- Recap: Lecture 22, 21st October 2015, 1530-1655 hrs.
  - Coriolis acceleration and its effect
  - Slip factor and its effect
  - Performance of centrifugal compressors
    - Corrected mass flow and pressure ratio characteristics
  - Characteristics of forward leaning, straight and backward leaning blades
  - Rotating stall and surge
  - Choking
    - Inlet, impeller and diffuser passages

- **Specific Speed**
  - Methodology originally developed for hydraulic turbomachines
  - A dimensional analysis will result in the following non-dimensional groups

$$\frac{Q}{ND^3} = \phi_1 = \text{constant}$$

$$\frac{gH}{N^2D^2} = \psi_1 = \text{constant}$$

$$\frac{P}{\rho N^3 D^5} = \hat{P}_1 = \text{constant}$$

- Combine any pair of these expressions in such a way as to eliminate the diameter.

– Thus,

$$N_s = \frac{\phi_1^{1/2}}{\psi_1^{3/4}} = \frac{NQ^{1/2}}{(gH)^{3/4}}$$

- In the case of a turbine the power specific speed N_{sp} is more useful and is defined by

$$N_{sp} = \frac{\hat{P}_1^{1/2}}{\psi_1^{5/4}} = \frac{N(P/\rho)^{1/2}}{(gH)^{5/4}}$$

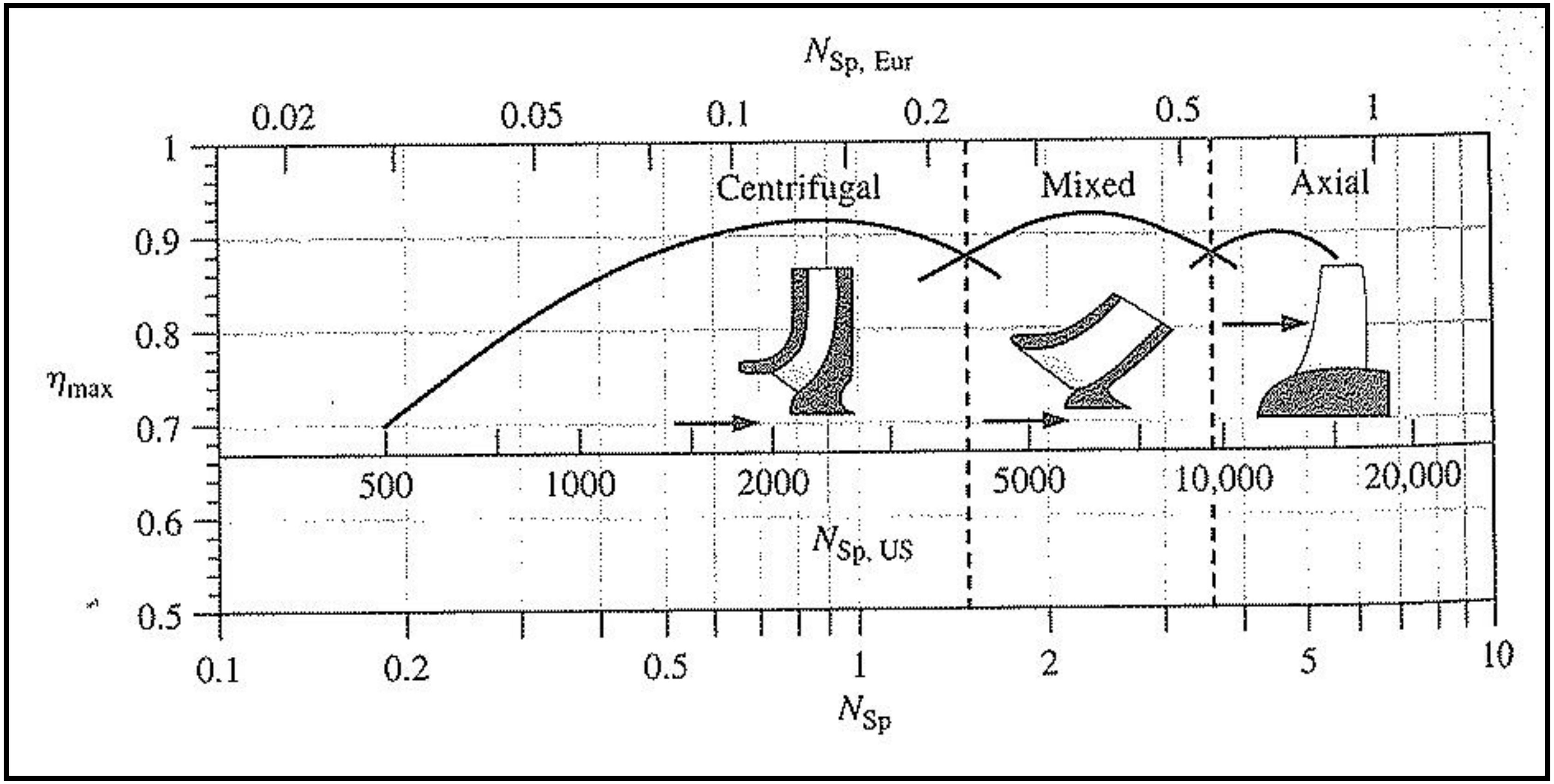
- N_s is non-dimensional when consistent units are used for N, Q & H.

