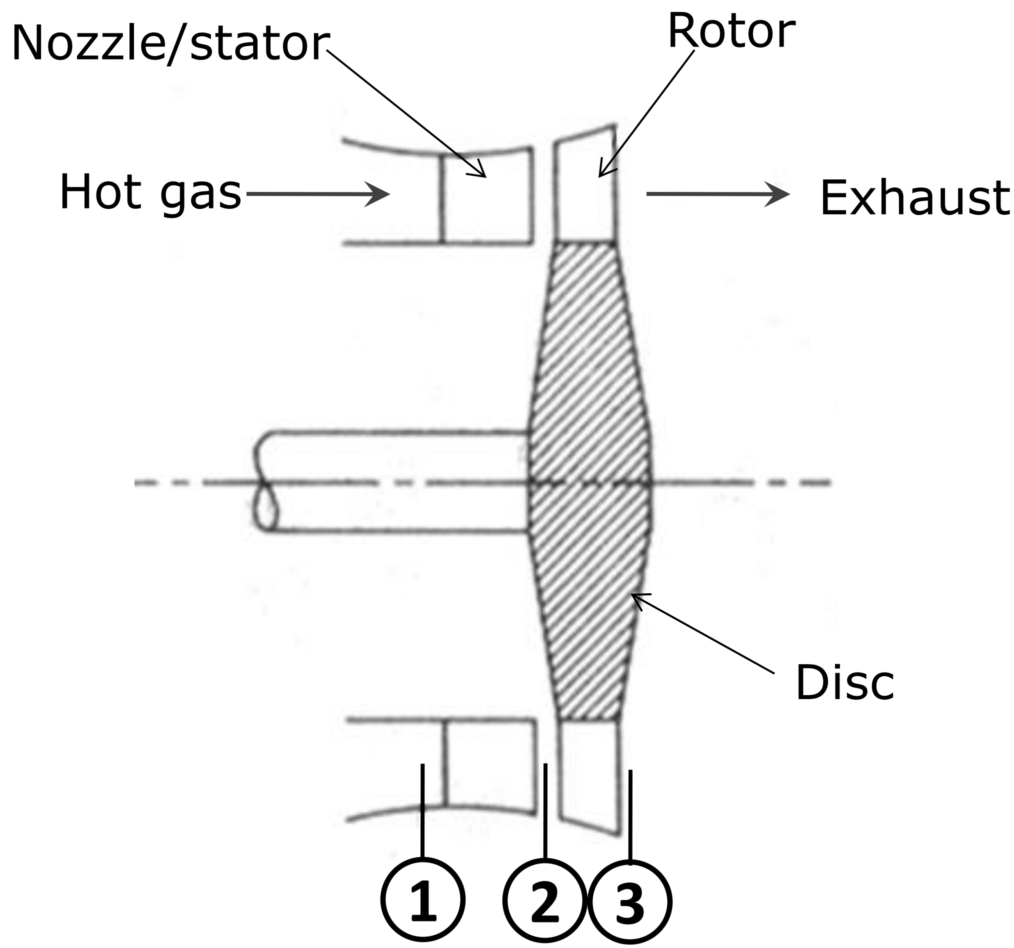
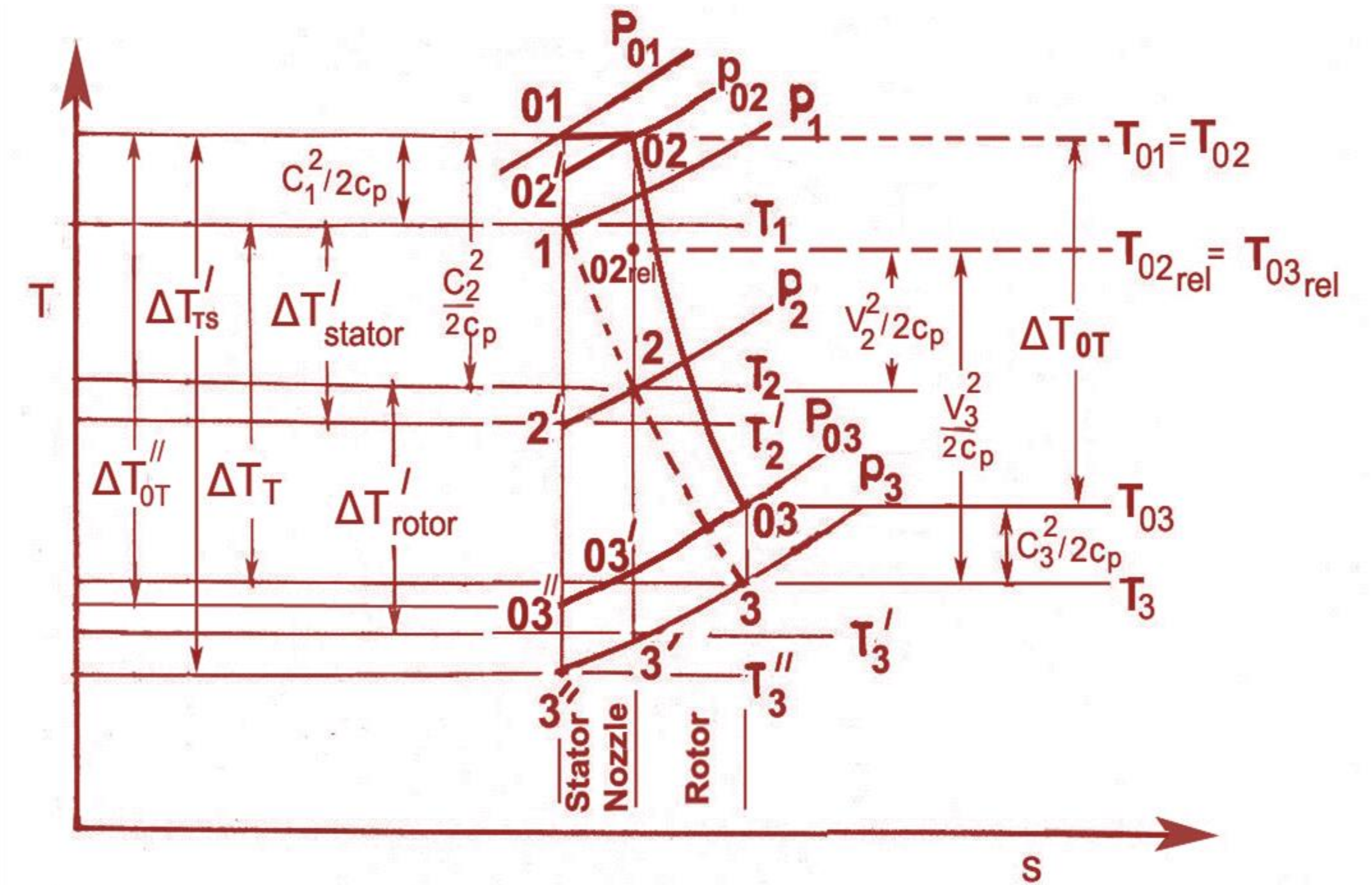
- Recap: Lecture 16, 26th September 2015, 1030-1135 hrs.
  - Introduction to axial flow turbines
    - Functions of axial turbines
    - Components of axial turbines
    - Thermodynamics
    - Velocity triangles
    - Impulse and reaction turbines

