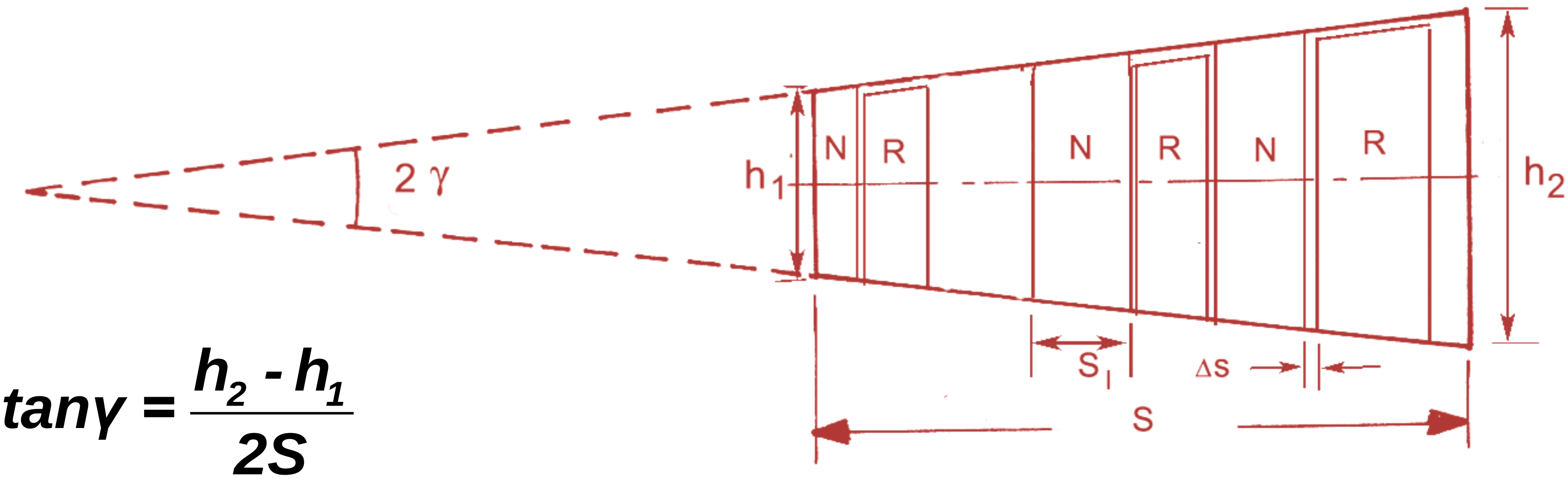
- Recap: Lecture 18, 6th October 2015, 1530-1655 hrs.
  - Losses in a turbine
    - 2D and 3D losses
    - Deviation: un-choked and choked turbine
  - Performance characteristics
  - Axial turbine blade geometries
  - Optimum number of blades: Zweifel criterion
  - Exit flow matching
    - Turbine-Nozzle
    - Turbine- Compressor
    - Inter-spool matching

Multi-staging

- Requirement for multi-staging of turbines comes from the aggregate of shaft work that needs to be produced.
- Typically if turbine pressure ratio requirement is more than 2.5 / 3.0 - multi-staging is required.
- As compression ratio over the years have kept on increasing, multi-staging has become inevitable in all aero-engines.
- Number of integer stages to be decided by the state of art of turbine design



