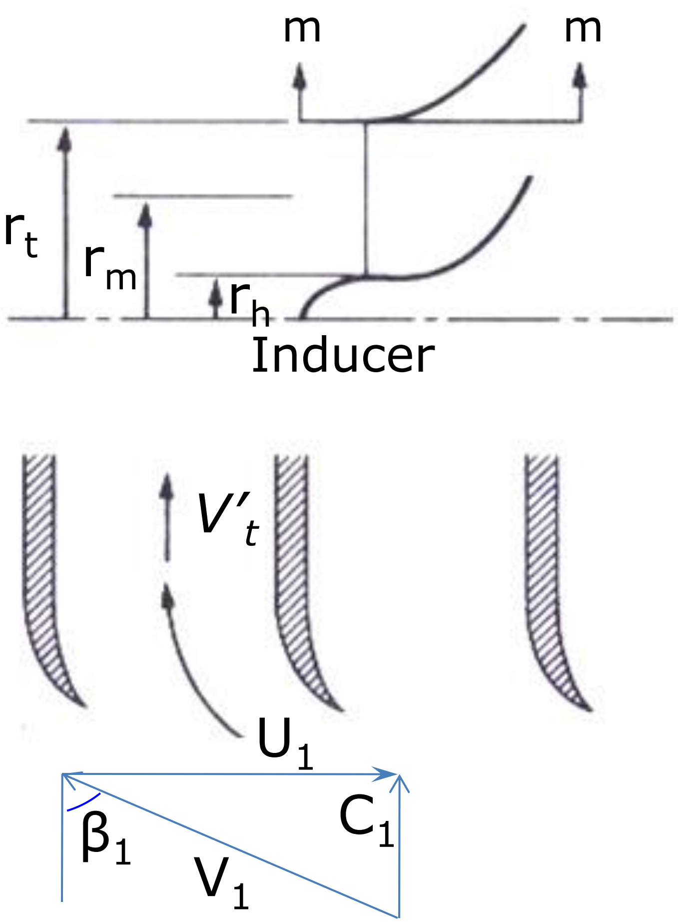
- Recap: Lecture 21, 16th October 2015, 1530-1655 hrs.
  - Centrifugal compressors
    - Introduction
    - Construction and components
    - Thermodynamics
    - Stage pressure ratio
    - Conservation of rothalpy
    - Components of a centrifugal compressor
      - Impeller
      - Inducer

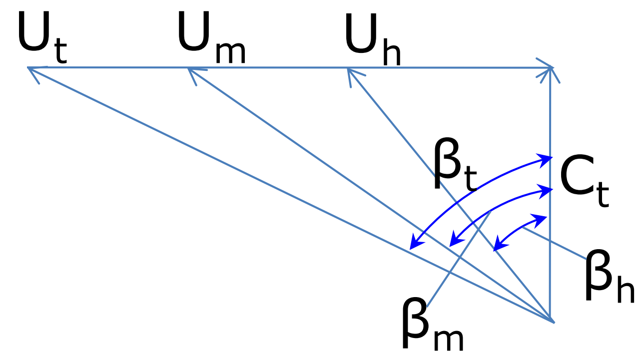
Inducer

- Inducer is the impeller entrance section where the tangential motion of the fluid is changed in the radial direction.
- This may occur with a little or no acceleration.
- Inducer ensures that the flow enters the impeller smoothly.
- Without inducers, the rotor operation would suffer from flow separation and high noise.

Inducer



Section m-m



Leading edge velocity triangles

Inducer

- It can be seen from the above that

  $V_t^{'} = V_{1t} \cos\beta_{1t}$

  Where, $V^{'}$ denotes the relative velocity at the inducer outlet.
- It can be seen that $V^{'} < V_1$, which indicates diffusion in the inducer.
- Similarly, we can see that the relative Mach number from the velocity triangle is,

$$M_{1rel} = M_1 / \cos\beta_{1t}$$

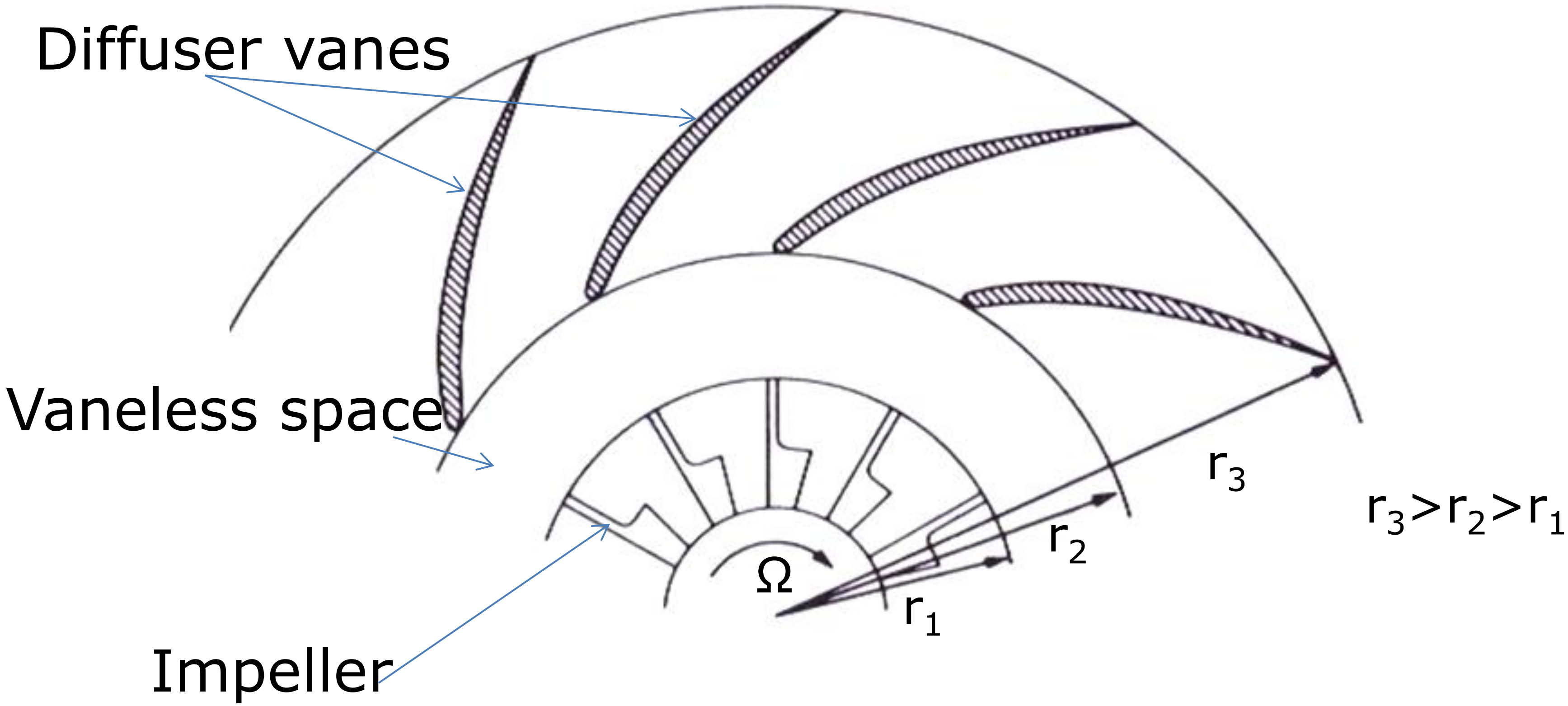
The diffuser

- High impeller speed results in a high absolute Mach number leaving the impeller.
- This high velocity is reduced (with an increase in pressure) in a diffuser.
- Diffuser represents the fixed or stationary part of the compressor.
- The diffuser decelerates the flow exiting the impeller and thus reduces the absolute velocity of the working fluid.
- The amount of deceleration depends upon the efficiency of the diffusion process.

