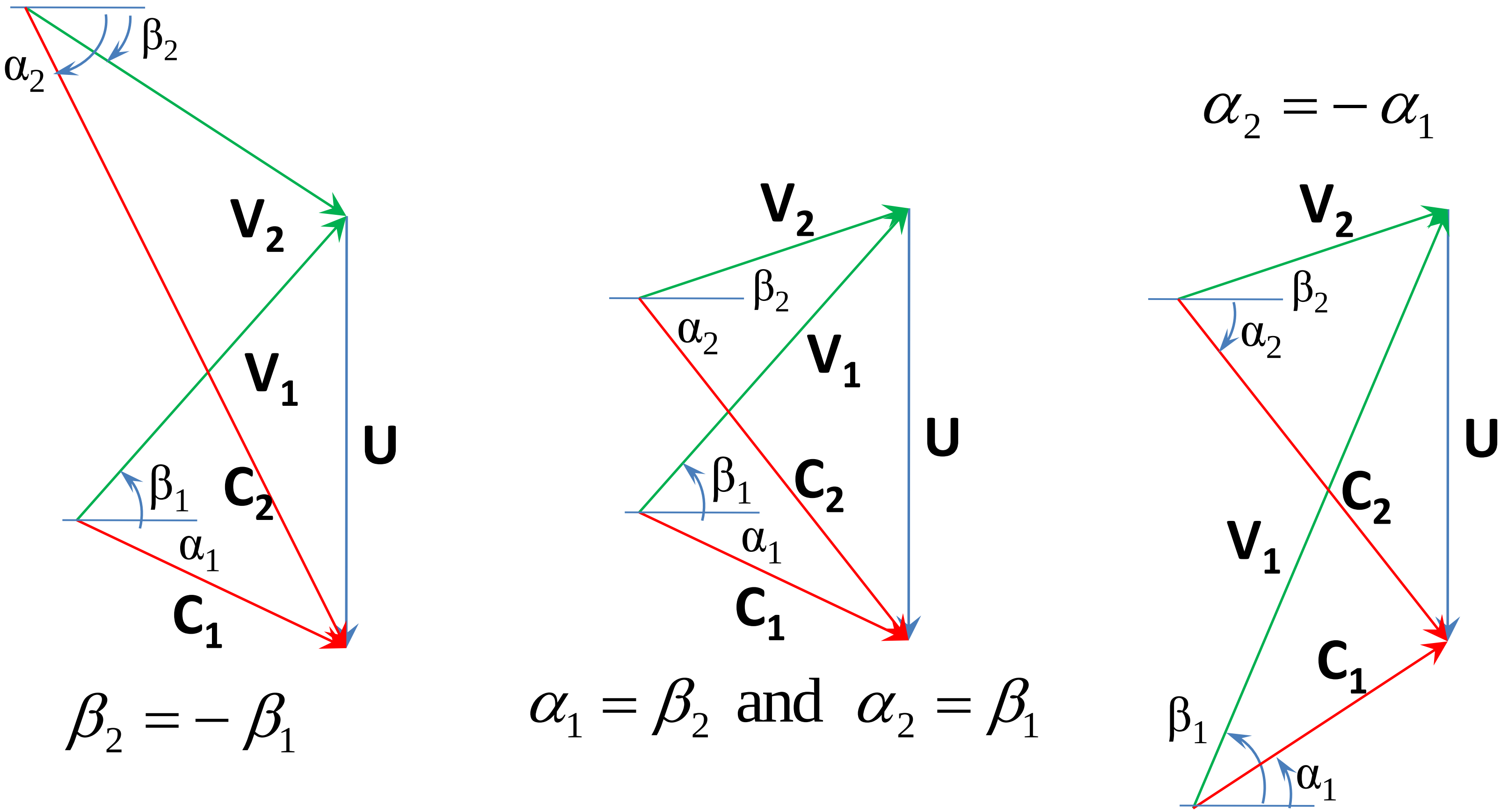
$$\tan\beta_1 = \frac{U}{C_a} - \tan\alpha_1 = \frac{\Omega r - K_1/r}{C_a} \text{ and}$$

$$\tan\beta_2 = \frac{U}{C_a} - \tan\alpha_2 = \frac{\Omega r - K_2/r}{C_a}$$

- We can express the degree of reaction in terms of the constants from the free vortex equation.
- $R_x = \frac{C_a}{2U}(tan\beta_1 + tan\beta_2)$
- Substituting for $(tan\beta_1 + tan\beta_2)$
- $R_x = 1 - \frac{(K_1+K_2)/2\Omega}{r^2}$
- For positive values of K, reaction increases from the root to tip.
- Since $C_w{}^2/r$ is always positive, the static pressure increases from root to tip.