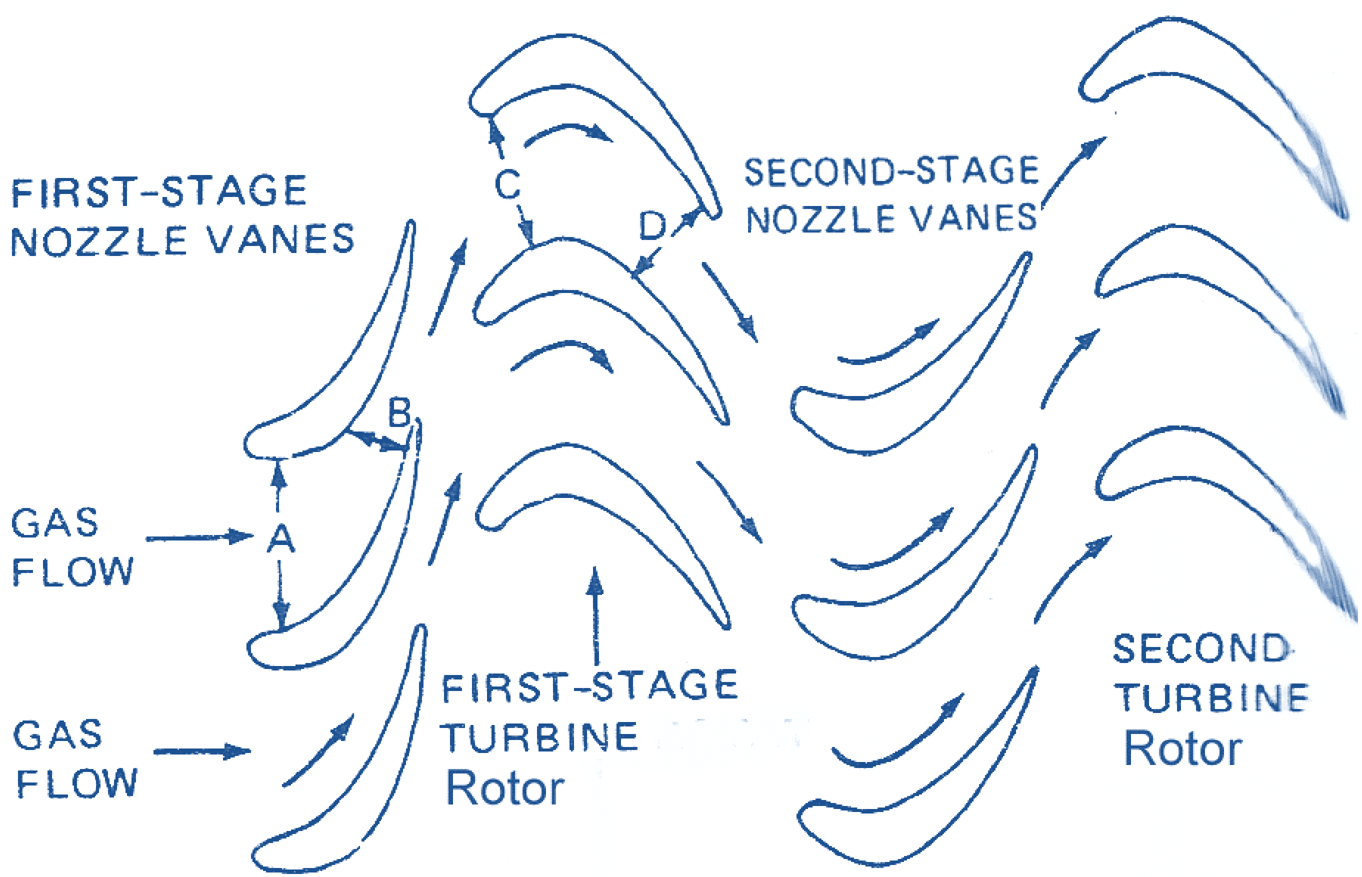
$$\tan\gamma = \frac{h_2 - h_1}{2S}$$

$$S = \Sigma \Delta S + \Sigma S_i = \Delta S . Z_p + S_i . Z$$

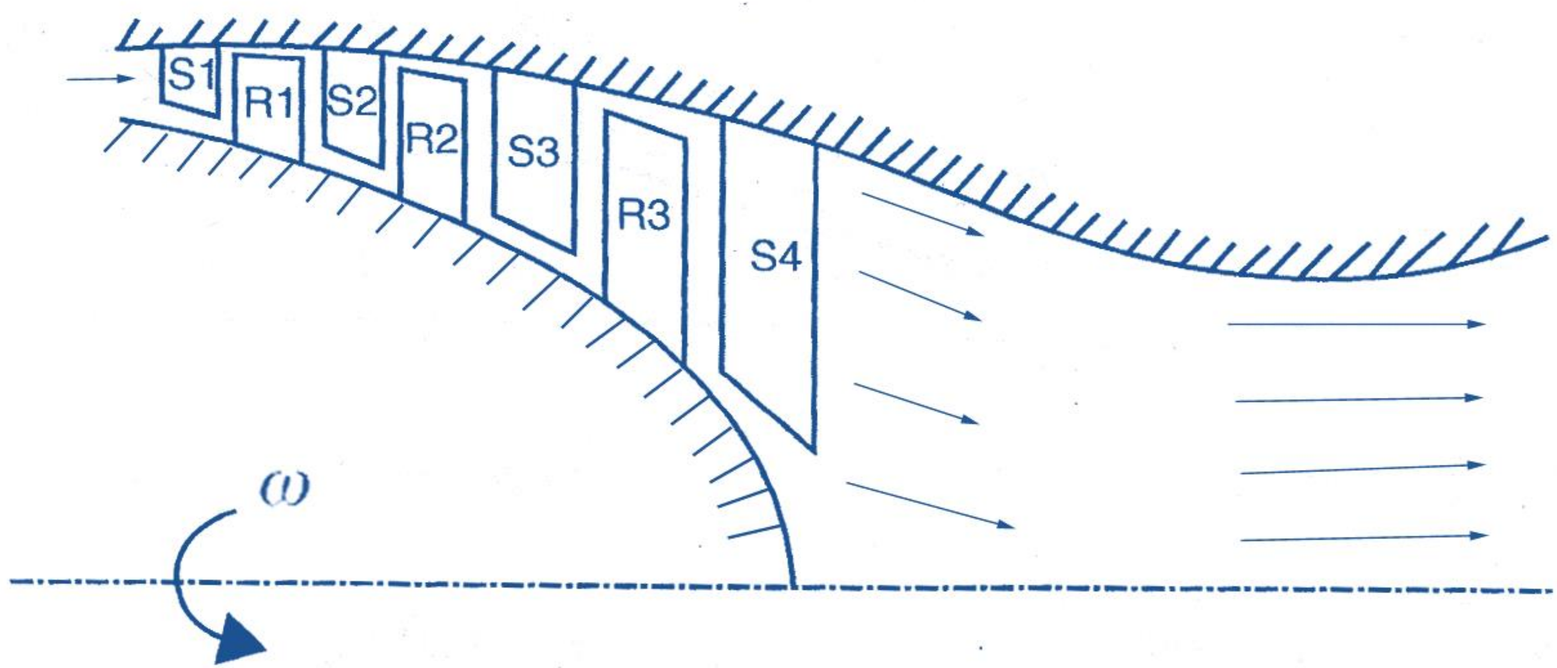
where Z is the number of stages, and $Z_p = 2.Z - 1$