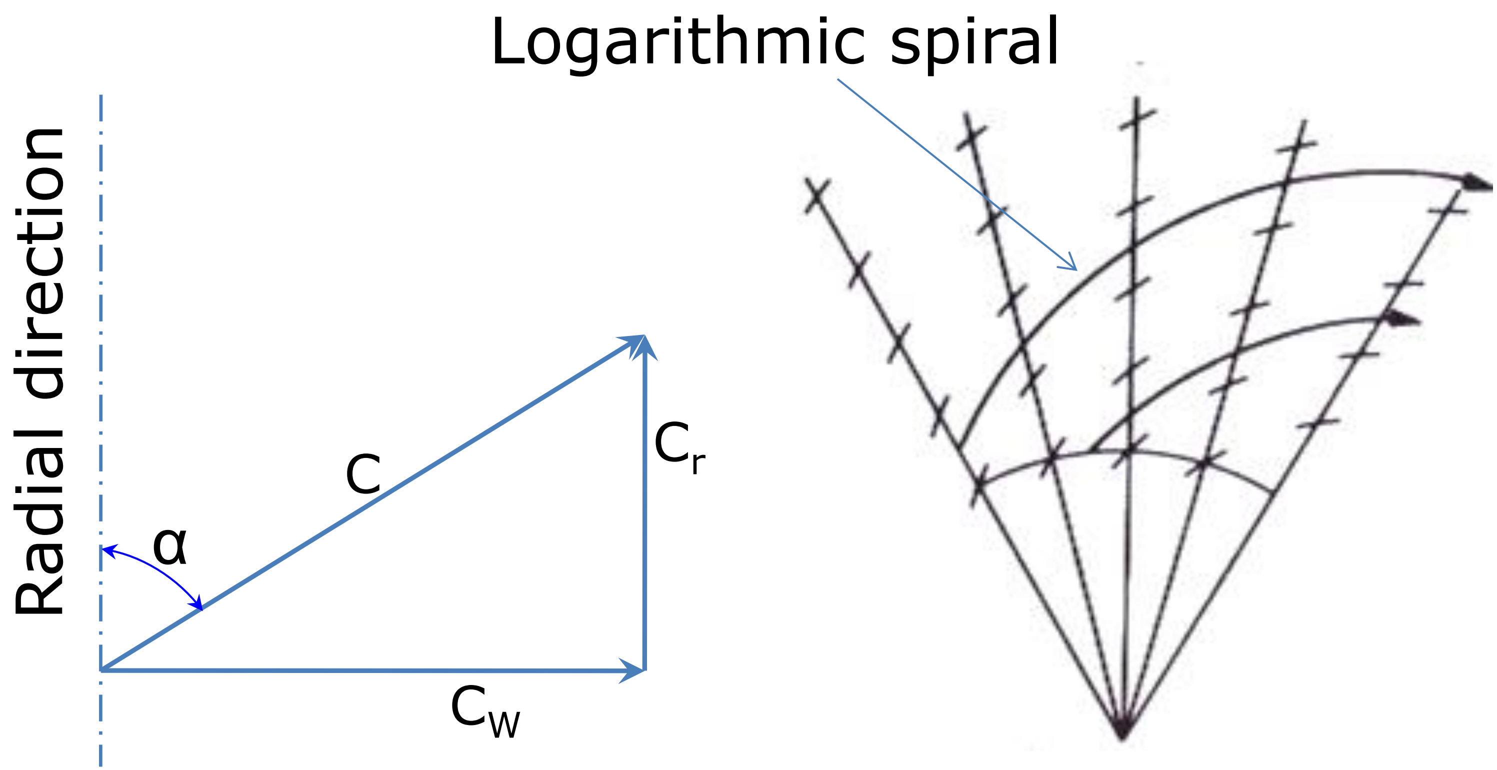
The diffuser

- The fluid flows radially outwards from the impeller, through a vaneless region and then through a vaned diffuser.
- Both vaned and the vaneless diffusers are controlled by boundary layer behaviour.
- Pipe and channel type diffusers are used in aero engines due to their compatibility with the combustors.

The diffuser



The diffuser



Streamlines in a radial diffuser

The diffuser

Let us consider an incompressible flow in a vaneless region of constant axial width.

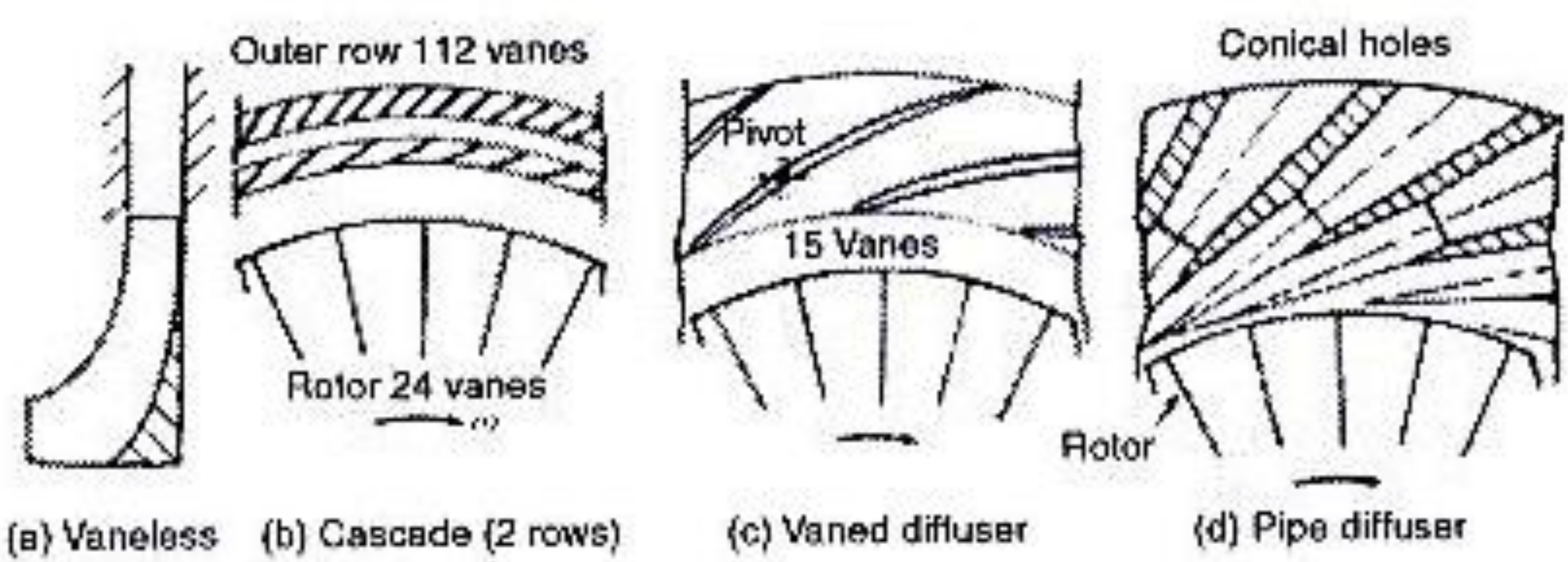
From continuity, $\dot{m} = \rho(2\pi rh)C_r =$ constant.