- U.S. Customary Units: H [ft], Q [gal/min], N [rpm]
- Europe Customary Units: H [m], Q [m^3/s], N [rot/sec – Hz]
- Conversion ratios

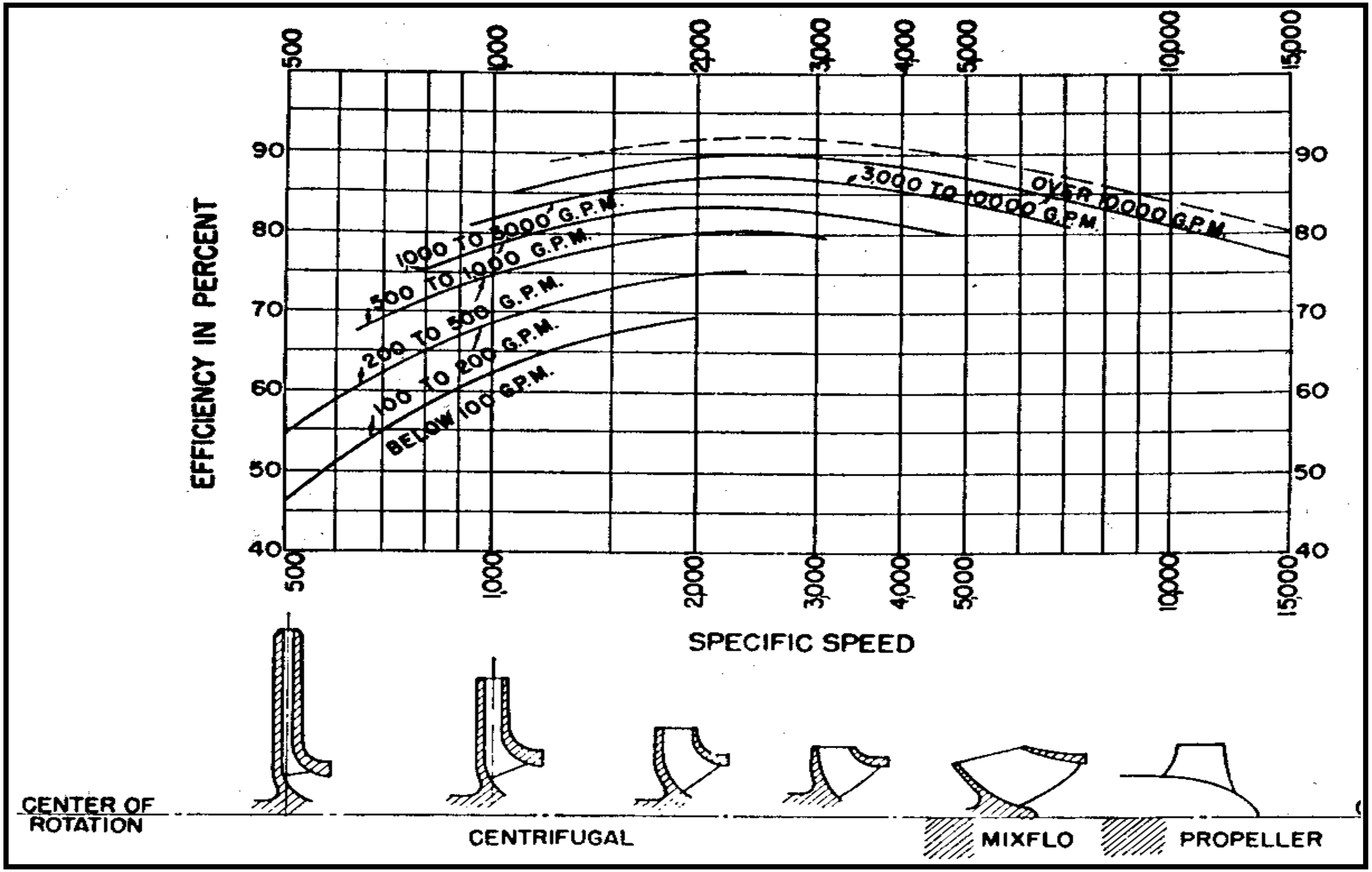
$$N_s / N_{s-US} = 3.568 \bullet 10^{-4}$$
$$N_s / N_{s-Eur} = 2\pi$$
$$N_{s-US} / N_{s-Eur} = 17{,}180$$