- The free vortex method, though is relatively easy to use, has several disadvantages.
  - The blades invariably have a large twist
  - The Mach number at the tip tends to be large
- Both these significantly affect the performance of the compressor.
- There are therefore, modifications to the free vortex method like forced vortex or exponential laws.

Effect of degree of reaction



$$\beta_2 = -\beta_1 \qquad \alpha_1 = \beta_2 \text{ and } \alpha_2 = \beta_1$$

R_x=0.0 **R_x=0.5** **R_x=1.0**

In view of the simplifications and the constraints of the free vortex design law, a generalized vortex law may be written as

$$C_w . r^n = constant$$

Where, $n = 1$ gives the free vortex law.

Normally, $-1 > n > 2$.

When $0.75 < n < 1.0$ it yields *near-free vortex or relaxed-free-vortex* designs in which the blade sections are slightly overloaded with respect to free vortex blade loading.

- when $n>1$ the blades are underloaded w.r.t. free vortex law;
- $n = -1$ is rarely used - known as the forced vortex design
- $n = 0$ is known as the Exponential design law and often is used to arrive at constant degree of reaction blade designs

1) Free Vortex Law : $C_w \,.r$ = constant

2) Forced vortex Law : $C_w \,/r$ = constant

3) Relaxed vortex law : $C_w.r^n$ = constant

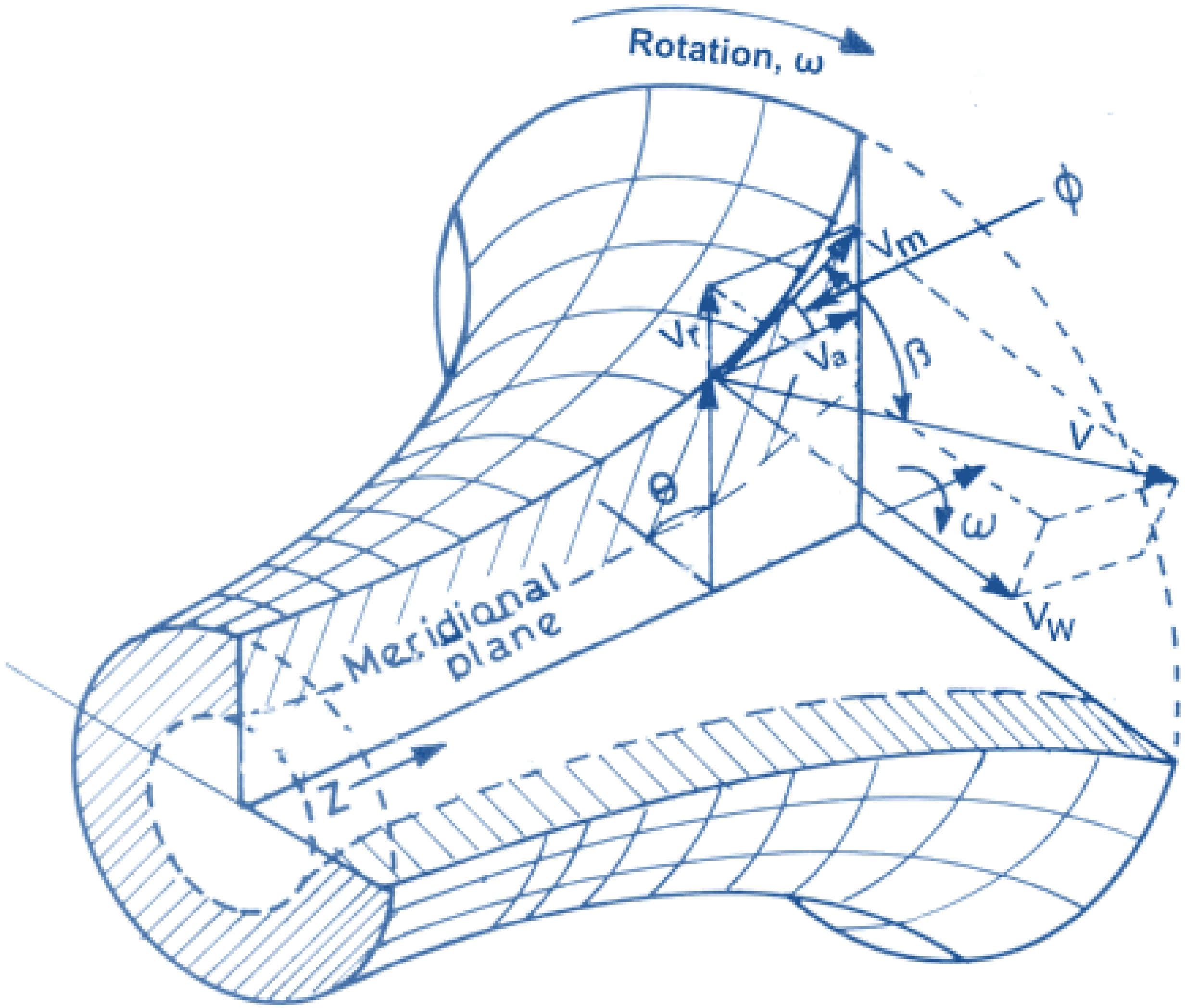
4) Exponential law : n = 0

A generalized version of the above laws may be stated as:

upstream: $C_{w1} = aR^{\boldsymbol{n}} - b/R$

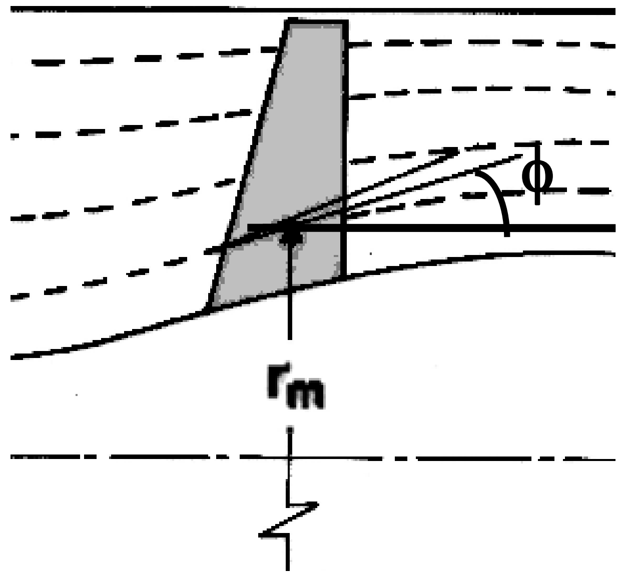
and, downstream: $C_{w2} = aR^{\boldsymbol{n}} + b/R$

