• Recap: Lecture 21, 16\textsuperscript{th} October 2015, 1530-1655 hrs.
  – Centrifugal compressors
    • Introduction
    • Construction and components
    • Thermodynamics
    • Stage pressure ratio
    • Conservation of rothalthy
    • Components of a centrifugal compressor
      – Impeller
      – Inducer
**Inducer**

- Inducer is the impeller entrance section where the tangential motion of the fluid is changed in the radial direction.
- This may occur with a little or no acceleration.
- Inducer ensures that the flow enters the impeller smoothly.
- Without inducers, the rotor operation would suffer from flow separation and high noise.
Inducer

Section m-m

Leading edge velocity triangles

$r_t$, $r_m$, $r_h$

$V'_t$, $U_1$

$U_t$, $U_m$, $U_h$

$C_t$, $C_1$

$\beta_1$, $\beta_m$, $\beta_h$, $\beta_t$
Inducer

• It can be seen from the above that
  \[ V_t' = V_{it} \cos \beta_{it} \]
  Where, \( V' \) denotes the relative velocity at the inducer outlet.

• It can be seen that \( V' < V_1 \), which indicates diffusion in the inducer.

• Similarly, we can see that the relative Mach number from the velocity triangle is,
  \[ M_{1rel} = M_1 / \cos \beta_{it} \]
The diffuser

• High impeller speed results in a high absolute Mach number leaving the impeller.
• This high velocity is reduced (with an increase in pressure) in a diffuser.
• Diffuser represents the fixed or stationary part of the compressor.
• The diffuser decelerates the flow exiting the impeller and thus reduces the absolute velocity of the working fluid.
• The amount of deceleration depends upon the efficiency of the diffusion process.
The diffuser

- The fluid flows radially outwards from the impeller, through a vaneless region and then through a vaned diffuser.
- Both vaned and the vaneless diffusers are controlled by boundary layer behaviour.
- Pipe and channel type diffusers are used in aero engines due to their compatibility with the combustors.
The diffuser

Diffuser vanes

Vaneless space

Impeller

$r_3 > r_2 > r_1$
The diffuser

Logarithmic spiral

Radial direction

Streamlines in a radial diffuser
The diffuser

Let us consider an incompressible flow in a vaneless region of constant axial width. From continuity, \( \dot{m} = \rho(2\pi rh)C_r = \text{constant} \). From conservation of angular momentum, \( rC_w = \text{constant} \)

\[ \therefore \frac{C_w}{C_r} = \text{constant} = \tan \alpha, \text{ where } \alpha \text{ is the angle between the velocity and the radial direction.} \]

Thus, the velocity is inversely proportional to radius. This means that there is diffusion taking place in the vaneless space.
Different types of diffuser geometries
Coriolis acceleration

• We have discussed earlier that pressure change due to the centrifugal force field is not a cause of boundary layer separation.
• This can also be explained by the Coriolis forces that are present in centrifugal compressor rotors.
• Let us consider a fluid element travelling radially outward in the passage of a rotor.
• We shall examine the velocity triangles of this fluid during a time period $dt$. 
Coriolis acceleration

\[ \Omega r \]

\[ \Omega (r + dr) \]

\[ \Omega dr \]

\[ V d\theta \]

\[ dC \]

\[ dC' \]

\[ C' - C \]

\[ V \]

\[ d\theta \]
Coriolis acceleration

• The magnitude of the relative velocity is "unchanged", but the particle has suffered an absolute change of velocity.

\[ dC_w = \Omega dr + V d\theta \]

or, \[ dC_w = \Omega V dt + V\Omega dt, \]

Thus, the Coriolis acceleration, \( a_\theta = 2\Omega V \)

and it requires a pressure gradient in the tangential direction of magnitude, \( \frac{1}{r} \frac{\partial P}{\partial \theta} = -2\rho \Omega V \)
Coriolis acceleration

• The existence of the tangential pressure gradient means that there will be a positive gradient of \( V \) in the tangential direction.

\[
\frac{1}{\rho} \frac{dP}{rd\theta} = - \frac{d\left(V^2 / 2\right)}{rd\theta} = - \frac{V}{r} \frac{dV}{d\theta}
\]