Specific Speed Used to Determine Turbomachine Type

$$N_s = \frac{N \cdot Q^{1/2}}{[gH]^{3/4}}$$



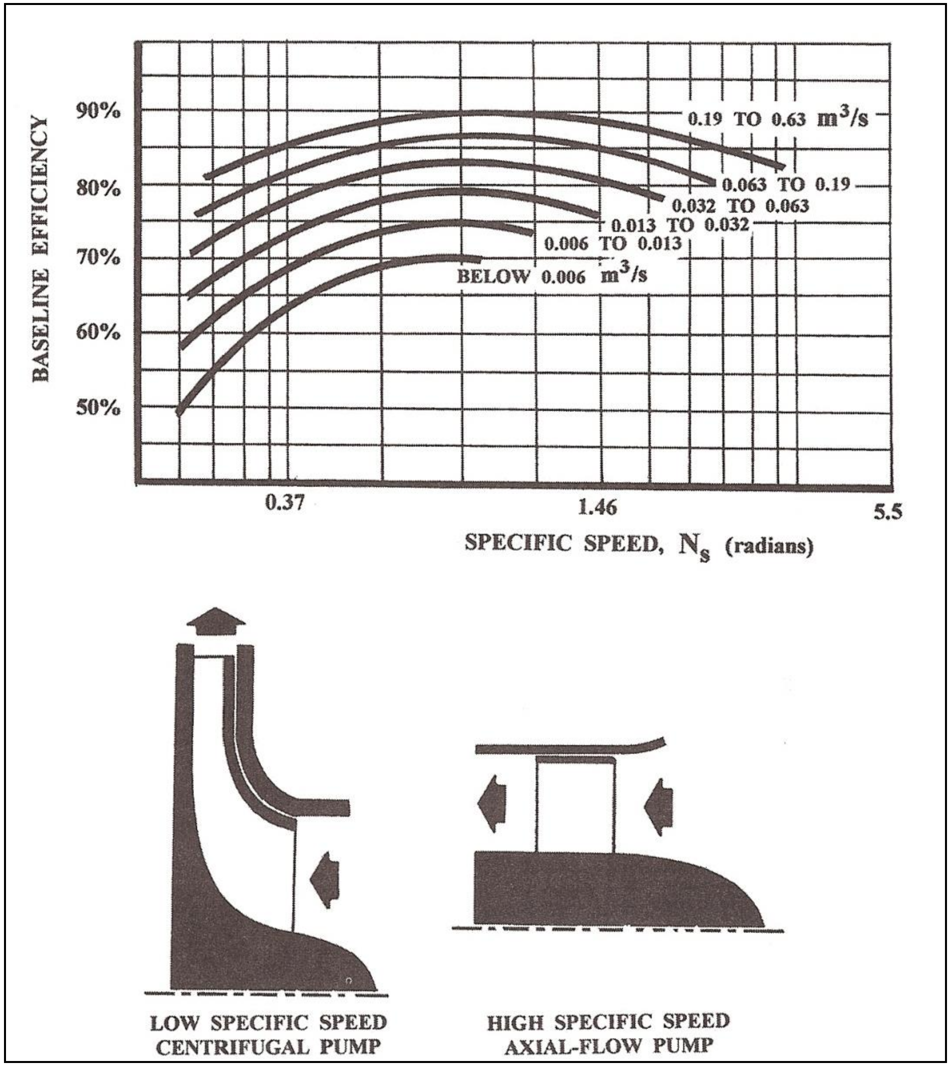
Variation of Efficiency with Ns for Various Pump Sizes