$$\tan\gamma = \frac{h_2 - h_1}{2Z_p\left(S_i + \Delta S\right)} = \frac{S_i\left(\overline{h_2} - \overline{h_1}\right)}{2Z_pS_i\left(1 + \dfrac{\Delta S}{S_i}\right)} = \frac{\overline{h_2}\left(1 - \dfrac{\overline{h_1}}{\overline{h_2}}\right)}{2Z_p\left(1 + \dfrac{\Delta S}{S_i}\right)}$$

Flow through the blades is non-axial and varies from the root to the tip



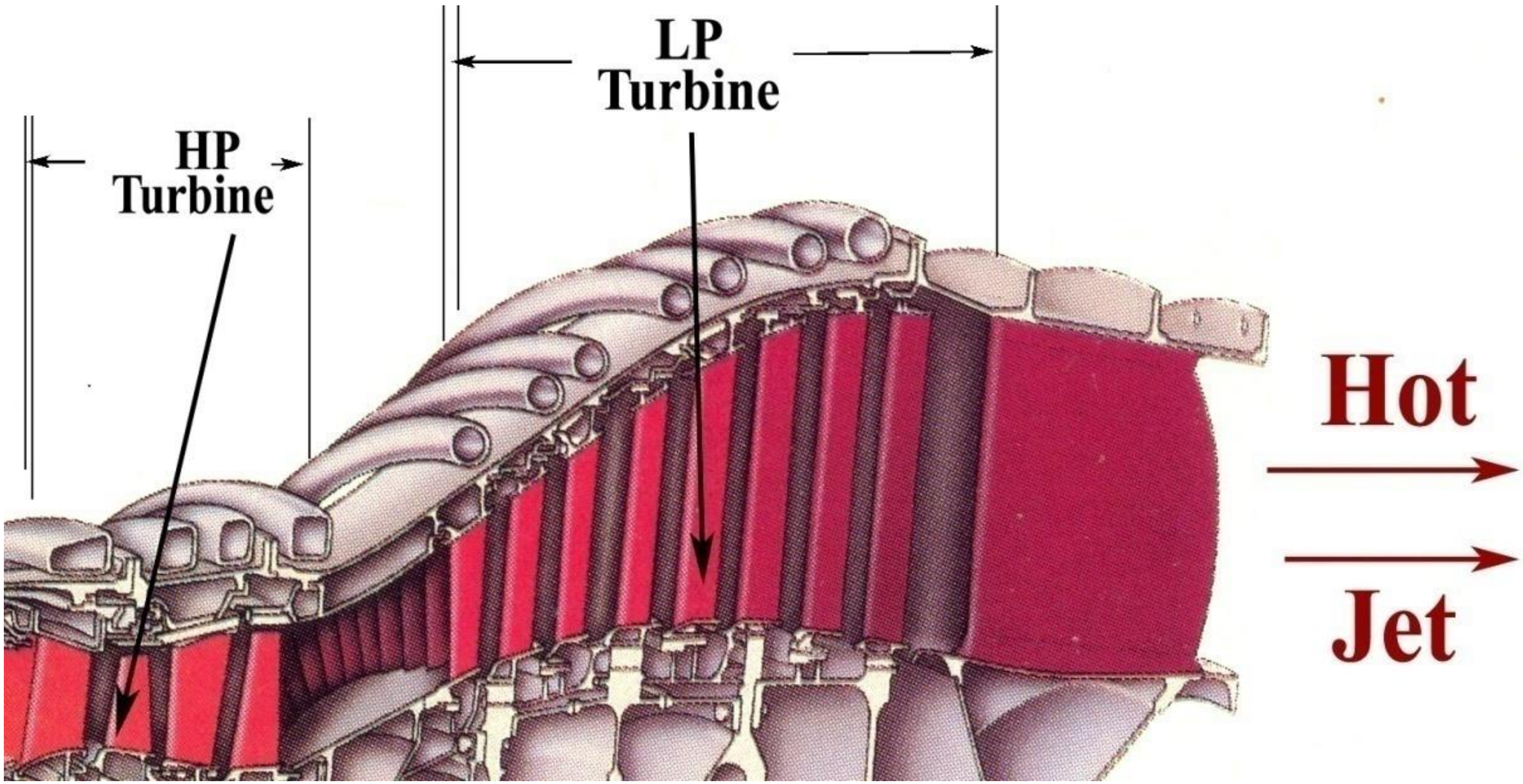
Axial Flow track in modern multi-stage turbines is often curved



Multi-stage flow analysis

- Flow track design decision comes from continuous application of continuity condition
- The track is diverging in axial direction
- Flow paths through the blades are generally in curved converging passages.
- This, thus, requires application of 3-D flow analysis to get accurate notion of the flow
- Most modern turbines are analyzed using 3-D CFD analytical techniques