Axial flow turbines

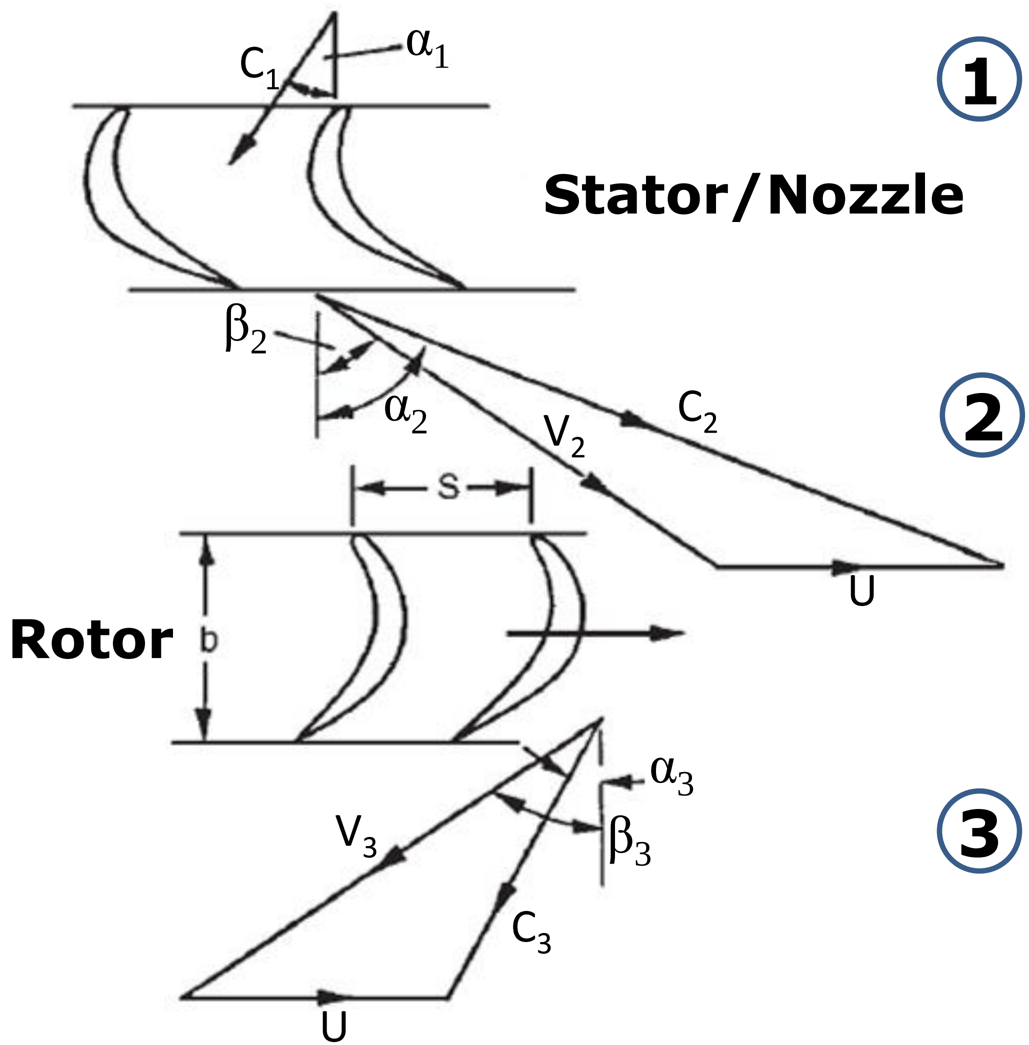


An axial turbine stage

Thermodynamic changes in Turbine in a GTE Cycle



Velocity triangles



Work and stage dynamics

Applying the angular momentum equation,

$$P = \dot{m}(U_2C_{w2} - U_3C_{w3})$$

In an axial turbine, $U_2 \approx U_3 = U$.

Therefore, the work per unit mass is

$$w_t = U(C_{w2} - C_{w3}) \quad \text{or} \quad w_t = c_p(T_{01} - T_{03})$$

Let $\Delta T_0 = T_{01} - T_{03} = T_{02} - T_{03}$

The stage work ratio is,

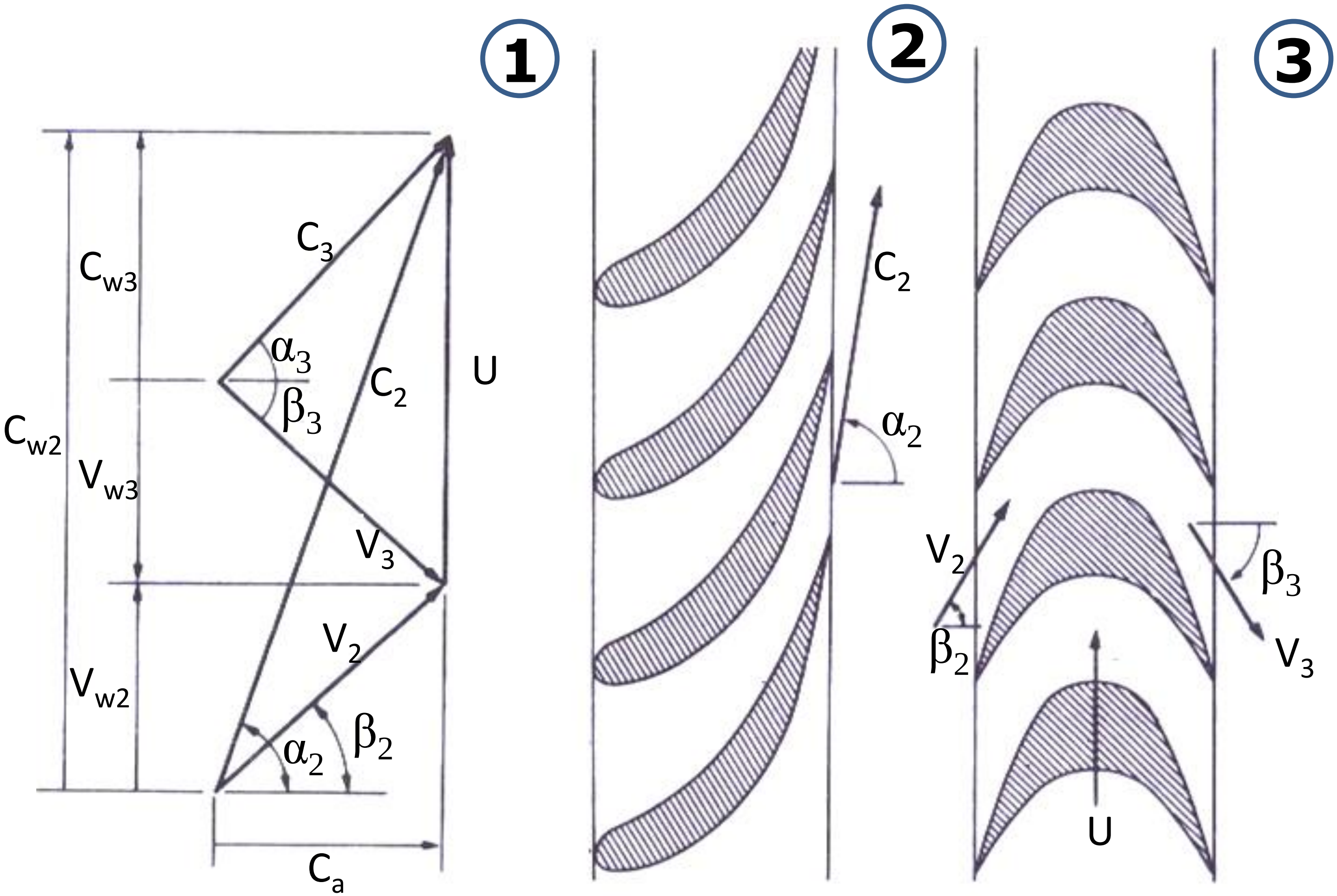
$$\frac{\Delta T_0}{T_{01}} = \frac{U(C_{w2} - C_{w3})}{c_pT_{01}}$$

Work and stage dynamics

- Turbine work per stage is limited by
  - Available pressure ratio
  - Allowable blade stresses and turning
- Unlike compressors, boundary layers are generally well behaved, except for local pockets of separation
- The turbine work ratio is also often defined in the following way:

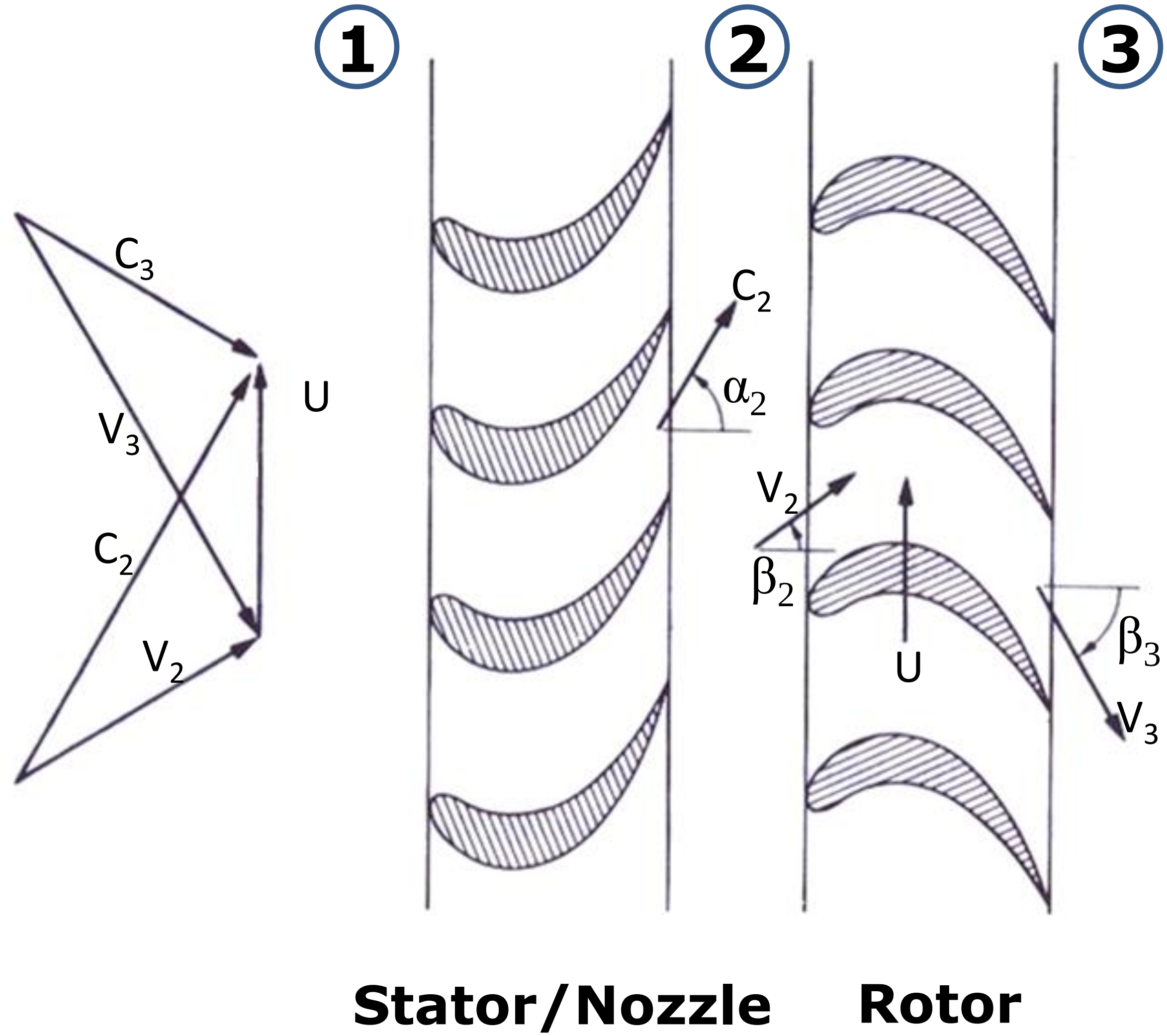
$$\frac{w_t}{U^2} = \frac{\Delta h_0}{U^2} = \frac{C_{w2} - C_{w3}}{U}$$

Impulse turbine stage



Stator/Nozzle **Rotor**

50% Reaction turbine stage



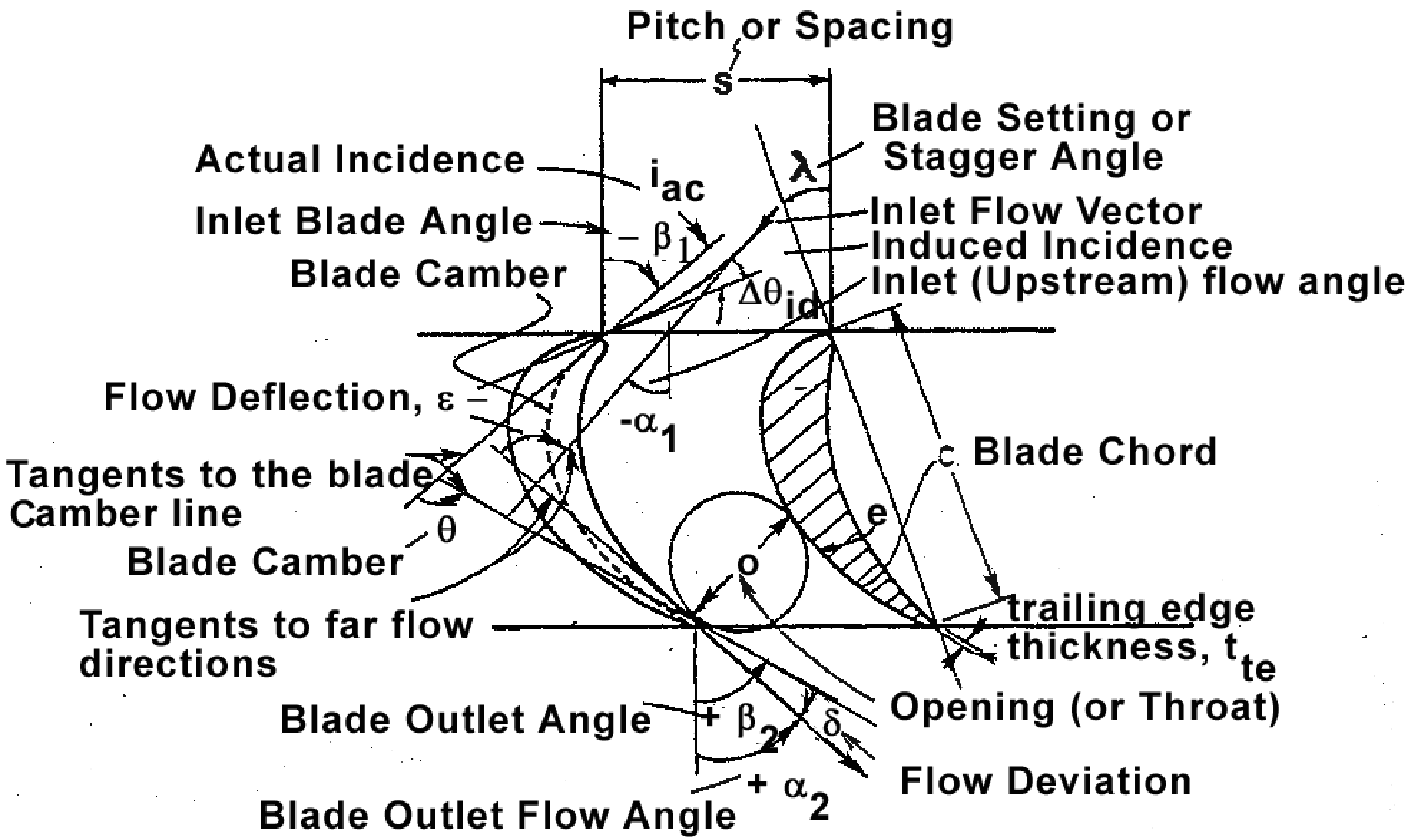
Turbine Cascade

- A cascade is a stationary array of blades.
- Cascade is constructed for measurement of performance similar to that used in axial turbines.
- Cascade usually has porous end-walls to remove boundary layer for a two-dimensional flow.
- Radial variations in the velocity field can therefore be excluded.
- Cascade analysis relates the fluid turning angles to blading geometry and measure losses in the stagnation pressure.

Turbine Cascade

- Turbine cascades are tested in wind tunnels similar to what was discussed for compressors.
- However, turbines operate in an accelerating flow and therefore, the wind tunnel flow driver needs to develop sufficient pressure to cause this acceleration.
- Turbine blades have much higher camber and are set at a negative stagger unlike compressor blades.
- Cascade analysis provides the blade loading from the surface static pressure distribution and the total pressure loss across the cascade.