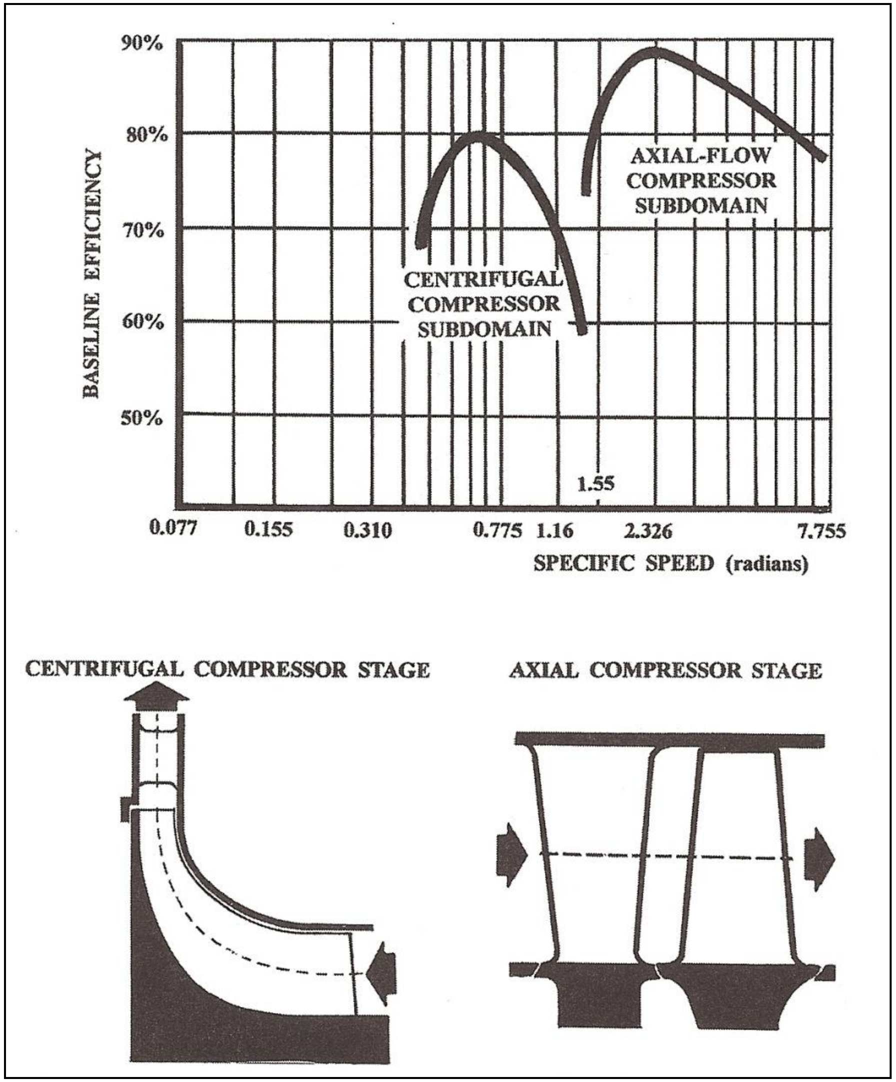


Low Ns