where R is radius ratio, r/r_{mean}



Meridional direction is defined as

$\tan\phi = V_r / V_a$ and $V_m = V \cos \beta$



- For axial flow compressor the flow track inside generally moves towards lesser ϕ or higher r_m , i.e. the flow later on flattens out.

$$\frac{1}{\rho}\frac{\partial P}{\partial r} = \frac{C_w^2}{r} + \cos\phi\,\frac{V_m^2}{r_m} - V_r\,\frac{dV_m}{dm}$$

- This is the generalized Radial Equilibrium Equation for circumferentially averaged properties.

For old fashioned compressor designs, V_m (instead of constant axial velocity V_a or C_a) is considered constant and the last term is eliminated. In the very early design of compressor the flow path was considered linear and hence even the 2nd term vanishes, giving us back the simple radial equilibrium equation

$$\frac{1}{\rho}\frac{dP}{dr} = \frac{1}{r}C_w^2$$

For modern compressor, this simple radial equilibrium equation relationship is inadequate and it becomes necessary to utilize the full radial equilibrium equation or a better approximation than the simple radial equilibrium equation.

- Wherever the flow is not experiencing the centrifugal force, the radial equilibrium can not be applied.
- Experiments have shown that in between the blade rows, in the axial gaps between the rotor and the stator, there could be radial shift of the meridional path. Hence for accurate design flow analysis the full radial equilibrium equation is be used.
- For using the full R.E.E., for computational purposes, further steps need to be taken.