Multi-staging of Turbine

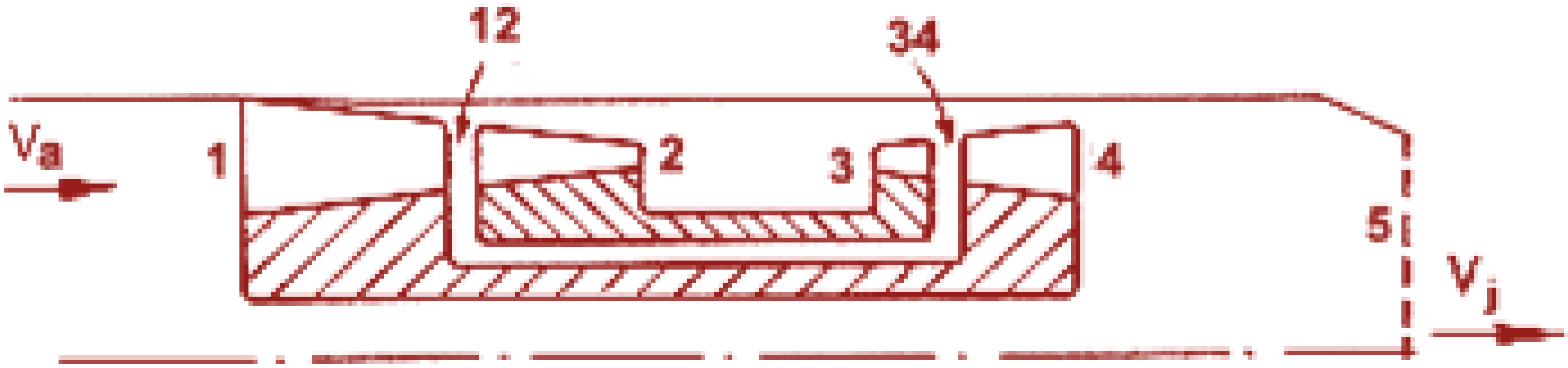


Multi-stage HP + LP turbine layout; Civil Engine

Compressor Turbine Matching

SINGLE SPOOL ENGINE

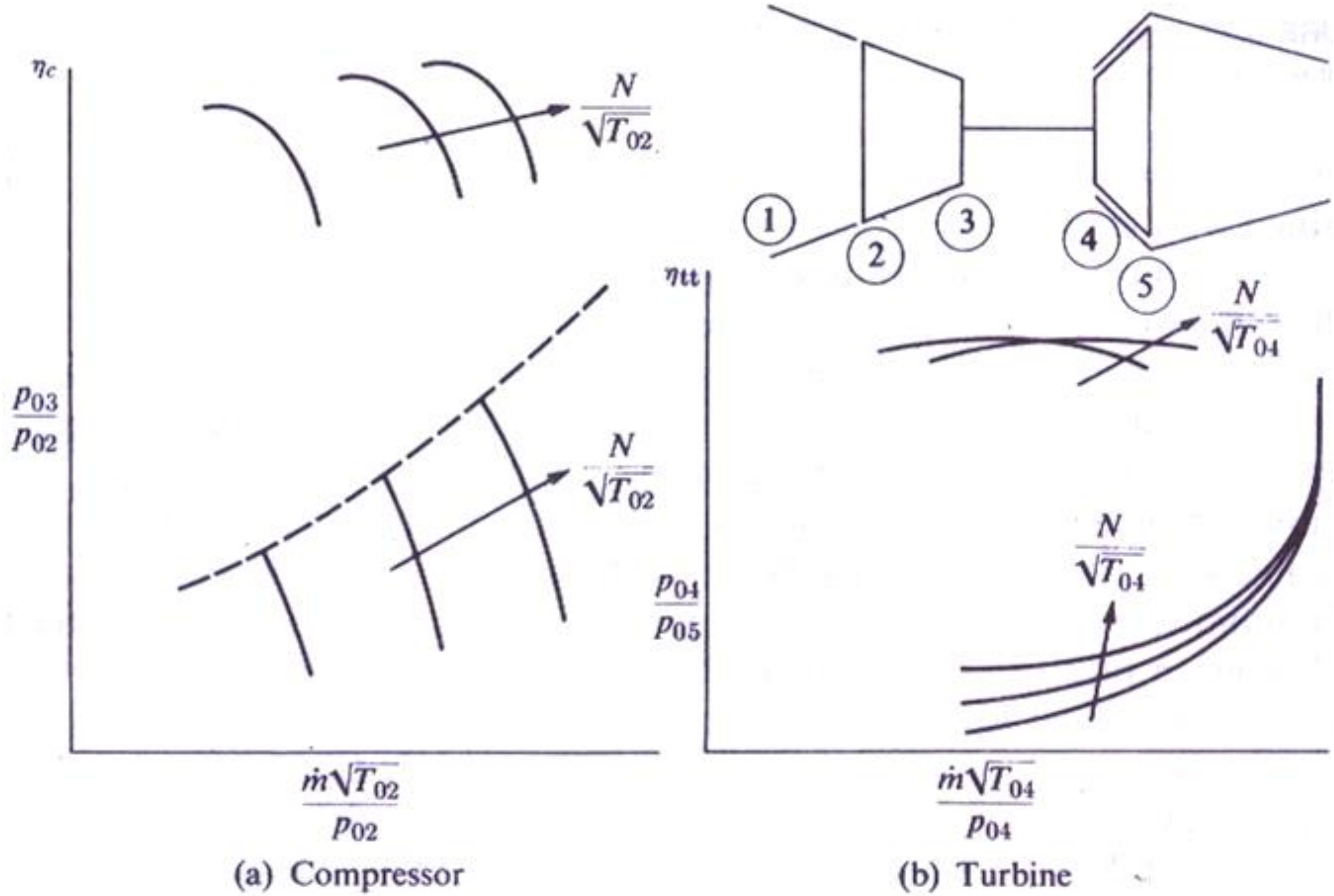
TWO-SPOOL TURBOJET



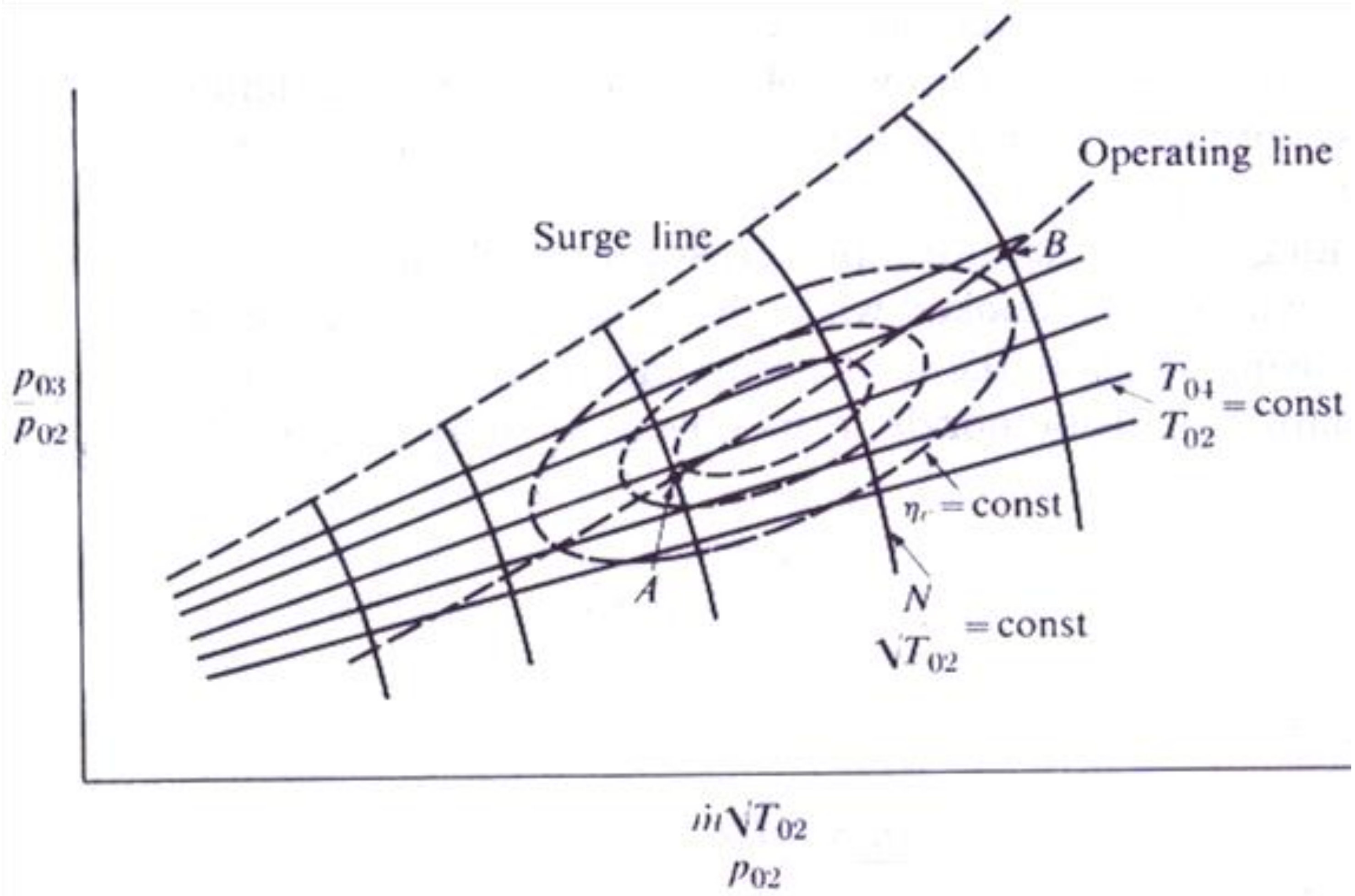
- Turbine – compressor matching
  - Mass flow
    - Turbine mass flow = compressor mass flow + Fuel mass flow – bleed mass flow
  - Power
    - Turbine power output = required compressor power
  - Compressor and turbine performance maps
  - Single spool and multi spool engine matching procedures

- Matching procedure
  1. Select an operating speed
  2. Assume turbine inlet temperature, T_{04}
  3. Assume compressor pressure ratio
  4. Calculate compressor work per unit mass
  5. Calculate turbine pressure ratio required to produce this work
  6. Check mass flow continuity, else repeat 4-6
  7. Now calculate pressure ratio across the jet nozzle from pressure ratios across diffuser, compressor and turbine
  8. Calculate area of the jet nozzle to pass turbine mass flow.
  9. If this area does not match with assumed area, assume a new value of T_{04} and repeat 2-9.