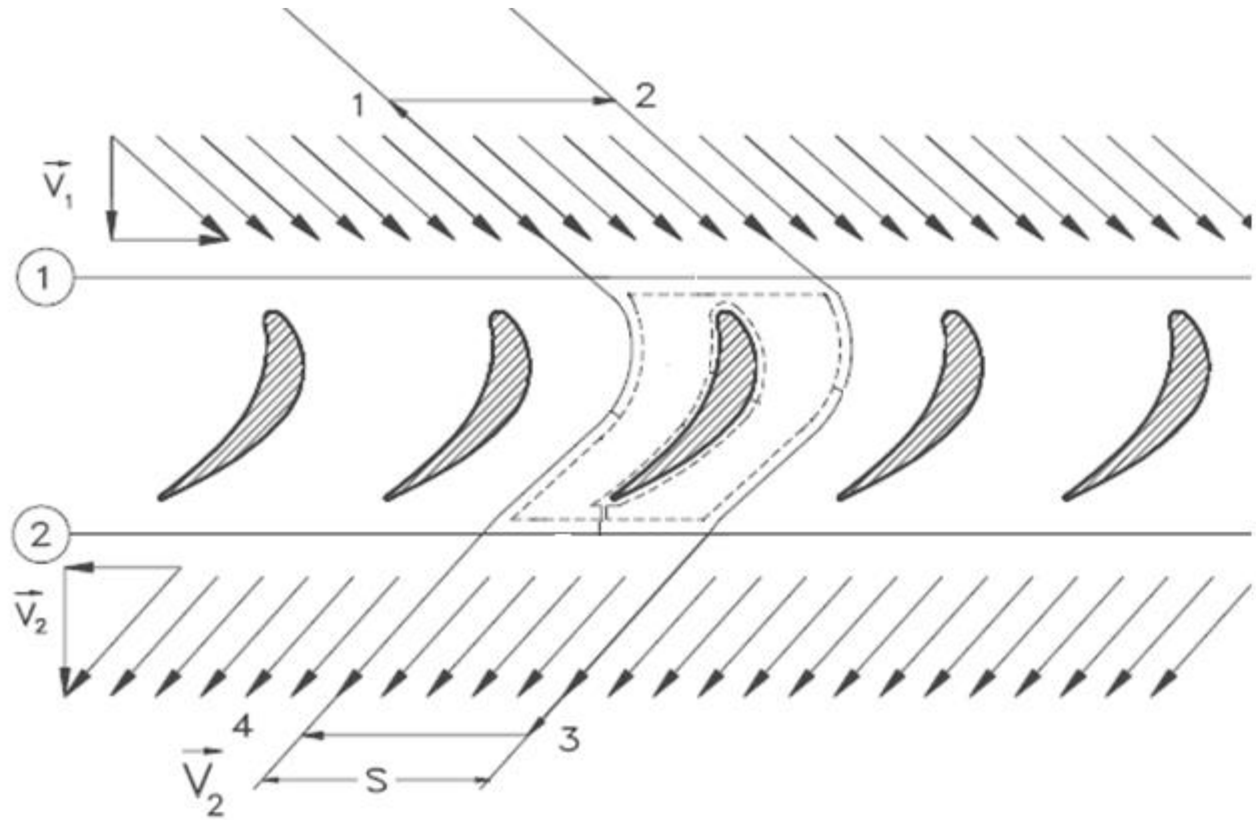
Turbine Cascade



Turbine Cascade

- From elementary analysis of the flow through a cascade, we can determine the lift and drag forces acting on the blades.
- This analysis could be done using inviscid or potential flow assumption or considering viscous effects (in a simple manner).
- Let us consider V_m as the mean velocity that makes and angle α_m with the axial direction.
- We shall determine the circulation developed on the blade and subsequently the lift force.
- In the inviscid analysis, lift is the only force.

Turbine Cascade



Inviscid flow through a turbine cascade

Turbine Cascade

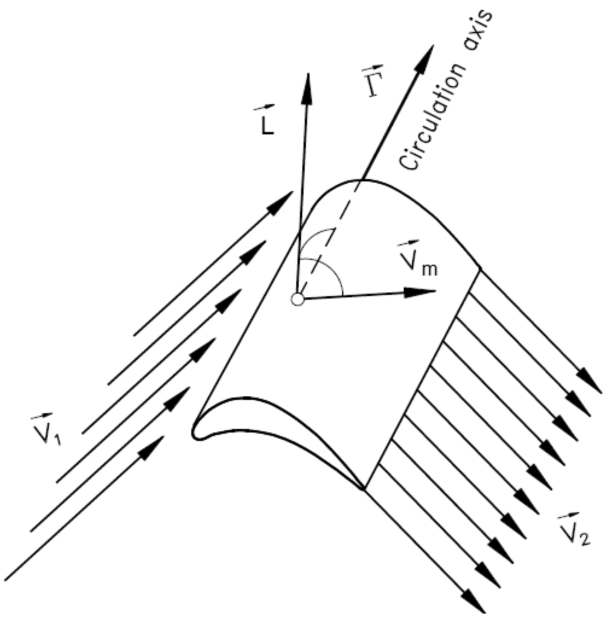
$$\text{Circulation}, \Gamma = S(V_{w2} - V_{w1})$$

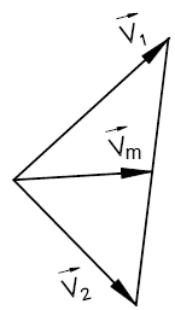
$$\text{and lift, } L = \rho V_m \Gamma = \rho V_m S(V_{w2} - V_{w1})$$

Expressing lift in a non-dimensional form,

$$\text{Lift coefficient, } C_L = \frac{L}{\frac{1}{2}\rho V_m^2 C} = \frac{\rho V_m S(V_{w2} - V_{w1})}{\frac{1}{2}\rho V_m^2 C}$$

$$= 2\frac{S}{C}(\tan\alpha_2 - \tan\alpha_1)\cos\alpha_m$$



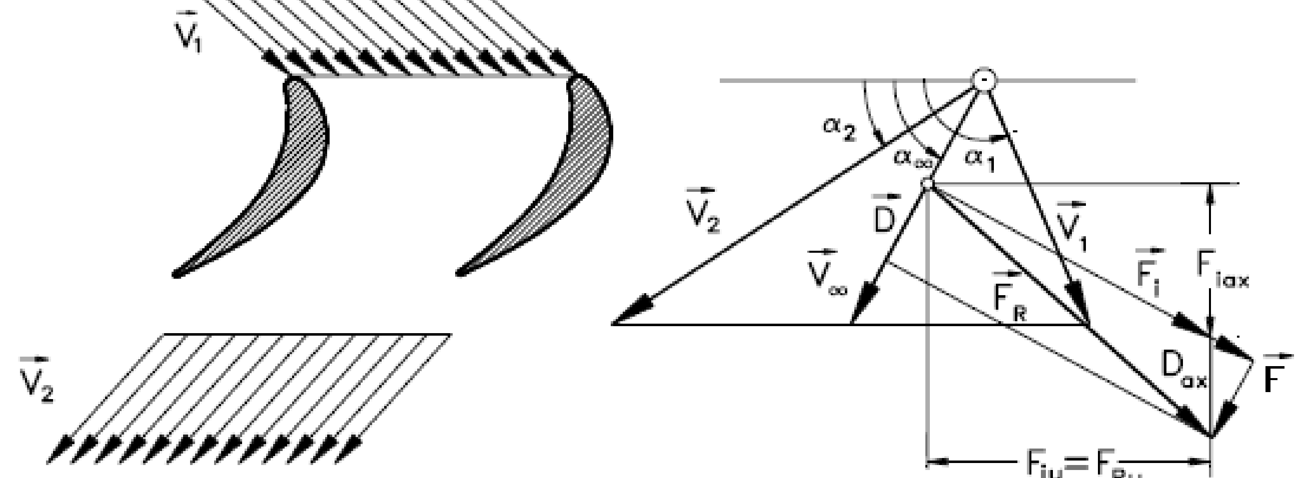


Turbine Cascade

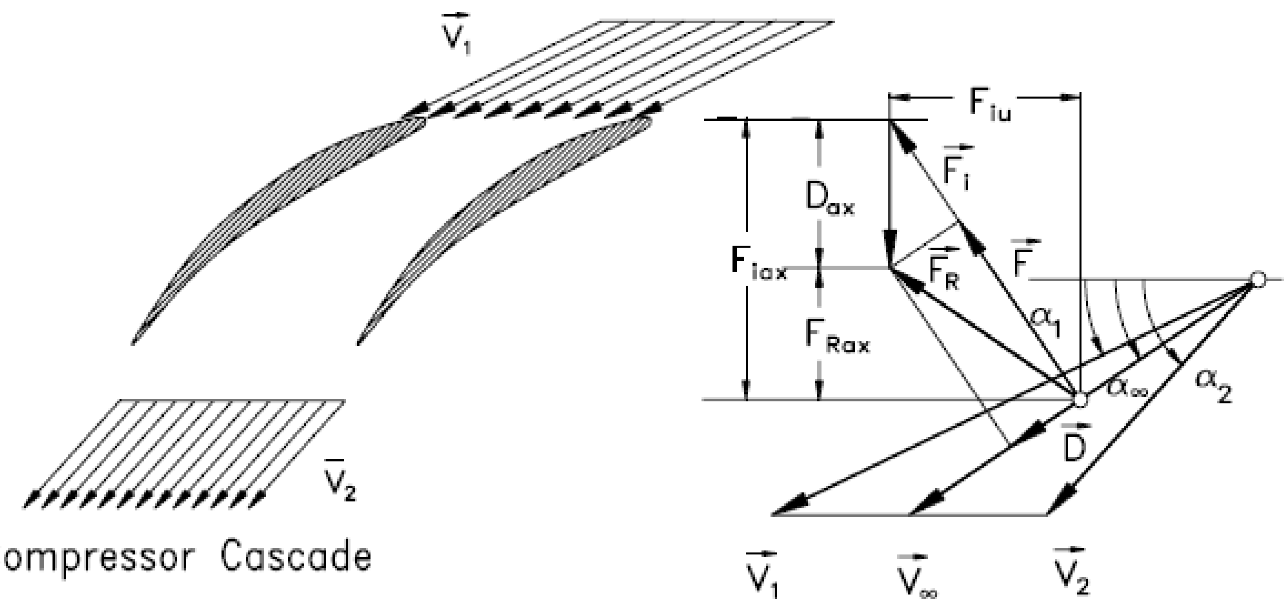
- Viscous effects manifest themselves in the form to total pressure losses.
- Wakes from the blade trailing edge lead to non-uniform velocity leaving the blades.
- In addition to lift, drag is another force that will be considered in the analysis.
- A component of drag actually contributes to the effective lift.
- We define total pressure loss coefficient as:

$$\overline{\omega} = \frac{P_{01} - P_{02}}{\frac{1}{2}\rho V_2^2}$$

Viscous Flow Forces

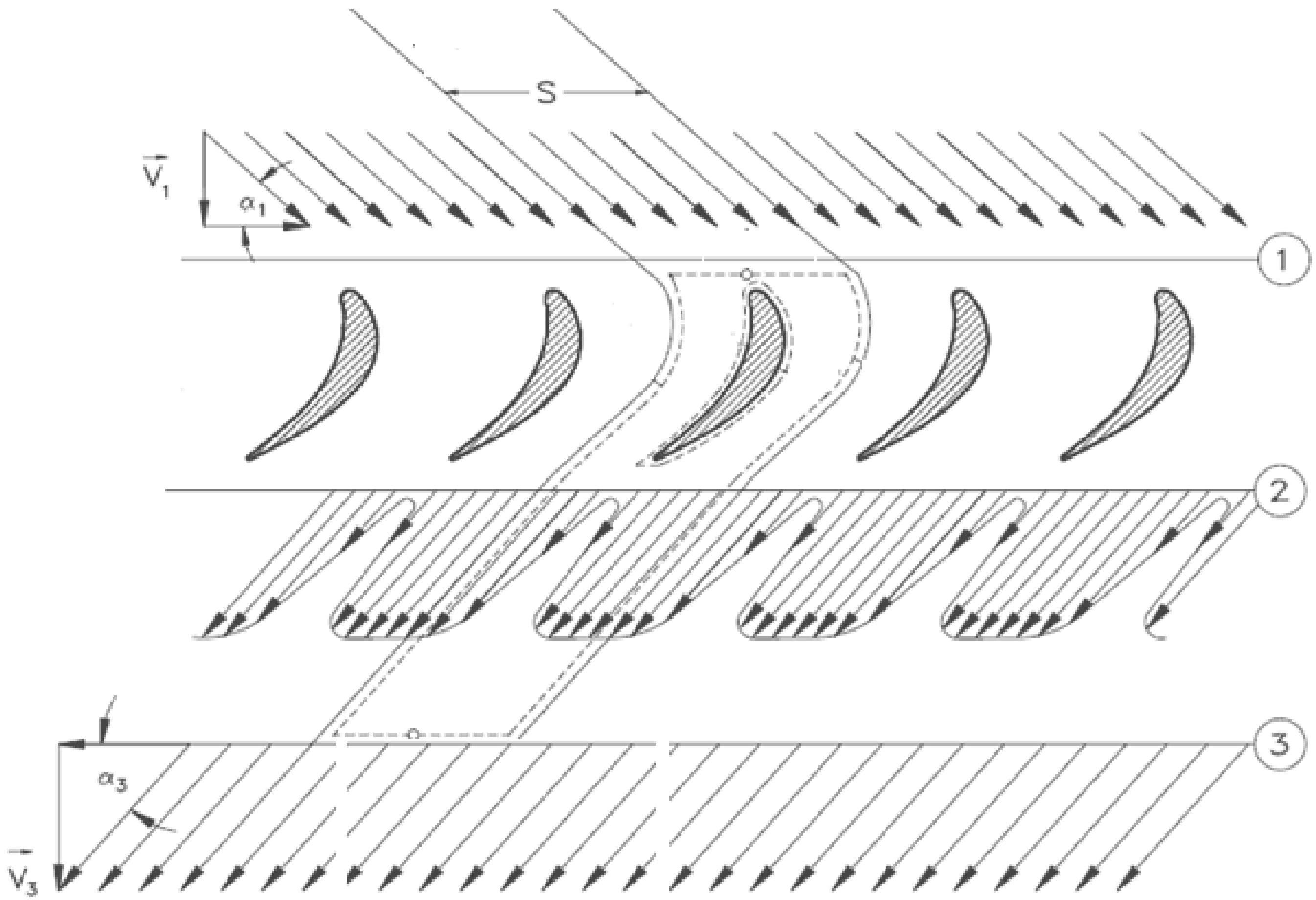


Turbine Cascade



Compressor Cascade

Turbine Cascade



Viscous flow through a turbine cascade

Turbine Cascade

Drag is given by, $D = \overline{\omega}(S/C)\cos\alpha_m$

The effective lift

$$L + \overline{\omega}(S/C)\cos\alpha_m = \rho V_m \Gamma + \overline{\omega}(S/C)\cos\alpha_m$$

Therefore, the lift coefficient,

$$C_L = 2\frac{S}{C}(\tan\alpha_2 - \tan\alpha_1)\cos\alpha_m + \overline{\omega}(S/C)\cos\alpha_m$$

$$C_L \frac{C}{S} = 2(\tan\alpha_2 - \tan\alpha_1)\cos\alpha_m + \overline{\omega}\cos\alpha_m$$