From conservation of angular momentum,

$rC_w =$ constant

$\therefore C_w/C_r =$ constant $= \tan\alpha$, where α is the angle between the velocity and the radial direction.

Thus, the velocity is inversely proportional to radius. This means that there is diffusion taking place in the vaneless space.

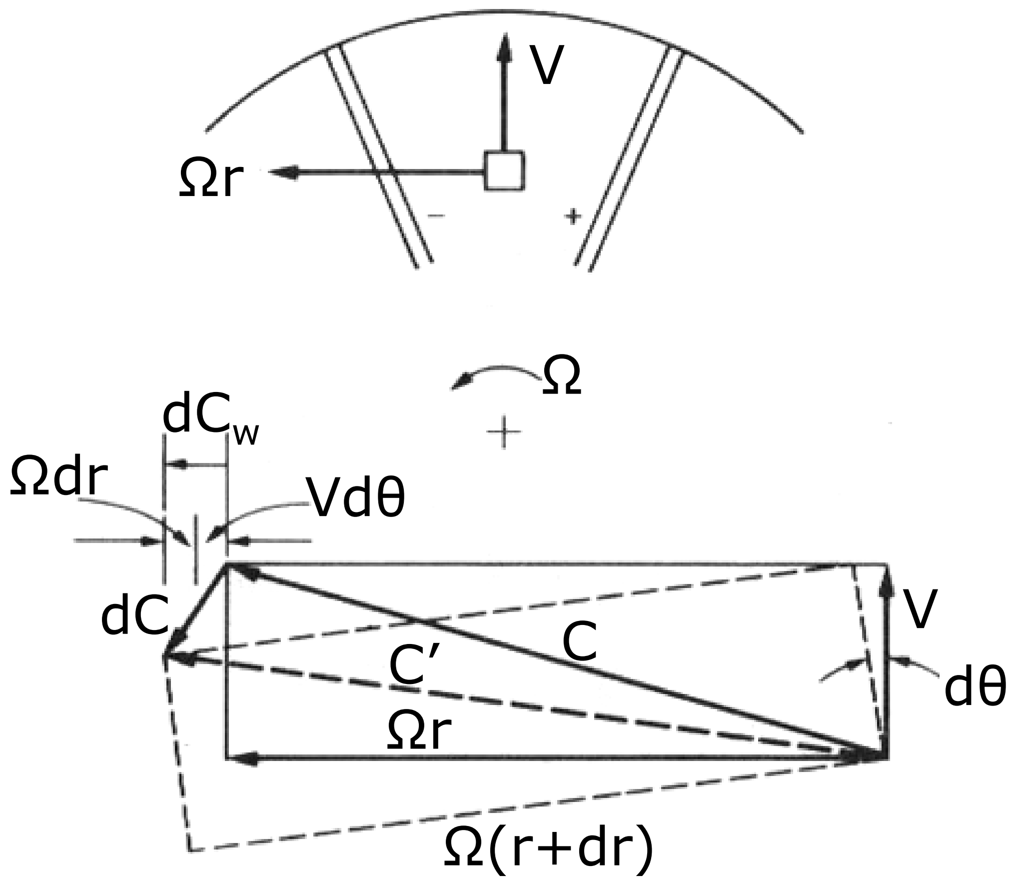


Different types of diffuser geometries

Coriolis acceleration

- We have discussed earlier that pressure change due to the centrifugal force field is not a cause of boundary layer separation.
- This can also be explained by the Coriolis forces that are present in centrifugal compressor rotors.
- Let us consider a fluid element travelling radially outward in the passage of a rotor.
- We shall examine the velocity triangles of this fluid during a time period *dt.*

Coriolis acceleration



Coriolis acceleration

- The magnitude of the relative velocity is "unchanged", but the particle has suffered an absolute change of velocity.

$$dC_w = \Omega dr + Vd\theta$$

$$\text{or, } dC_w = \Omega V dt + V\Omega dt,$$

$$\text{Thus, the Coriolis acceleration, } a_\theta = 2\Omega V$$

$$\text{and it requires a pressure gradient in the tangential}$$

$$\text{direction of magnitude, } \frac{1}{r}\frac{\partial P}{\partial \theta} = -2\rho\Omega V$$

Coriolis acceleration

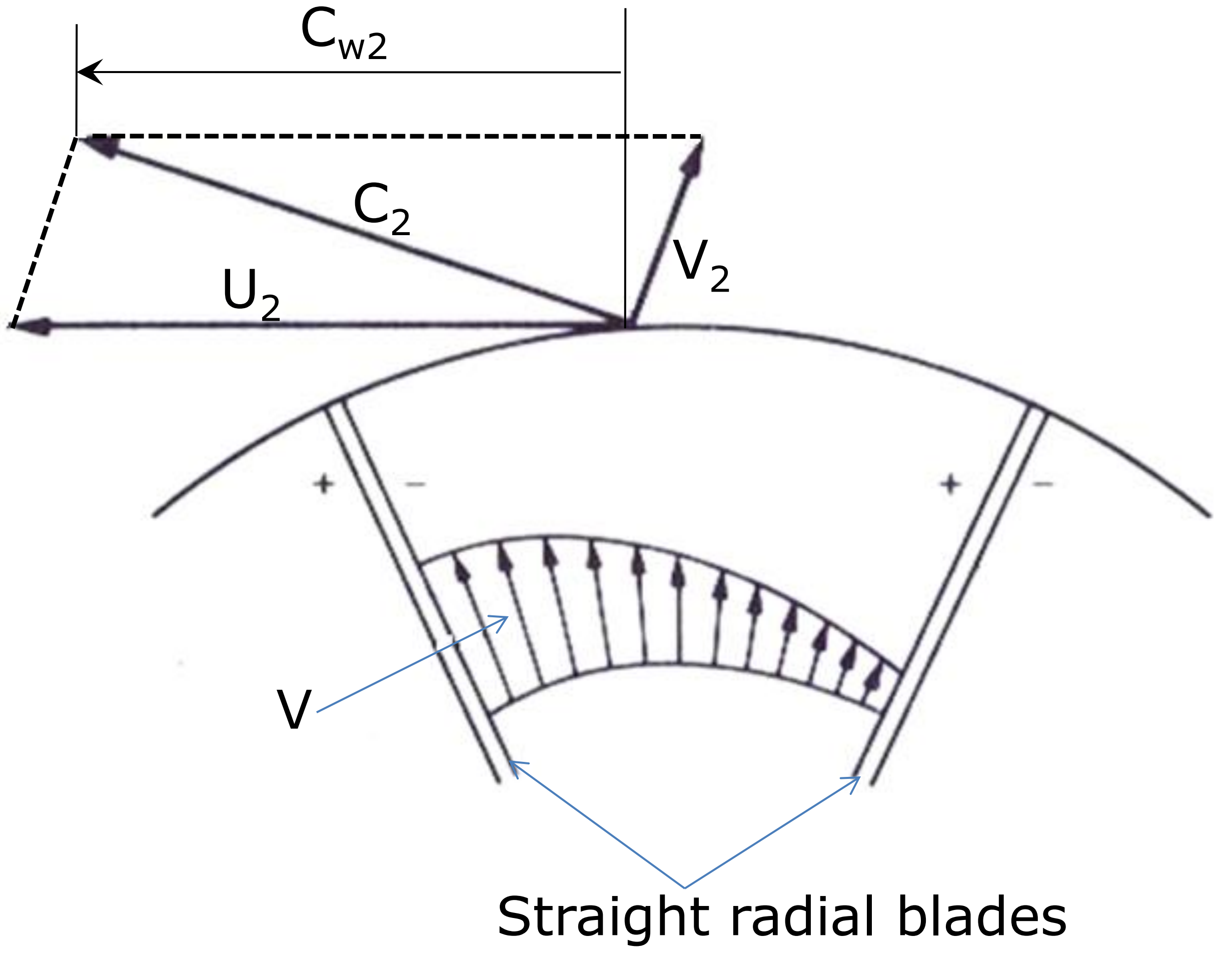
- The existence of the tangential pressure gradient means that there will be a positive gradient of V in the tangential direction.