  This procedure is then repeated for various operating speeds



Typical compressor and turbine performance maps



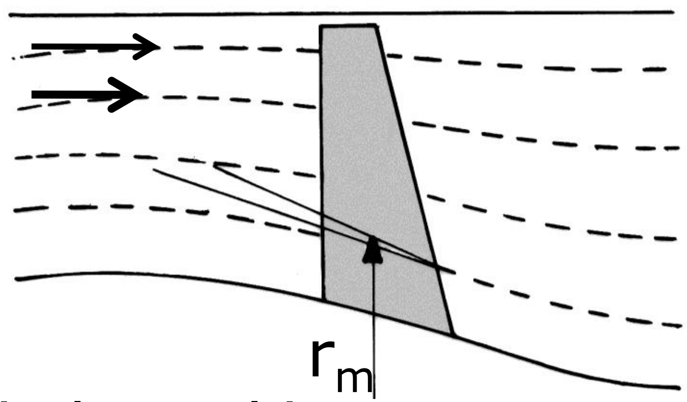
Compressor map with operating line and throttle characteristics

3-D flow in axial flow turbines

- It is assumed that radial motion takes place in the blade passage only

$$C_r << C_a \; ; \quad C_r << C_w$$

- The stream surface has a cylindrical shape



- Simplified radial equilibrium equation is valid

$$\frac{1}{\rho}\frac{dp}{dr} = \frac{1}{r}.C_w^2$$

Following three 3-D flow models in axial turbines are often used for design and analysis

1) Free Vortex flow

2) Constant nozzle exit angle, α_2

3) Arbitrary vortex case, $C_w = r^n$

1) Free Vortex Flow model

$C_w.r$ = constant, applied on the rotor flow which normally entails a few assumptions:

At turbine rotor entry ,

$dH_{02}/dr = 0$; $C_{w2}.r$ = const.; C_{a2} = const.

Rotor specific work done :

$$H_{02} - H_{03} = U\,(C_{w2} + C_{w3}) = \omega(r_2.C_{w2} - r_3.C_{w3})$$

$$= \text{const.}$$

With $C_{w3}.r$ = constant, it follows C_{a3} = const.

Hence, for obtaining various parameters along blade length following may be adopted:

1) All thermodynamic properties are constant in the annulus

2) $\tan\alpha_2 = (r_m/r)_2 \tan \alpha_{2m}$

3) $\tan\beta_2 = (r_m/r)_2 \tan \alpha_{2m} - (r/r_m)_2 . U_m/C_{a2}$

4) $C_{w3}.r$ = constant, C_{a3} = const= C_{a2}

5) $\tan\alpha_3 = (r_m/r)_3 \tan \alpha_{3m}$

6) $\tan\beta_3 = (r_m/r)_3 \tan \alpha_{3m} + (r/r_m)_3 . U_m/C_{a3}$

Constant nozzle exit angle model

This model has been utilized for the practical purpose of creating stator-nozzle blades with zero twist. When stator-nozzles are facing very high inlet temperature elaborate cooling mechanism is embedded inside the blades; to facilitate efficient cooling of the blades, it is thought that such blades may not be twisted at all.

$$\alpha_2 = \text{constant}$$

$$\cot\alpha_2 = \frac{C_{a2}}{C_{w2}} = const$$

$$C_{a2} = C_{w2}.\cot\alpha_2; \quad which yields \frac{dC_{a2}}{dr} = \frac{dC_{w2}}{dr}.\cot\alpha_2$$

Now invoking the *radial equilibrium equation* in energy eqn.

$$\frac{dH}{dr} = C_a \frac{dC_a}{dr} + C_w \frac{dC_w}{dr} + \frac{C_w^2}{r}$$ and, $$\frac{dH}{dr} = 0$$

We get, $$C_a \frac{dC_a}{dr} + C_w \frac{dC_w}{dr} + \frac{C_w^2}{r} = 0$$

$$C_{w2}.\cot^2\alpha_2.\frac{dC_{w2}}{dr}+C_{w2}.\frac{dC_{w2}}{dr}+\frac{C_{w2}^{\,2}}{r}=0$$

$$C_{w2}(1+\cot^2\alpha_2).\frac{dC_{w2}}{dr}+\frac{C_{w2}^{\,2}}{r}=0$$

$$\frac{dC_{w2}}{dr}=-\sin^2\alpha_2.\frac{dr}{r}$$

which, on integration yields

$$C_{w2}.r^{\sin^2\alpha_2} = const \ ;$$

and then $C_{w2} = C_{w2m}\left(\frac{r_m}{r}\right)^{\sin^2\alpha_2}$

alternately, $C_{a2}.r^{\sin^2\alpha_2} = const$

and then $C_{a2} = C_{a2m}\left(\frac{r_m}{r}\right)^{\sin^2\alpha_2}$

and finally in terms of absolute velocity,

$$C_2 = C_{2m}\left(\frac{r_m}{r}\right)^{\sin^2\alpha_2}$$

So, at the rotor inlet station one can say,

if

$$\alpha_2 = constant$$

then,

$$\frac{c_{w2}}{c_{w2m}} = \frac{c_{a2}}{c_{a2m}} = \frac{c_2}{c_{2m}} = \frac{r}{r_m}$$