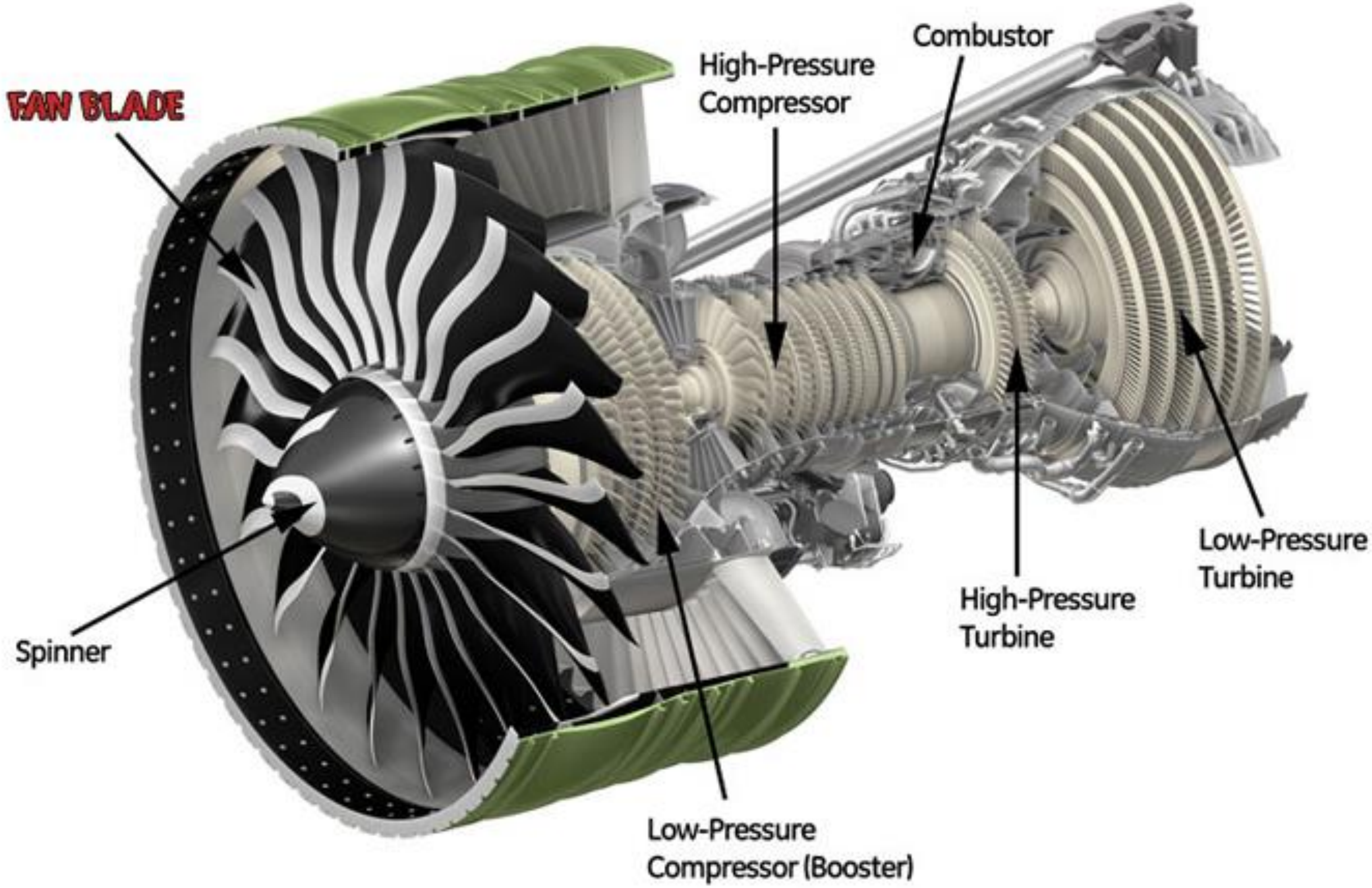
  i) The R.E.E. is to be transformed into a form that contains partial derivatives of all parameters with respect to r and θ

  ii) Next, the circumferential average of those parameters is taken by integrating over θ from pressure side of one blade to the suction side of the other blade.

  iii) The flow is analysed at various axial stations with a) Energy equation, b) Continuity condition and c) R.E.E.