$$\frac{1}{\rho}\frac{dP}{rd\theta} = -\frac{d\left(V^2/2\right)}{rd\theta} = -\frac{V}{r}\frac{dV}{d\theta}$$

Therefore, $\frac{1}{r}\frac{dV}{d\theta} = 2\Omega$

- This means that there will be a tangential variation in relative velocity.

Coriolis acceleration



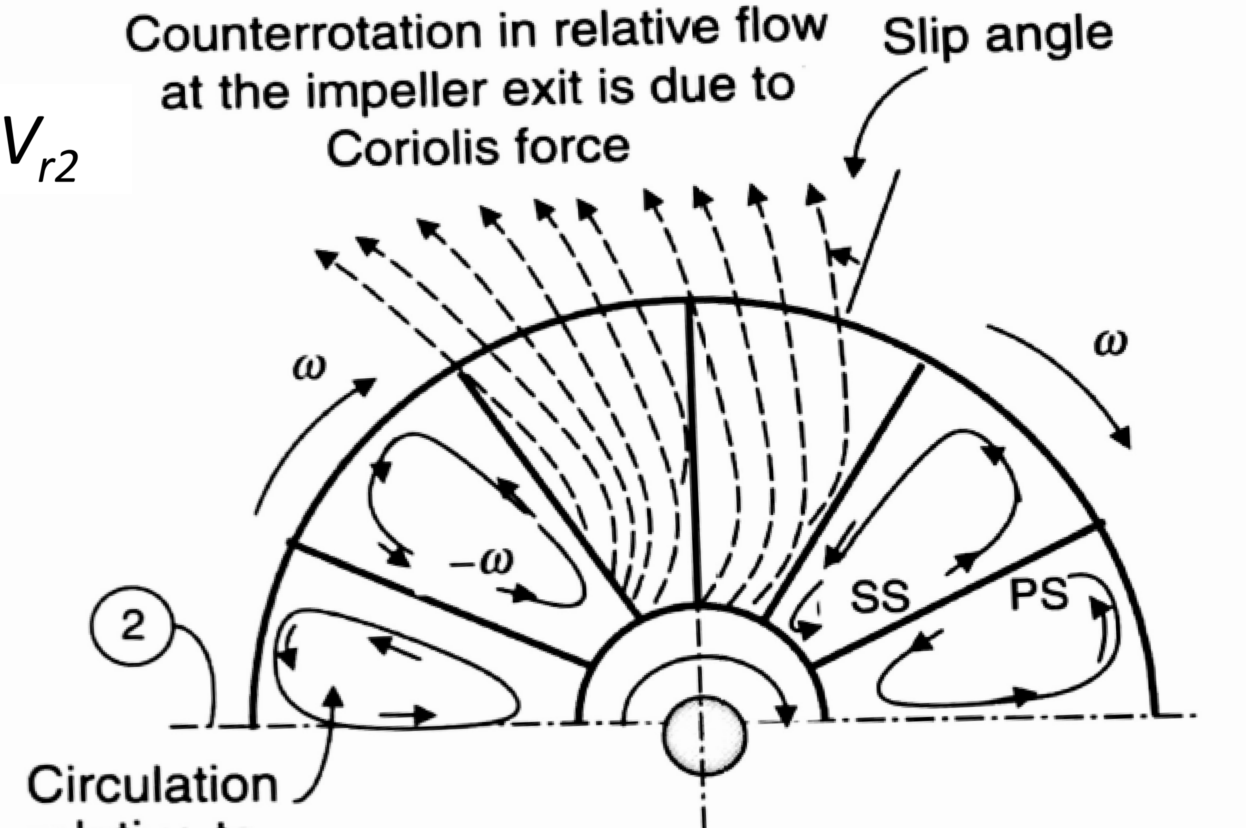
Slip factor

- Towards the outlet of the impeller, as the Coriolis pressure gradient disappears, there will be a difference between C_{w2} and U_2.
- This difference in the velocities is expressed as slip factor, $\sigma_s = C_{w2} / U_2$
- Generalised expression for slip:

$$(C_{w2})_{actual}/(C_{w2})_{ideal}$$

- The slip factor is approximately related to the number of blades of the impeller.
- For a straight radial blade, the slip factor is empirically expressed as $\sigma_s \approx 1 - 2/N$, where N is the number of blades.

$C_{r2}=V_{r2}$



(d) Flow circulation in the impeller passage causes “slip flow” at the exit [dashed lines are the relative streamlines that are bent backward]

Slip factor

- As the number of blades increases, the slip factor also increases and thus the slip lag at the tip of the impeller reduces.
- The effect of slip is to reduce the magnitude of swirl velocity and therefore the pressure ratio.
- The presence of slip means that to deliver the same pressure ratio, either the impeller diameter or the rotational must be increased.
- This in turn may lead to either increase in frictional losses or stresses on the impeller.

Performance characteristics

- The centrifugal compressor performance characteristics can be derived in the same way as an axial compressor.
- Performance is evaluated based on the dependence of pressure ratio and efficiency on the mass flow at different operating speeds.
- Centrifugal compressors also suffer from instability problems like surge and rotating stall.