Now, there are three possibilities:

a) Constant H_{03} at the rotor outlet

b) Zero whirl velocity at the outlet, i.e.

$$\alpha_3 = 0$$

a) Free Vortex at the outlet

a) Constant Total Enthalpy at the outlet condition, if applied

$$U\,(C_{w2} + C_{w3}) = \Delta H_0$$

And, whirl component of the velocity at rotor outlet is found from :

$$C_{w3} = \frac{\Delta H_0}{U} - C_{w2} = \frac{K}{r} - C_{a2}\tan\alpha_2$$

$$\text{where, } K = \Delta H_0 / \omega$$

And, subsequently C_{a3} may be also computed Both of which are computed from root to tip, using the variation shown previously.

b) Zero rotor exit whirl velocity $\boldsymbol{\alpha_3}$ **=** 0

this means, $dH/dr = C_{a3} \, .dC_{a3}/dr$

And ,

$$H_{03} = H_{02} - U\,C_{w2} = H_{02} - U.C_{w2m}(r_m/r)^{\sin^2\alpha}$$

Which, produces the enthalpy distribution radially at exit :

$$\frac{dH_{03}}{dr} = \frac{d}{dr}\left[U.C_{w2m}.\left(\frac{r_m}{r}\right)^{\sin^2\alpha_2}\right]$$

Turbine blade cooling

- For a given pressure ratio and adiabatic efficiency, the turbine work per unit mass is proportional to the inlet stagnation temperature.
- Therefore, typically a 1% increase in the turbine inlet temperature can cause 2-3% increase in the engine output.
- Therefore there are elaborate methods used for cooling the turbine nozzle and rotor blades.
- Turbine blades with cooling can withstand temperatures higher than that permissible by the blade materials.

Turbine blade cooling

- Thrust of a jet engine is a direct function of the turbine inlet temperature.
- Brayton cycle analysis, effect of maximum cycle temperature on work output and efficiency.
- Materials that are presently available cannot withstand a temperature in excess of 1300 K.
- However, the turbine inlet temperature can be raised to temperatures higher than this by employing blade cooling techniques.
- Associated with the gain in performance is the mechanical, aerodynamic and thermodynamic complexities involved in design and analysis of these cooling techniques.

Turbine blade cooling

- The environment in which the nozzles and rotors operate are very extreme.
- In addition to high temperatures, turbine stages are also subjected to significant variations in temperature.
- The flow is unsteady and highly turbulent resulting in random fluctuations in temperatures.
- The nozzle is subjected to the most severe operating conditions.