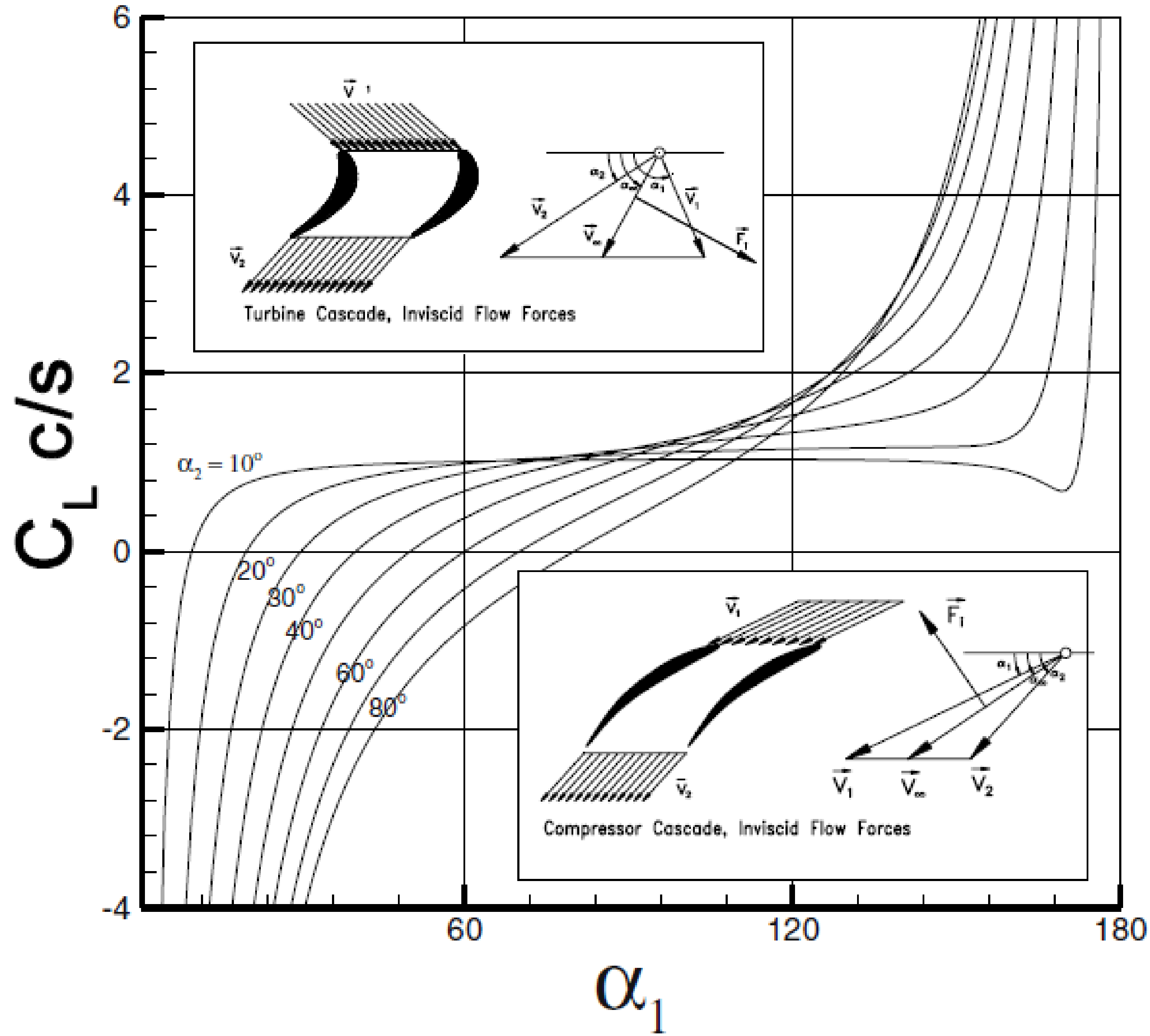
Turbine Cascade

- Based on the calculation of the lift and drag coefficients, it is possible to determine the blade efficiency.
- Blade efficiency is defined as the ratio of ideal static pressure drop to obtain a certain change in KE to the actual static pressure drop to produce the same change in KE.

$$\eta_b = \frac{1 - \frac{C_D}{C_L} \tan \alpha_m}{1 + \frac{C_D}{C_L} \cot \alpha_m}$$

If we neglect the C_D term in the lift definition,

$$\eta_b = \frac{1}{1 + \dfrac{2C_D}{C_L \sin 2\alpha_m}}$$

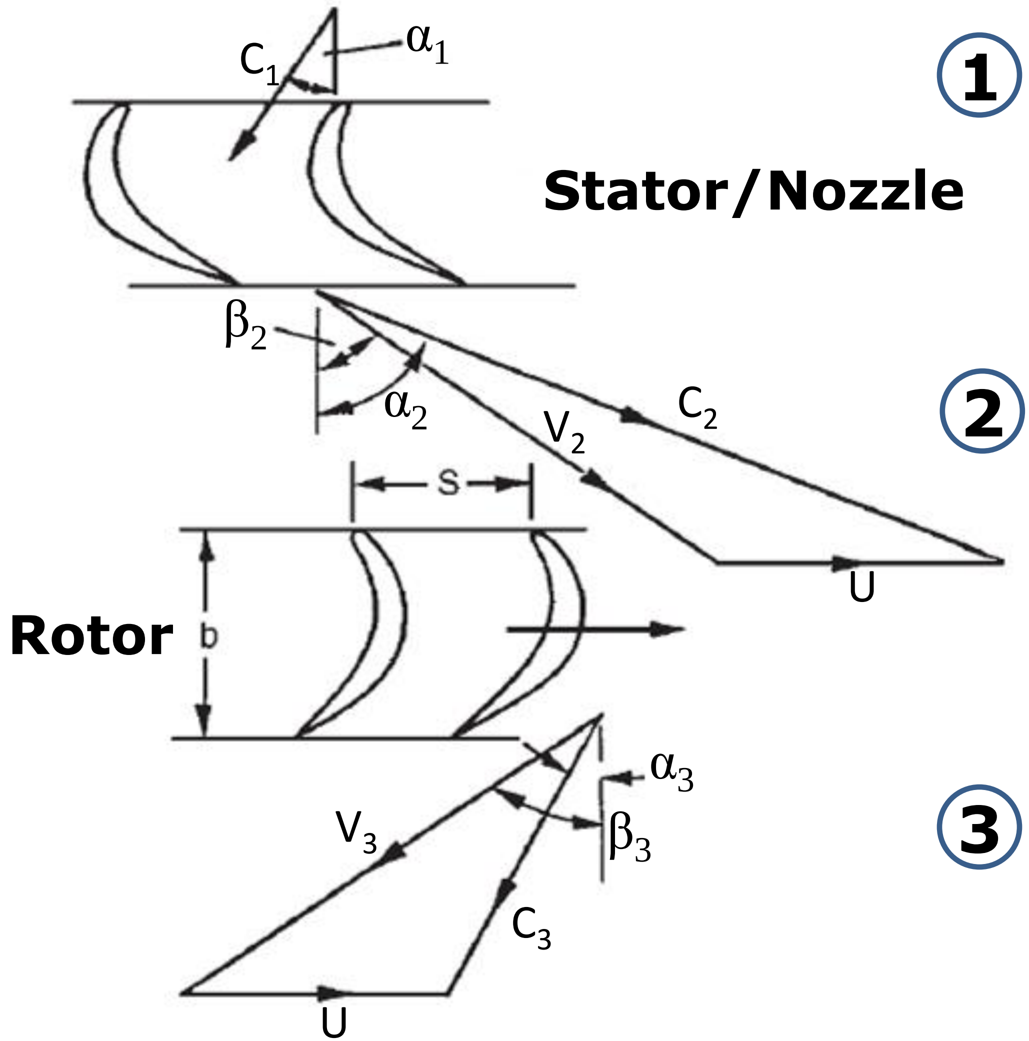


Turbine Cascade, Inviscid Flow Forces
Compressor Cascade, Inviscid Flow Forces
$\alpha_2 = 10°$
20°
30°
40°
60°
80°
$C_L\ c/s$
α_1
6
4
2
0
-2
-4
60
120
180

Degree of reaction

- Acceleration takes place in both rotor and the stator.
- Enthalpy drop in the rotor as well as the stator.
- Degree of reaction provides a measure of the extent to which the rotor contributes to the overall enthalpy drop in the stage.

Velocity triangles



Degree of reaction

$$R_x = \frac{\text{Static enthalpy drop in the rotor}}{\text{Stagnation enthalpy drop in the stage}}$$

$$= \frac{h_2 - h_3}{h_{01} - h_{03}}$$

Since, in a coordinate system fixed to the rotor, the apparent stagnation enthalpy is constant,

$$h_2 - h_3 = \frac{V_3^2}{2} - \frac{V_2^2}{2}$$

If the axial velocity is the same upstream and downstream of the rotor, this becomes,

$$h_2 - h_3 = \tfrac{1}{2}(V_{w3}^2 - V_{w2}^2) = \tfrac{1}{2}(V_{w3} - V_{w2})(V_{w3} + V_{w2})$$

Also, since $h_{01} - h_{03} = U(C_{w2} - C_{w3})$

Degree of reaction

$$R_X = \frac{(V_{w3} - V_{w2})(V_{w3} + V_{w2})}{2U(C_{w2} - C_{w3})}$$

$$\text{Since, } (V_{w3} - V_{w2}) = (C_{w3} - C_{w2})$$

$$\text{Therefore, } R_X = -\frac{(V_{w3} + V_{w2})}{2U}$$

$$\text{We know that, } V_{w3} = C_a \tan\beta_3$$

$$\text{and } V_{w2} = C_a \tan\alpha_2 - U$$

$$\text{so that } R_X = \tfrac{1}{2}\left[1 - \frac{C_a}{U}(\tan\alpha_2 + \tan\beta_3)\right]$$