Therefore

\[
\frac{1}{r} \frac{dV}{d\theta} = 2\Omega
\]

• This means that there will be a tangential variation in relative velocity.
Coriolis acceleration

\[ C_{w2} \]

\[ \mathbf{C}_2 \]

\[ \mathbf{U}_2 \]

\[ \mathbf{V}_2 \]

\[ \mathbf{V} \]

Straight radial blades
**Slip factor**

- Towards the outlet of the impeller, as the Coriolis pressure gradient disappears, there will be a difference between $C_{w2}$ and $U_2$.
- This difference in the velocities is expressed as slip factor, $\sigma_s = C_{w2} / U_2$.
- Generalised expression for slip:
  \[
  \frac{(C_{w2})_{\text{actual}}}{(C_{w2})_{\text{ideal}}}
  \]
- The slip factor is approximately related to the number of blades of the impeller.
- For a straight radial blade, the slip factor is empirically expressed as $\sigma_s \approx 1 - 2/N$, where $N$ is the number of blades.
$C_{r2}=V_{r2}$

Counterrotation in relative flow at the impeller exit is due to Coriolis force.

(d) Flow circulation in the impeller passage causes "slip flow" at the exit [dashed lines are the relative streamlines that are bent backward]
Slip factor

- As the number of blades increases, the slip factor also increases and thus the slip lag at the tip of the impeller reduces.
- The effect of slip is to reduce the magnitude of swirl velocity and therefore the pressure ratio.
- The presence of slip means that to deliver the same pressure ratio, either the impeller diameter or the rotational must be increased.
- This in turn may lead to either increase in frictional losses or stresses on the impeller.
Performance characteristics

• The centrifugal compressor performance characteristics can be derived in the same way as an axial compressor.

• Performance is evaluated based on the dependence of pressure ratio and efficiency on the mass flow at different operating speeds.

• Centrifugal compressors also suffer from instability problems like surge and rotating stall.
Performance characteristics

- The compressor outlet pressure, $P_{02}$, and the isentropic efficiency, $\eta_C$, depend upon several physical variables:

$$P_{02}, \eta_C = f(m, P_{01}, T_{01}, \Omega, \gamma, R, \nu, \text{design}, D)$$

In terms of non-dimensional parameters:

$$\frac{P_{02}}{P_{01}}, \eta_C = f\left(\frac{m\sqrt{\gamma RT_{01}}}{P_{01}D^2}, \frac{\Omega D}{\sqrt{\gamma RT_{01}}}, \frac{\Omega D^2}{\nu}, \gamma, \text{design}\right)$$

The above reduces to:

$$\frac{P_{02}}{P_{01}}, \eta_C = f\left(\frac{m\sqrt{T_{01}}}{P_{01}}, \frac{N}{\sqrt{T_{01}}}\right)$$
Performance characteristics

Usually, this is further processed in terms of the standard day pressure and temperature.

\[
\frac{P_{02}}{P_{01}}, \eta_C = f\left(\frac{\dot{m}\sqrt{\theta}}{\delta}, \frac{N}{\sqrt{\theta}}\right)
\]

Where, \(\theta = \frac{T_{01}}{(T_{01})_{Std.\ day}}\) and \(\delta = \frac{P_{01}}{(P_{01})_{Std.\ day}}\)

\((T_{01})_{Std.\ day} = 288.15\, K\) and \((P_{01})_{Std.\ day} = 101.325\, kPa\)
Performance characteristics

Pressure ratio vs Mass flow diagram:
- **Surging limit**
- **Choking limit**
- **Constant speed line**
- Points: A, B, C, D, E
Performance characteristics

Surge line

Locus of points of maximum efficiency
Performance characteristics

\[ \eta \]

\[ \frac{N}{\sqrt{\theta}} \]

\[ \frac{m\sqrt{\theta}}{\delta} \]
Performance characteristics

• There are two limits to the operation of the compressor.
• Operation between A and B are limited due to occurrence of surge.
• Surging: sudden drop in delivery pressure and violent aerodynamic pulsations.
• Operation on the positive slope of the performance characteristics: unstable.
• Surging usually starts to occur in the diffuser passages.
Performance characteristics