Performance characteristics

- The compressor outlet pressure, P_{02}, and the isentropic efficiency, $\eta_{C,}$ depend upon several physical variables

$$P_{02}, \eta_C = f(\dot{m}, P_{01}, T_{01}, \Omega, \gamma, R, \nu, \text{design}, D)$$

In terms of non-dimensionless parameters,

$$\frac{P_{02}}{P_{01}}, \eta_C = f\left(\frac{\dot{m}\sqrt{\gamma R T_{01}}}{P_{01} D^2}, \frac{\Omega D}{\sqrt{\gamma R T_{01}}}, \frac{\Omega D^2}{\nu}, \gamma, \text{design}\right)$$

The above reduces to $\dfrac{P_{02}}{P_{01}}, \eta_C = f\left(\dfrac{\dot{m}\sqrt{T_{01}}}{P_{01}}, \dfrac{N}{\sqrt{T_{01}}}\right)$

Performance characteristics

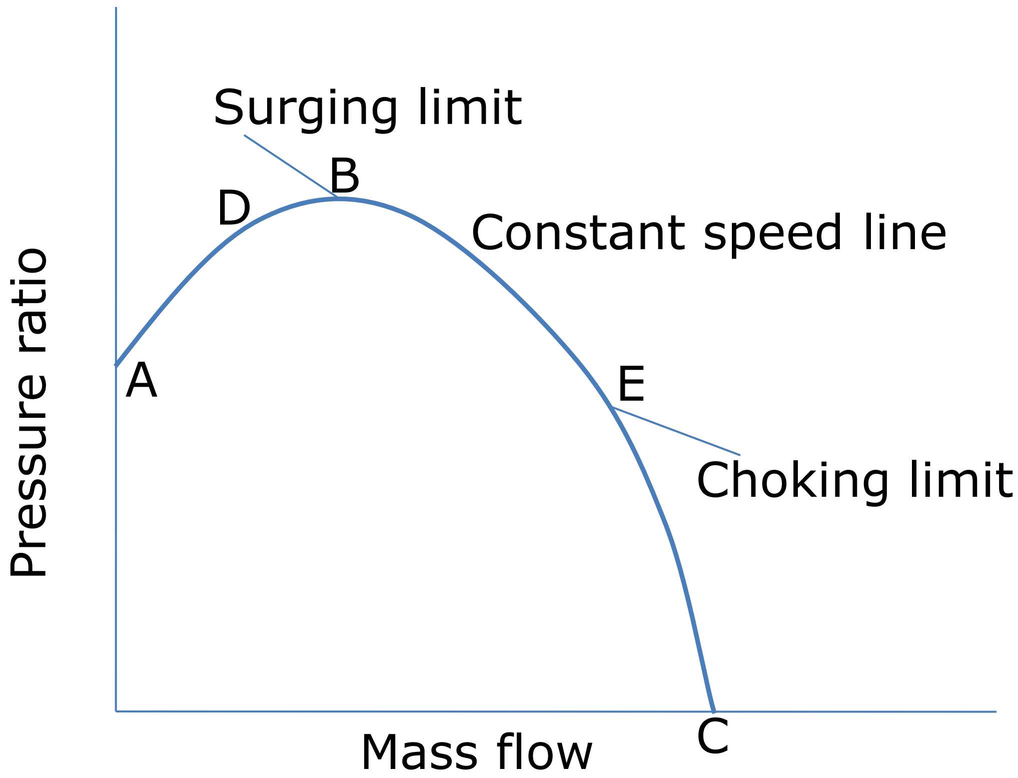
Usually, this is further processed in terms of the standard day pressure and temperature.

$$\frac{P_{02}}{P_{01}}, \eta_C = f\left(\frac{\dot{m}\sqrt{\theta}}{\delta}, \frac{N}{\sqrt{\theta}}\right)$$

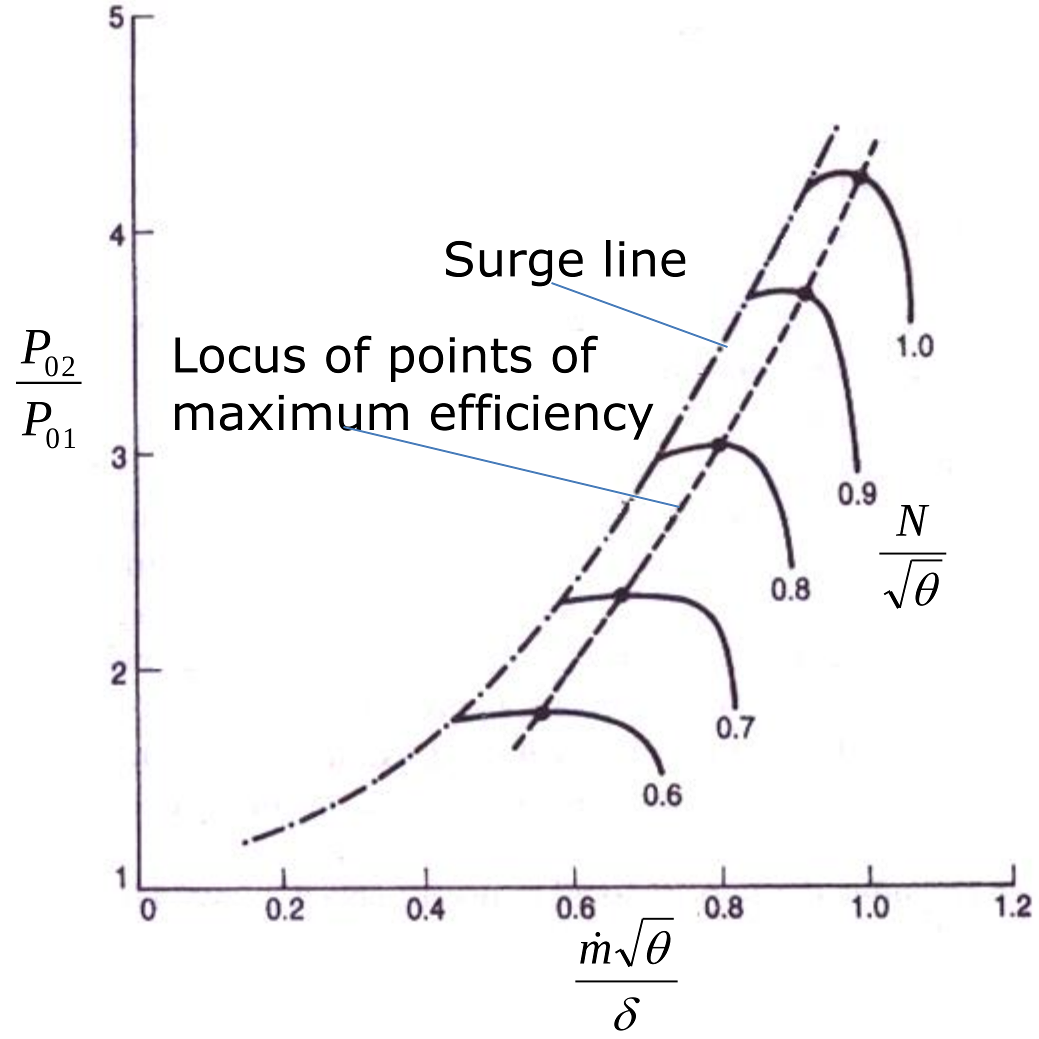
Where, $\theta = \dfrac{T_{01}}{(T_{01})_{\text{Std. day}}}$ and $\delta = \dfrac{P_{01}}{(P_{01})_{\text{Std. day}}}$

$$(T_{01})_{\text{Std. day}} = 288.15\,\text{K} \quad \text{and} \quad (P_{01})_{\text{Std. day}} = 101.325\ \text{kPa}$$