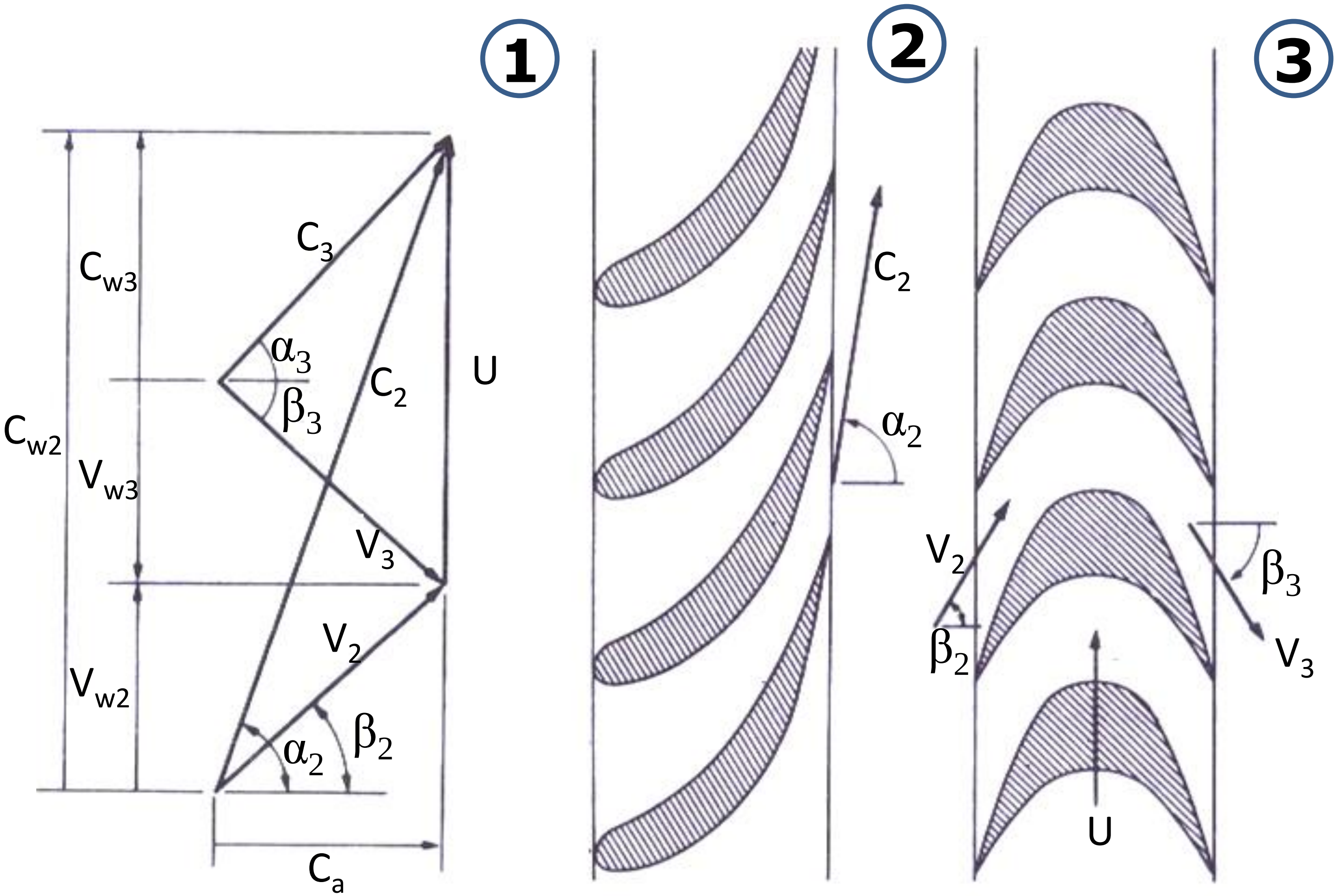
Degree of reaction

It can be seen that for a special case of symmetrical triangles, $\alpha_2 = -\beta_3$, $R_X = 0.5$.

When, $V_{w3} = -V_{w2}$, $R_X = 0 \rightarrow$ Impulse turbine

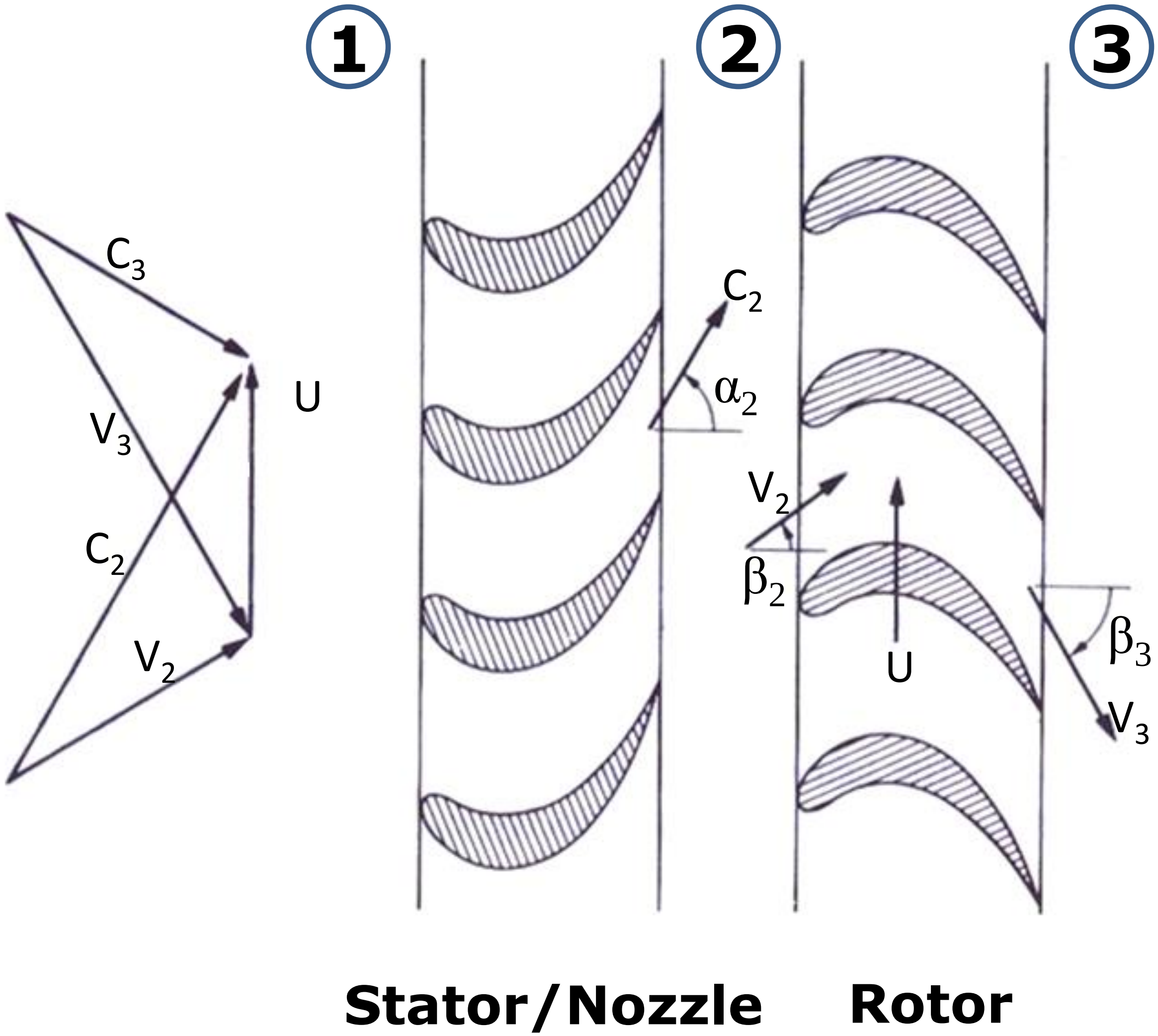
For a given stator outlet angle, the impulse turbine stage requires a much higher axial velocity ratio than does the 50% reaction stage. In the impulse turbine stage, all the flow velocities are higher and that is one of the reason why its efficiency is lower than that of a 50% reaction stage.