Compulsions of these choices are often present depending on whether one is designing a:

i) Small sized axial compressor
ii) Large sized axial fan (in a bypass turbofan)
iii) First stage of a multi-stage axial compressor
iv) Middle stage of a multi-stage compressor
v) End stage of a multi-stage compressor
vi) High hub/tip radius ratio stage
vii) Low hub/tip radius ratio stage



General Electric J85-GE-17A Turbofan Engine

Source: http://www.geaviation.com

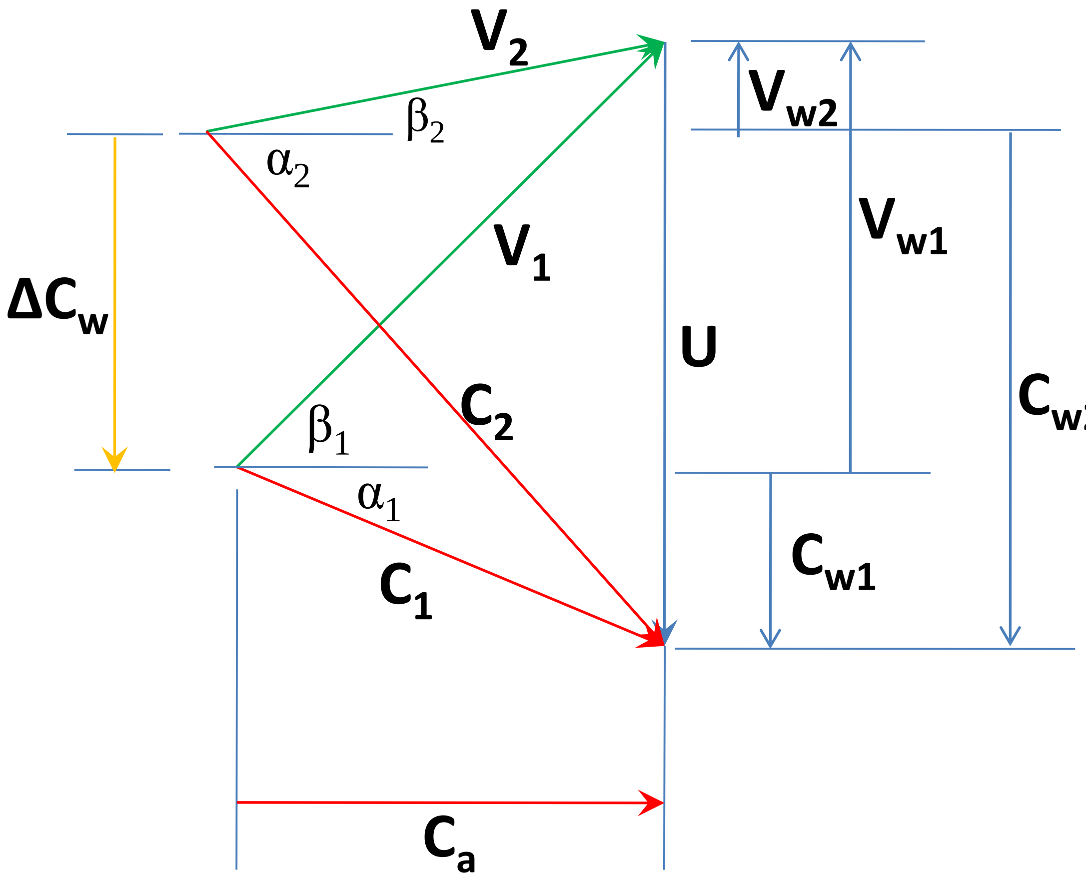
Axial Distribution of the specific work (w) and efficiency (η) amongst the individual stages of a typical multi-stage compressor must be completed and are arrived at from early design choices :

	Initial Stages	**Middle stages**	**Last stages**
η	0.86	0.92	0.88
π	1.5-1.8	1.3-1.4	1.1-1.2
$\Delta T_0 {}^oC$	40-75	30-50	15-30

The radial distribution of these parameters are then taken up for each stage design

- The 3-D flow computations has provided immense assistance to engine designers.
- It has cut down on design time and has reduced dependence on costly experimental analysis.
- The 3-D methods have helped understand various flow phenomena e.g. secondary flow development, choking in the stages, effects of end-wall flows etc.
- However, the designer uses these solutions in conjunction with many empirical relations and experimental data to make the design.
- There is still scope for improvement in these methods and for reducing dependence on empirical relations