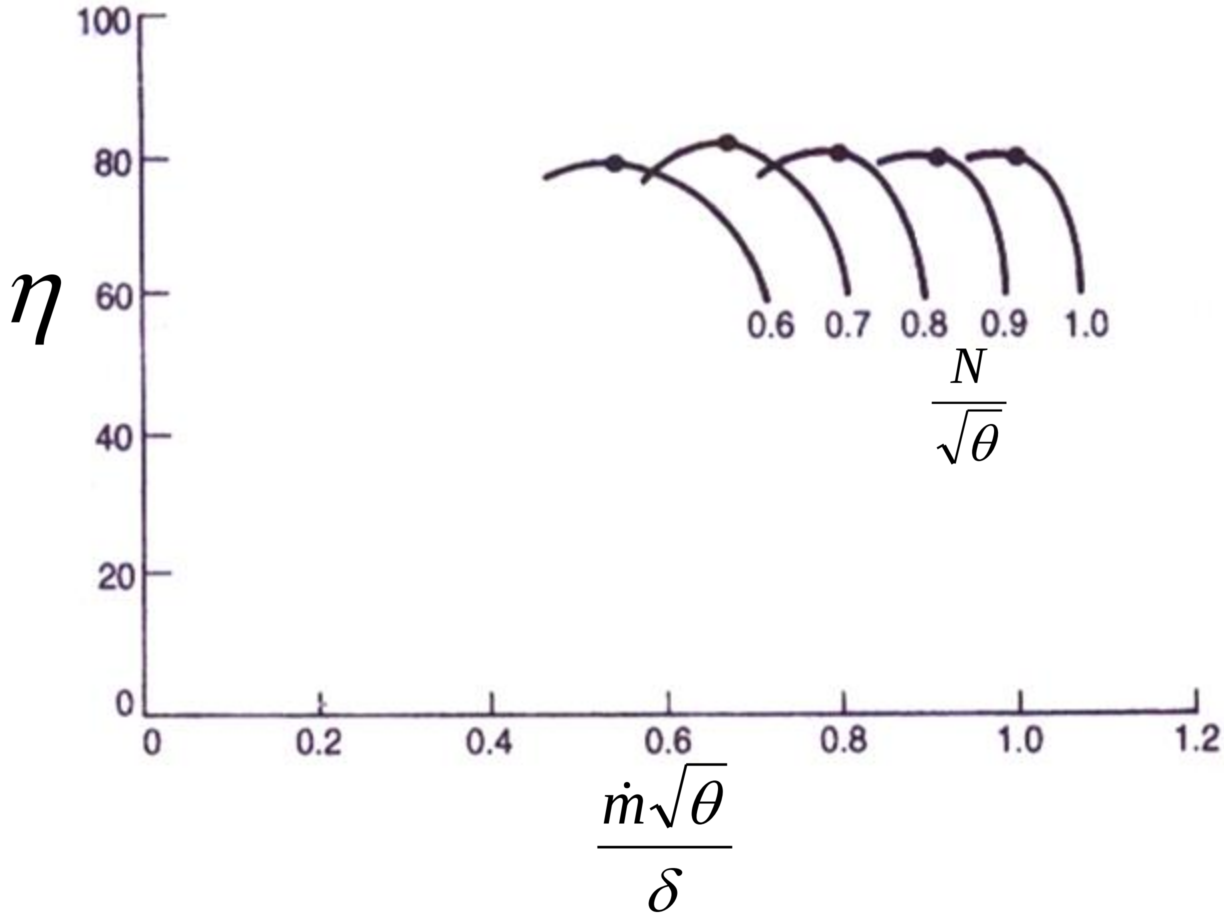
Performance characteristics



Performance characteristics



Performance characteristics



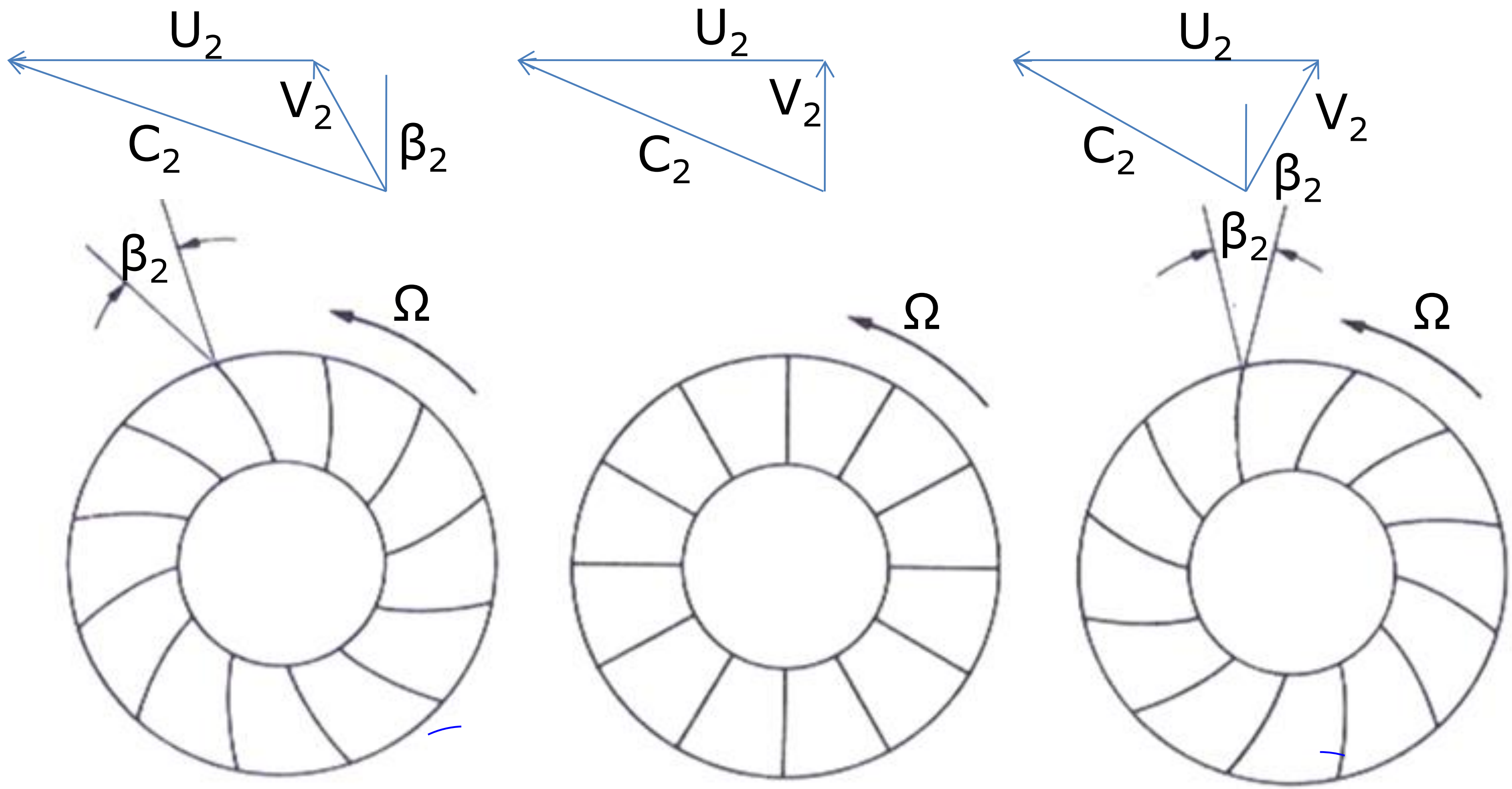
Performance characteristics

- There are two limits to the operation of the compressor.
- Operation between A and B are limited due to occurrence of surge.
- Surging: sudden drop in delivery pressure and violent aerodynamic pulsations.
- Operation on the positive slope of the performance characteristics: unstable
- Surging usually starts to occur in the diffuser passages.