Impulse turbine stage



Stator/Nozzle **Rotor**

50% Reaction turbine stage



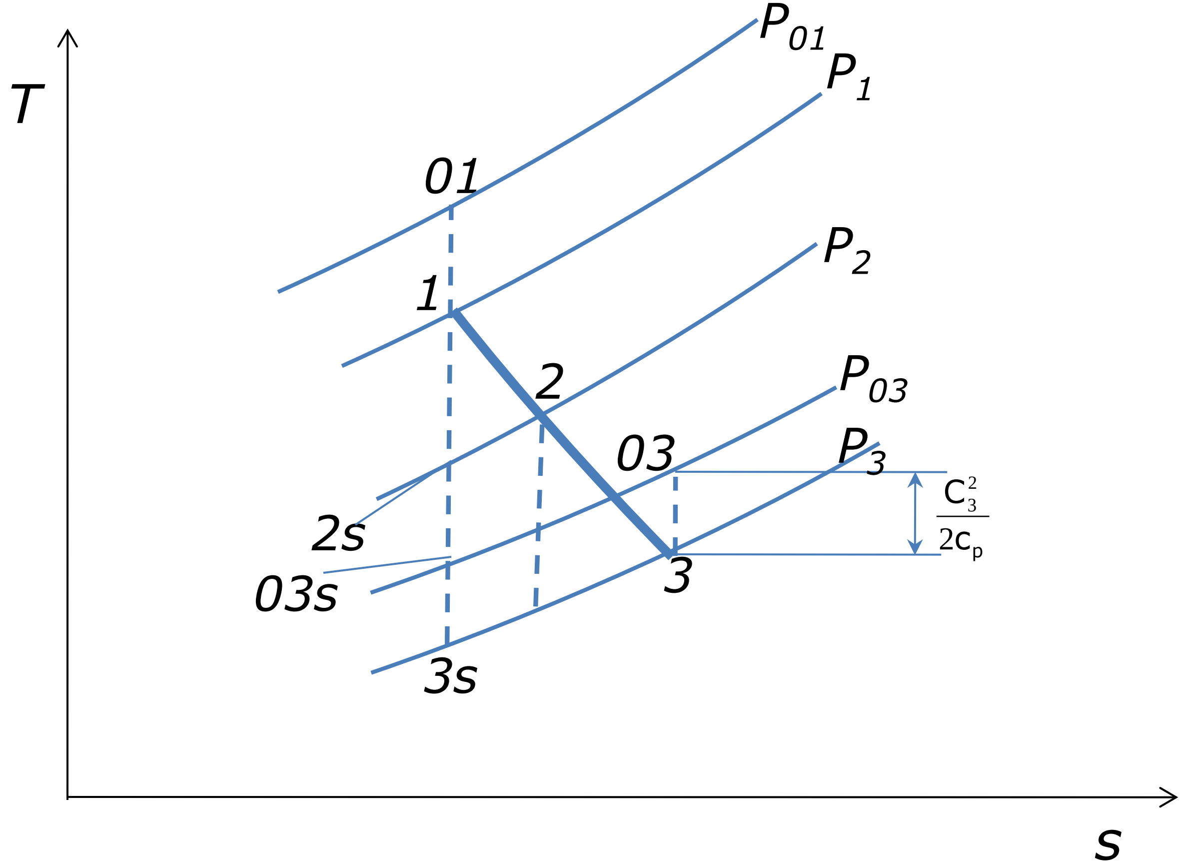
Efficiency

- We noted that the aerodynamic losses in the turbine differ with the stage configuration, or the degree of reaction.
- Improved efficiency is associated with higher reaction, which implies less work per stage and therefore a higher number of stages for a given overall pressure ratio.
- The understanding of losses is important to design, not only in the choice of the configuration, but also on methods to control these losses.

Efficiency

- There are two commonly used turbine efficiency definitions.
  - Total-to-static efficiency
  - Total-to-total efficiency
- The usage of the efficiency definition depends upon the application.
- In land-based power plants, the useful turbine output is in the form of shaft power and exhaust KE is a loss.
- In this case the ideal turbine process would be isentropic such that there is no exhaust KE.

Efficiency



Expansion process in a turbine stage

Efficiency

The ideal turbine work with no exhaust KE would be

$$w_{\mathrm{T,\,ideal}} = c_P (T_{01} - T_{3s})$$

The total-to-static efficiency is defined as

$$\eta_{\mathrm{ts}} = \frac{T_{01} - T_{03}}{T_{01} - T_{3s}}$$

$$= \frac{T_{01} - T_{03}}{T_{01}\left[1 - (P_3 / P_{01})^{(\gamma-1)/\gamma}\right]} = \frac{1 - (T_{03} / T_{01})}{\left[1 - (P_3 / P_{01})^{(\gamma-1)/\gamma}\right]}$$

Efficiency

In many applications (turbojets), the exhaust KE is not considered a loss as this is converted to thrust in such machines.

The ideal turbine work in such cases would be

$$\mathrm{w}_{\mathrm{T,\,ideal}} = c_P (T_{01} - T_{03s})$$

The total - to - total efficiency is defined as

$$\eta_{\mathrm{tt}} = \frac{T_{01} - T_{03}}{T_{01} - T_{03s}}$$

$$= \frac{T_{01} - T_{03}}{T_{01}\left[1 - (P_{03}/P_{01})^{(\gamma-1)/\gamma}\right]} = \frac{1 - (T_{03}/T_{01})}{\left[1 - (P_{03}/P_{01})^{(\gamma-1)/\gamma}\right]}$$

Efficiency

We can compare the two definitions of efficiency by making an approximation:

$$T_{03s} - T_{3s} \cong T_{03s} - T_3 = C_3^2 / 2c_p$$

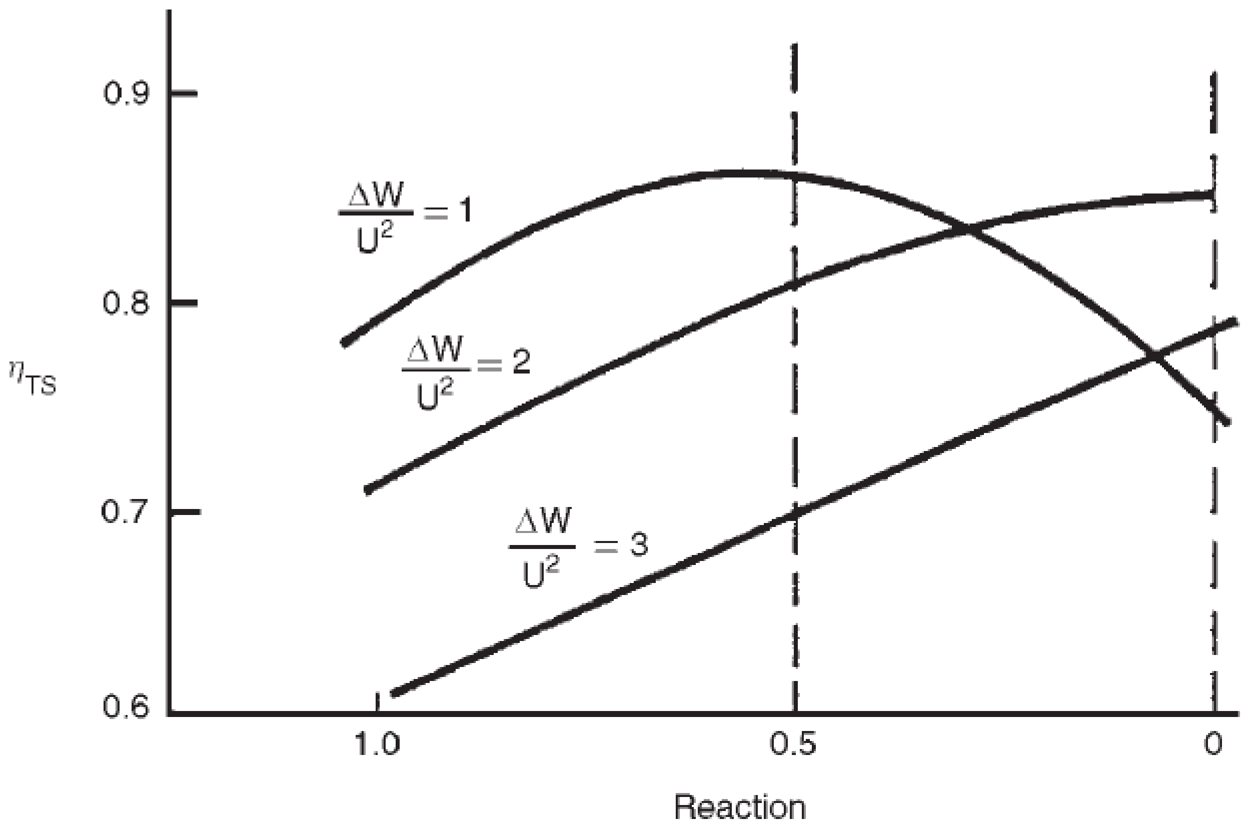
Therefore, $\eta_{tt} = \dfrac{\eta_{ts}}{1 - C_3^2 \left[2c_p(T_{01} - T_{3s})\right]}$

We can see that, $\eta_{tt} > \eta_{ts}$

The efficiency definitions can also be related to the specific work done in the following way:

$$w_t = \eta_{tt} c_p T_{01} \left[1 - \left(\frac{P_{03}}{P_{01}} \right)^{(\gamma - 1)/\gamma} \right] \text{ and } w_t = \eta_{ts} c_p T_{01} \left[1 - \left(\frac{P_3}{P_{01}} \right)^{(\gamma - 1)/\gamma} \right]$$

Efficiency



Influence of loading on the total-to-static efficiency