Single stage performance characteristics



Single stage performance characteristics

- From the above velocity triangles,

$$C_{w2} = U - C_a \tan\beta_2 \quad and \quad C_{w1} = C_a \tan\alpha_1$$

$$\text{Since, } \Delta h_0 = U\Delta C_w$$

$$\Delta h_0 = U\left[U - C_a\left(\tan\alpha_1 + \tan\beta_2\right)\right]$$

$$or,\ \frac{\Delta C_w}{U} = \frac{\Delta h_0}{U^2} = 1 - \frac{C_a}{U}\left(\tan\alpha_1 + \tan\beta_2\right)$$

Single stage performance characteristics

- Change in the design mass flow rate affects C_a, change in rotor speed affects U.
- Change of either C_a or U changes the inlet angle β_1 at which the flow approaches the rotor.
- The above equation shows that the blade performance depends upon the ratio C_a/U.

The stage performance is a function of the loading coefficient, flow coefficient and the efficiency.

Thus,

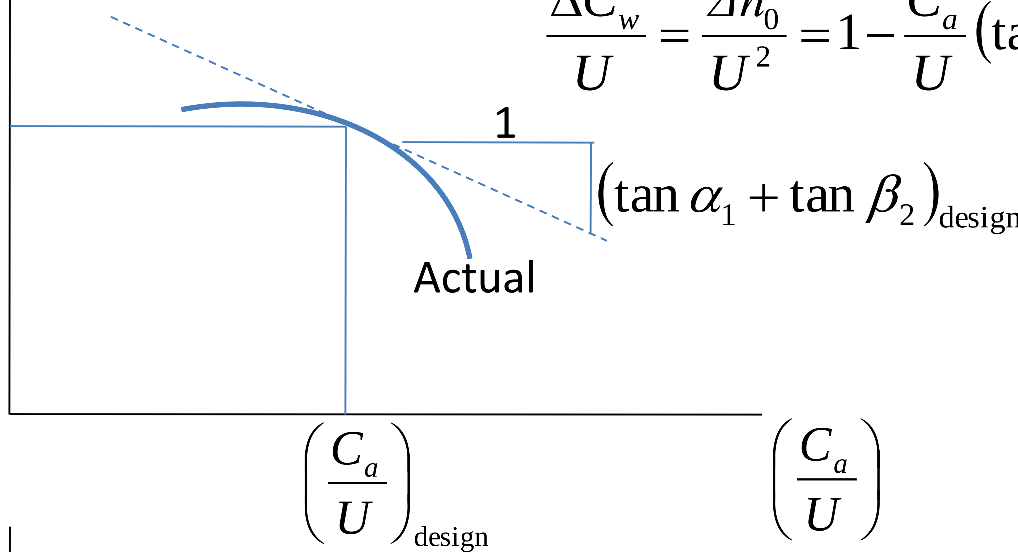
$$\text{Stage performance} = \mathrm{f}(\psi, \phi, \eta)$$

Single stage performance characteristics

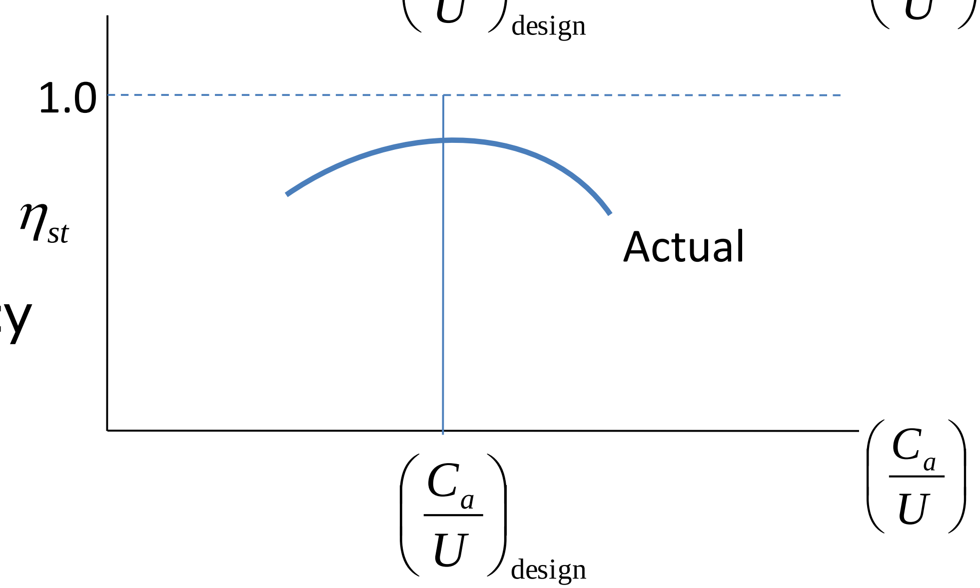
$$\frac{\Delta h_0}{U^2}=\frac{\Delta C_w}{U}$$

$$\frac{\Delta C_w}{U}=\frac{\Delta h_0}{U^2}=1-\frac{C_a}{U}\left(\tan\alpha_1+\tan\beta_2\right)$$