Performance characteristics

- The pressure ratio or the temperature rise in a centrifugal compressor also depends upon the blade shaping.
- There are three possible types of blade shapes: forward leaning, straight radial and backward leaning.
- Theoretically, the forward leaning blading produces higher pressure ratio for a given flow coefficient.
- However such a blading has inherent dynamic instability.
- Therefore, straight radial or backward leaning blades are popularly used.

Impeller

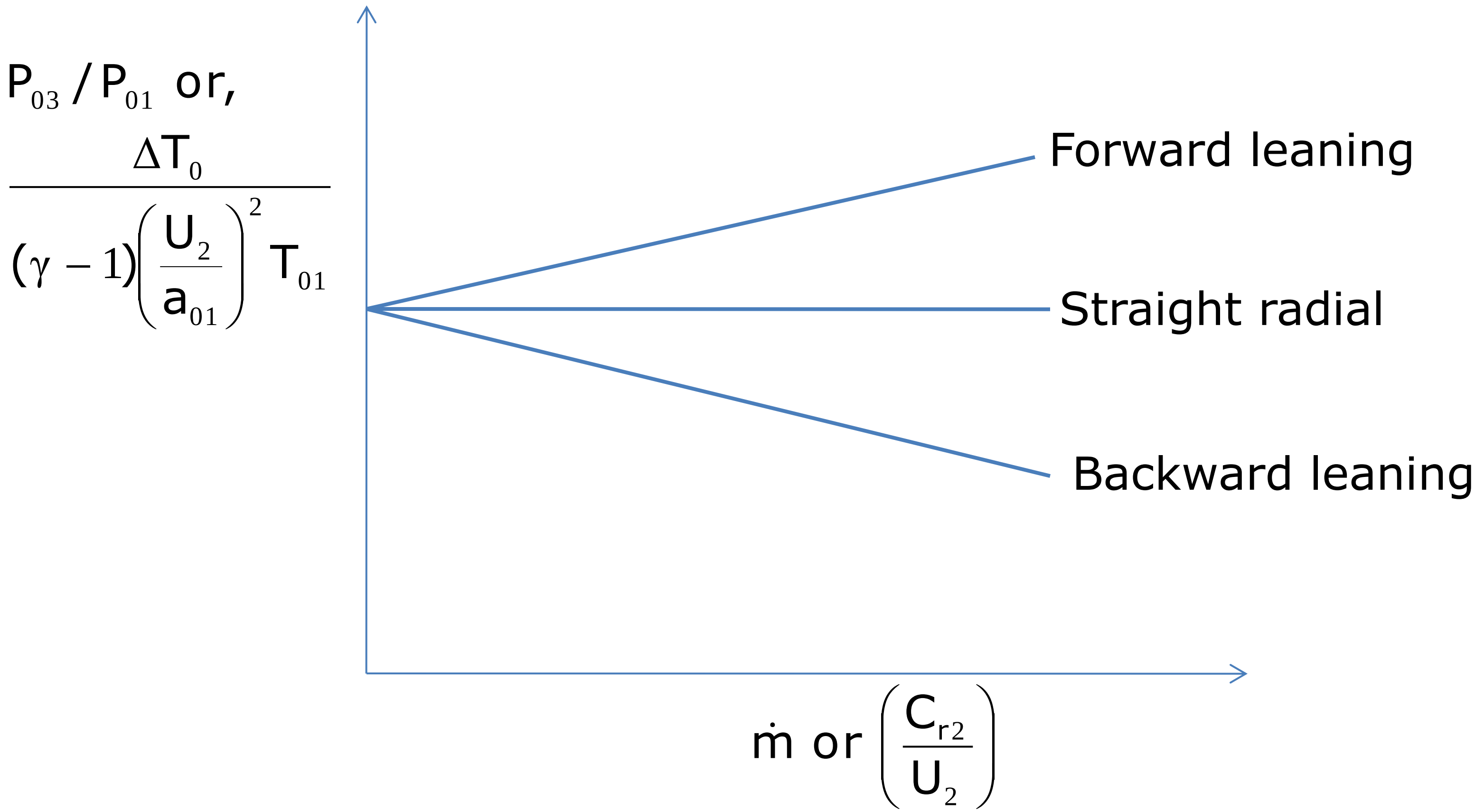


Forward leaning blades
(β_2 is negative)

Straight radial

Backward leaning blades
(β_2 is positive)

Performance characteristics

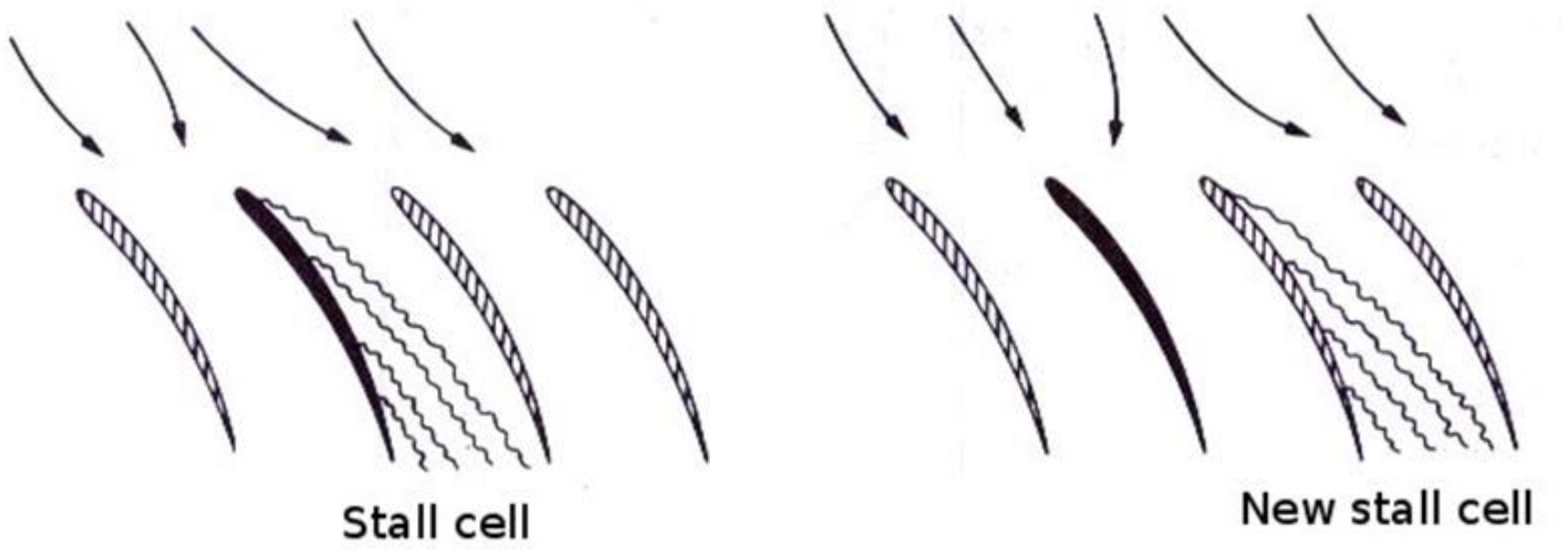


Performance characteristics for different blade geometries

Rotating stall

- Rotating stall might also affect the compressor performance.
- In this case a stall cell (that might cover one or more adjacent blades) rotates within the annulus.
- Full annulus rotating stall may eventually lead to surge.
- Rotating stall may also lead to aerodynamically induced vibrations and fatigue failure of the compressor components.