• The pressure ratio or the temperature rise in a centrifugal compressor also depends upon the blade shaping.
• There are three possible types of blade shapes: forward leaning, straight radial and backward leaning.
• Theoretically, the forward leaning blading produces higher pressure ratio for a given flow coefficient.
• However such a blading has inherent dynamic instability.
• Therefore, straight radial or backward leaning blades are popularly used.
Impeller

Forward leaning blades
($\beta_2$ is negative)

Straight radial

Backward leaning blades
($\beta_2$ is positive)
Performance characteristics for different blade geometries

Performance characteristics

\[ \frac{P_{03}}{P_{01}} \text{ or,} \]

\[ \frac{\Delta T_0}{(\gamma - 1) \left( \frac{U_2}{a_{01}} \right)^2 T_{01}} \]

- Forward leaning
- Straight radial
- Backward leaning

\[ \dot{m} \text{ or } \left( \frac{C_{r2}}{U_2} \right) \]
Rotating stall

• Rotating stall might also affect the compressor performance.
• In this case a stall cell (that might cover one or more adjacent blades) rotates within the annulus.
• Full annulus rotating stall may eventually lead to surge.
• Rotating stall may also lead to aerodynamically induced vibrations and fatigue failure of the compressor components.
Rotating stall

Propagation of rotating stall
Choking in a compressor stage

- The other limiting aspect of centrifugal compressors is choking.
- As the mass flow increases, the pressure decreases, density reduces.
- After a certain point, no further increase in mass flow will be possible.
- The compressor is then said to have choked.
- The right hand side of the constant speed lines together form the choking line.
Choking in a compressor stage

• Choking behaviour for rotating passages is different from that of stationary passages.

• Inlet:
  • Choking takes place when $M=1$

  \[
  \frac{T}{T_0} = \frac{2}{\gamma + 1}
  \]

  Assuming an isentropic flow, the choking mass flow rate is

  \[
  \frac{\dot{m}}{A} = \rho_0 a_0 \left( \frac{2}{\gamma + 1} \right)^{(\gamma+1)/2(\gamma-1)}
  \]

  • Since $\rho_0$, $a_0$ refer to the inlet stagnation conditions and are constant, the mass flow rate is also a constant: choking mass flow.
Choking in a compressor stage

• Impeller:
  • In rotating passages, the flow conditions are referred through rothalpy, I.
  • During choking, it is the relative velocity, V, that becomes equal to the speed of sound.

\[
I = h + \frac{1}{2} (V^2 - U^2) \rightarrow T_{01} = T + (\gamma RT / 2c_p) - (U^2 / 2c_p)
\]

\[
\therefore \frac{T}{T_{01}} = \left( \frac{2}{\gamma + 1} \right) \left( 1 + \frac{U^2}{2c_p T_{01}} \right) \quad \text{and} \quad \frac{\dot{m}}{A} = \rho_{01} a_{01} \left( \frac{T}{T_{01}} \right)^{(\gamma+1) / 2(\gamma-1)}
\]

or, \[
\frac{\dot{m}}{A} = \rho_{01} a_{01} \left[ \frac{2 + (\gamma - 1)U^2 / a_{01}^2}{\gamma + 1} \right]^{(\gamma+1) / 2(\gamma-1)}
\]
Choking in a compressor stage

- In an impeller, the choking mass flow is a function of the rotational speed.
- Therefore, the compressor can, in principle, handle a higher mass flow with an increase in speed.
- This also requires that no other component like the inlet or the diffuser undergoes choking at this new rotational speed.
Choking in a compressor stage

• Diffuser:
  • The choking mass flow in a diffuser has an equation similar to that of an inlet:

\[
\frac{\dot{m}}{A} = \rho_0 a_0 \left( \frac{2}{\gamma + 1} \right)^{(\gamma+1)/2(\gamma-1)}
\]

• The stagnation conditions at the inlet of diffuser depend upon the impeller exit conditions.

• It can be shown that the choking mass flow is a function of the rotational speed and therefore can be varied by changing the rotational speed.
Losses in a centrifugal compressor