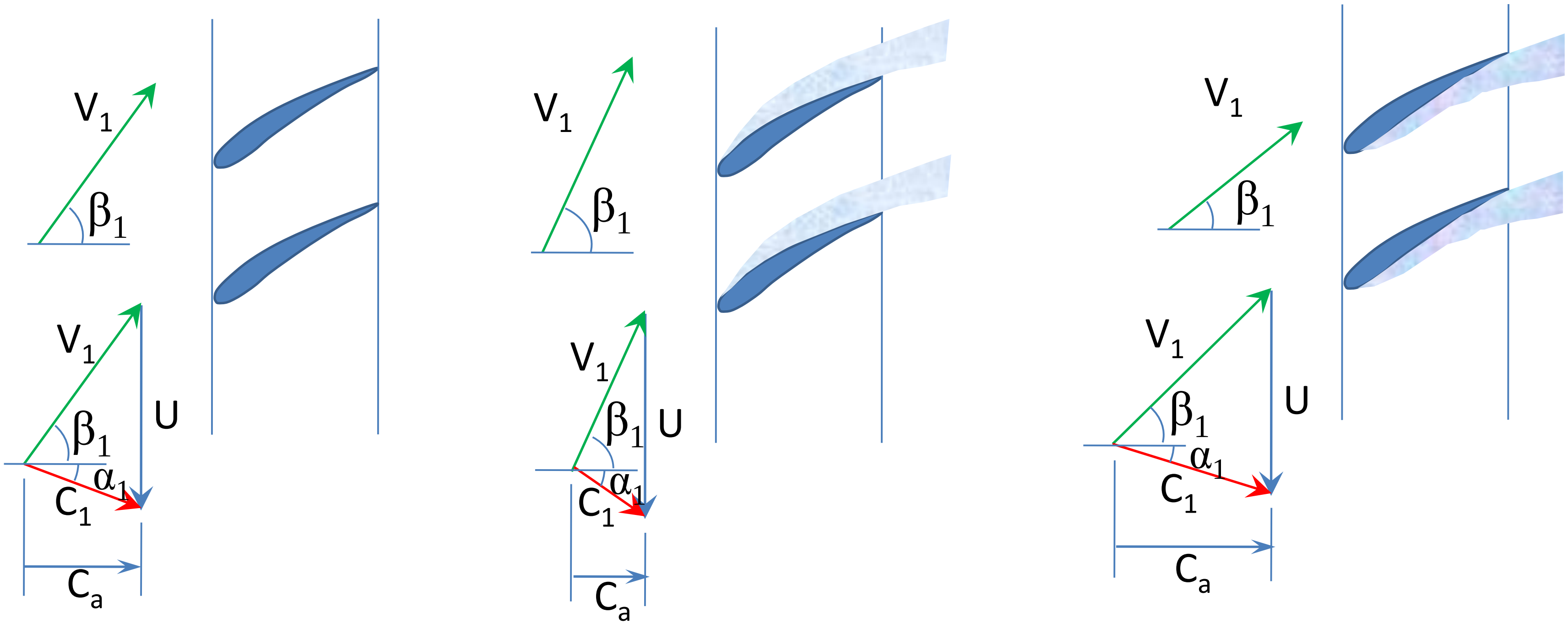
Stage loading



Stage efficiency



Single stage performance characteristics



Design condition:

Normal operation

$$\left(\frac{C_a}{U}\right) = \left(\frac{C_a}{U}\right)_{\text{design}}$$

Off-design condition:

Positive incidence flow separation

$$\left(\frac{C_a}{U}\right) < \left(\frac{C_a}{U}\right)_{\text{design}}$$

Off-design condition:

Negative incidence flow separation

$$\left(\frac{C_a}{U}\right) > \left(\frac{C_a}{U}\right)_{\text{design}}$$