Rotating stall



Propagation of rotating stall

Choking in a compressor stage

- The other limiting aspect of centrifugal compressors is choking.
- As the mass flow increases, the pressure decreases, density reduces.
- After a certain point, no further increase in mass flow will be possible.
- The compressor is then said to have choked.
- The right hand side of the constant speed lines together form the choking line.

Choking in a compressor stage

- Choking behaviour for rotating passages is different from that of stationary passages.
- Inlet:
  - Choking takes place when M=1

$$\frac{T}{T_0} = \frac{2}{\gamma + 1}$$

Assuming an isentropic flow, the choking mass flow rate is

$$\frac{\dot{m}}{A} = \rho_0 a_0 \left(\frac{2}{\gamma + 1}\right)^{(\gamma+1)/2(\gamma-1)}$$

  - Since ρ_0, a_0 refer to the inlet stagnation conditions and are constant, the mass flow rate is also a constant: choking mass flow.

Choking in a compressor stage

- Impeller:
  - In rotating passages, the flow conditions are referred through rothalpy, I.
  - During choking, it is the relative velocity, V, that becomes equal to the speed of sound.

$$I = h + \tfrac{1}{2}\left(V^2 - U^2\right) \rightarrow T_{01} = T + \left(\gamma R T / 2c_p\right) - \left(U^2 / 2c_p\right)$$

$$\therefore \frac{T}{T_{01}} = \left(\frac{2}{\gamma+1}\right)\left(1 + \frac{U^2}{2c_p T_{01}}\right) \text{ and } \frac{\dot{m}}{A} = \rho_{01} a_{01} \left(\frac{T}{T_{01}}\right)^{(\gamma+1)/2(\gamma-1)}$$

$$\text{or, } \frac{\dot{m}}{A} = \rho_{01} a_{01} \left[\frac{2 + (\gamma-1) U^2 / a_{01}^2}{\gamma+1}\right]^{(\gamma+1)/2(\gamma-1)}$$

Choking in a compressor stage

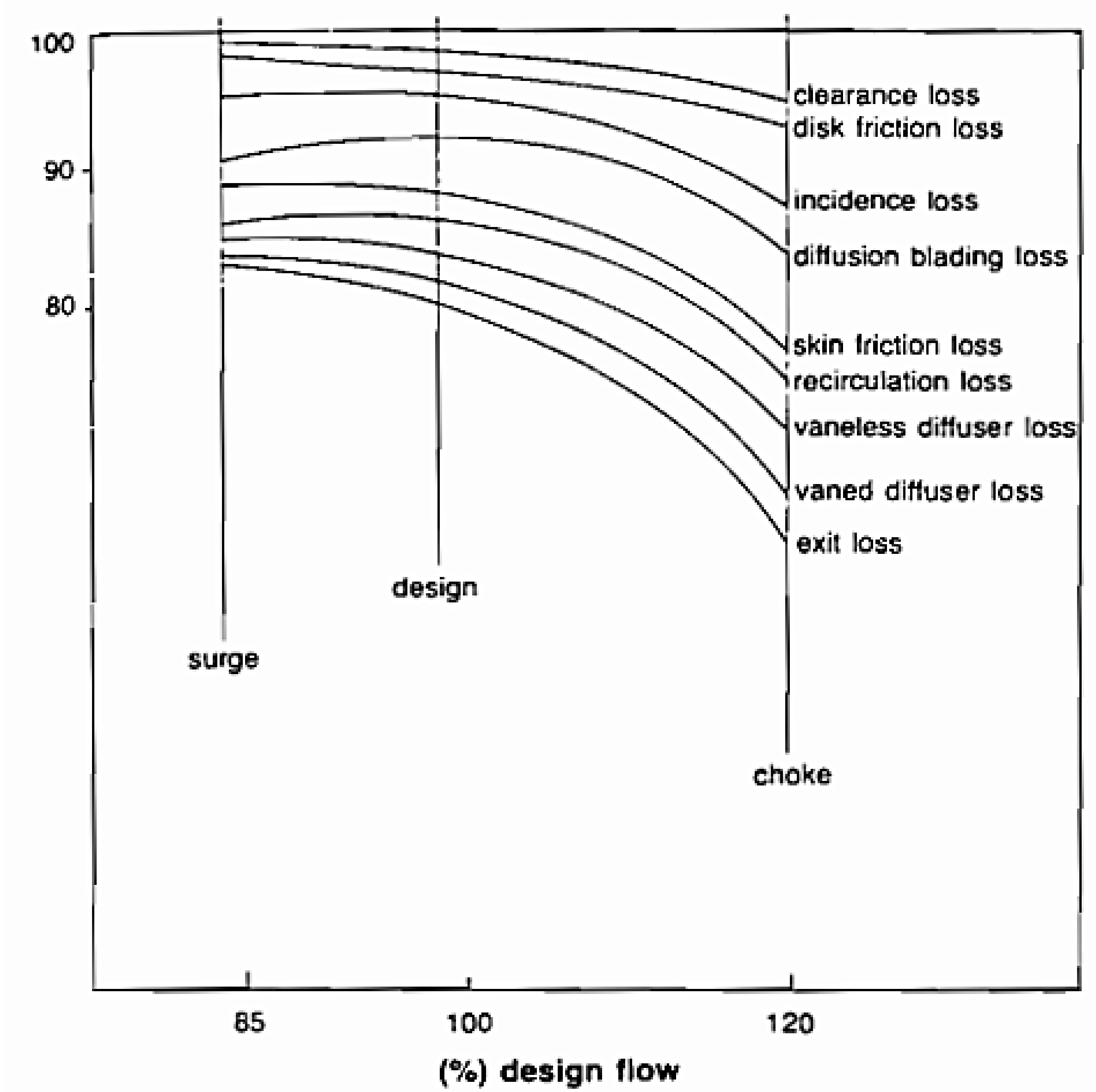
- In an impeller, the choking mass flow is a function of the rotational speed.
- Therefore, the compressor can, in principle, handle a higher mass flow with an increase in speed.
- This also requires that no other component like the inlet or the diffuser undergoes choking at this new rotational speed.

Choking in a compressor stage

- Diffuser:
  - The choking mass flow in a diffuser has an equation similar to that of an inlet:

$$\frac{\dot{m}}{A} = \rho_0 a_0 \left(\frac{2}{\gamma + 1} \right)^{(\gamma+1)/2(\gamma-1)}$$

  - The stagnation conditions at the inlet of diffuser depend upon the impeller exit conditions.
  - It can be shown that the choking mass flow is a function of the rotational speed and therefore can be varied by changing the rotational speed.



Losses in a centrifugal compressor