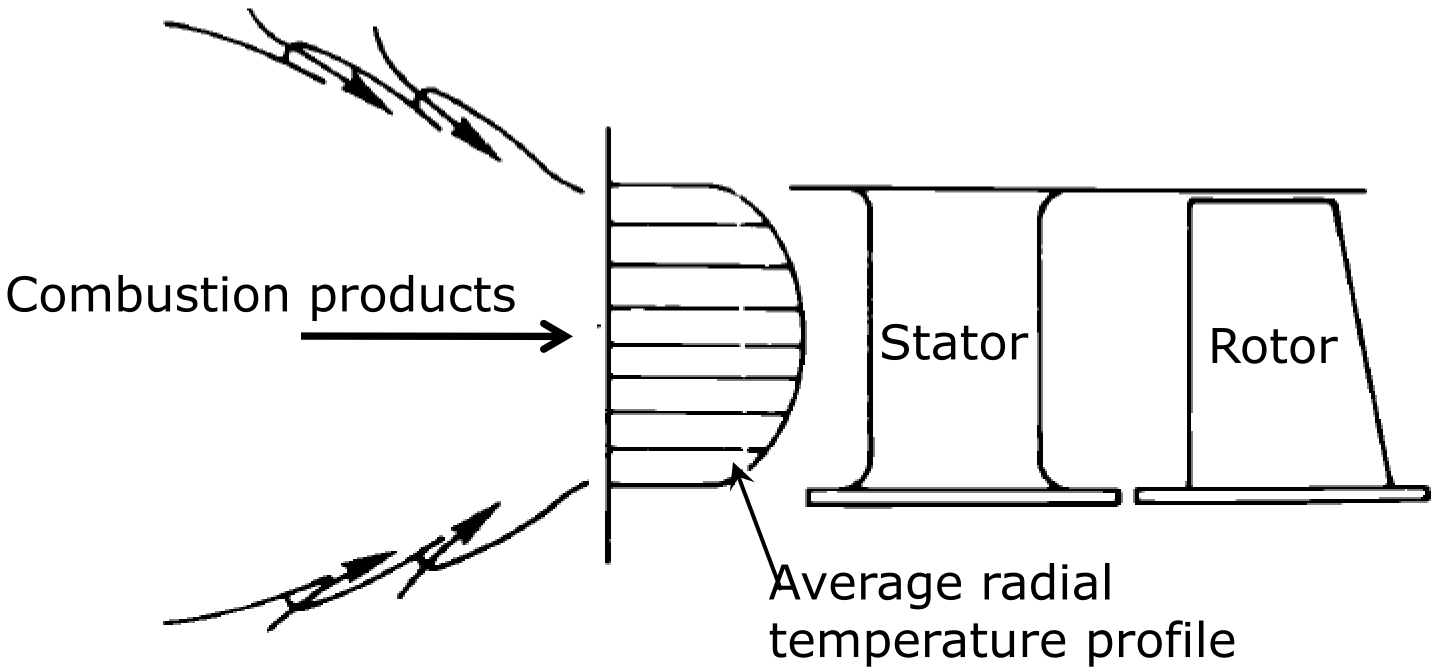
Turbine blade cooling

- Because the relative Mach number that the rotor experiences, it perceives lower stagnation temperatures (about 200-300 K) than the nozzle.
- However the rotor experience far more stresses due to the high rotational speeds.
- The highest temperatures are felt primarily by the first stage.
- Cooling problems are less complicated in later stages of the turbine.

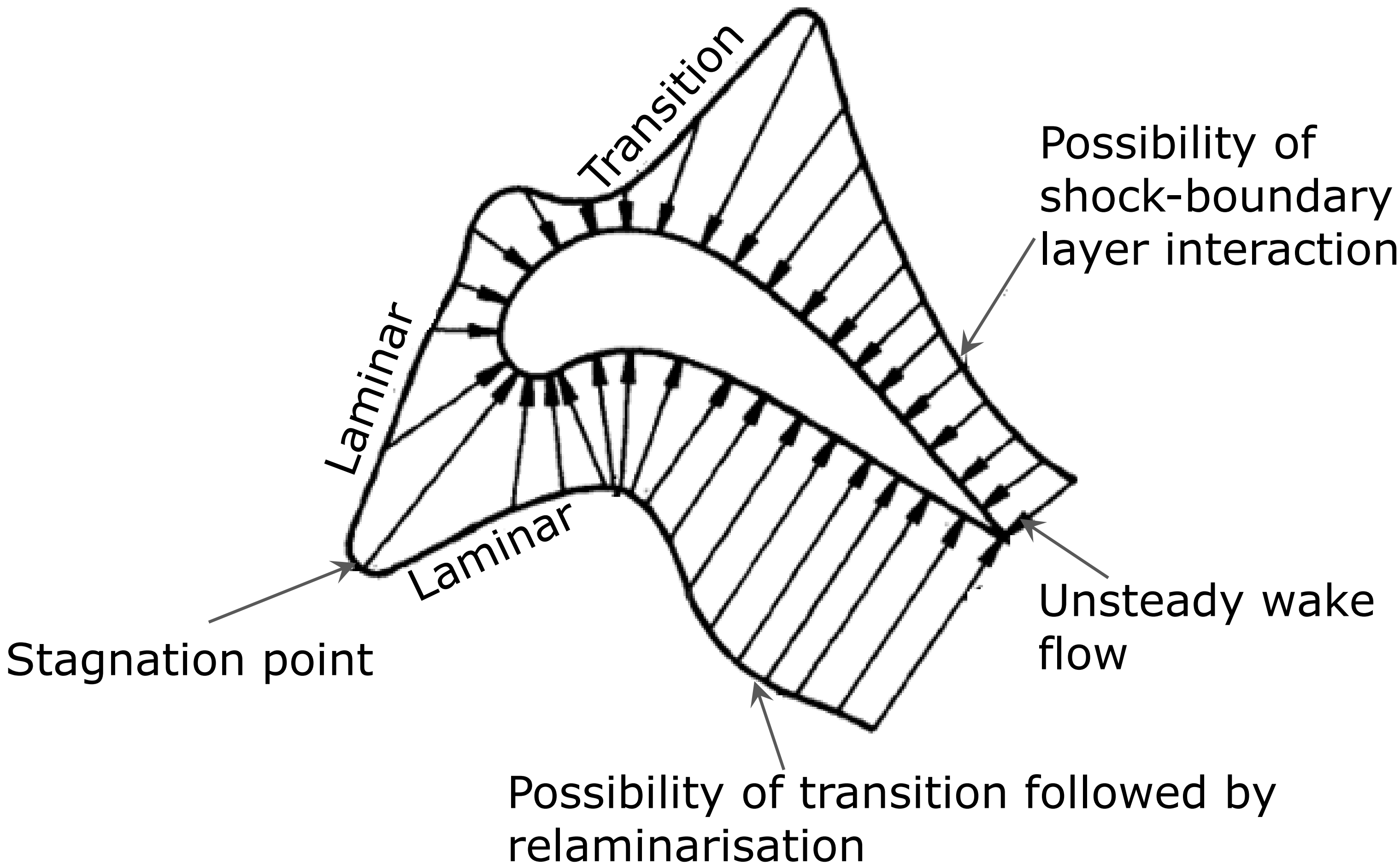
Turbine blade cooling

- There are several modes of failure of a turbine blade.
  - Oxidation/erosion/corrosion
    - Occurs due to chemical and particulate attack from the hot gases.
  - Creep
    - Occurs as a result of prolonged exposure to high temperatures.
  - Thermal fatigue
    - As a result of repeated cycling through high thermal stresses.

Turbine blade cooling



Average temperature profile entering a turbine stage



Variation of heat transfer around a turbine blade