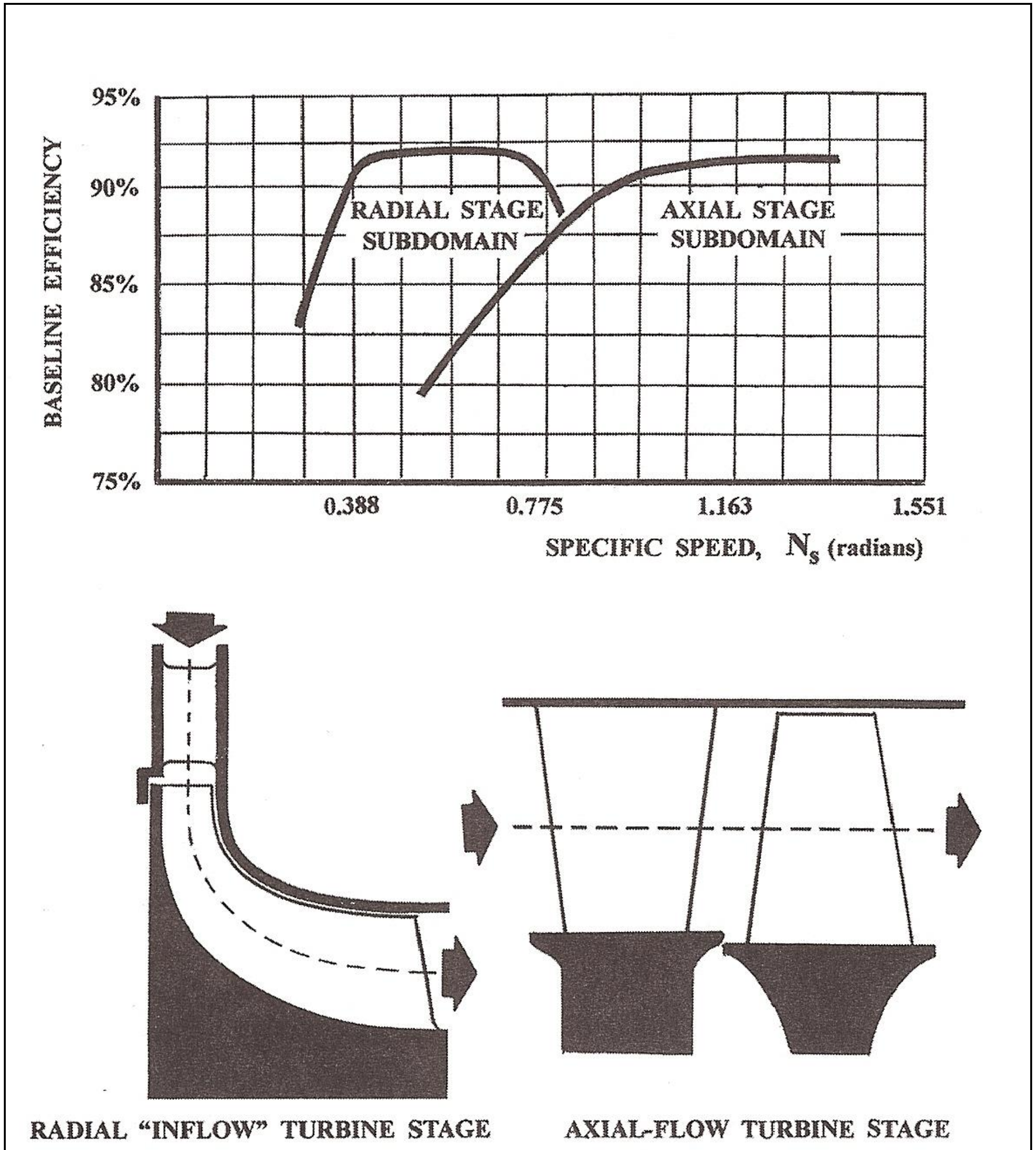
High Ns

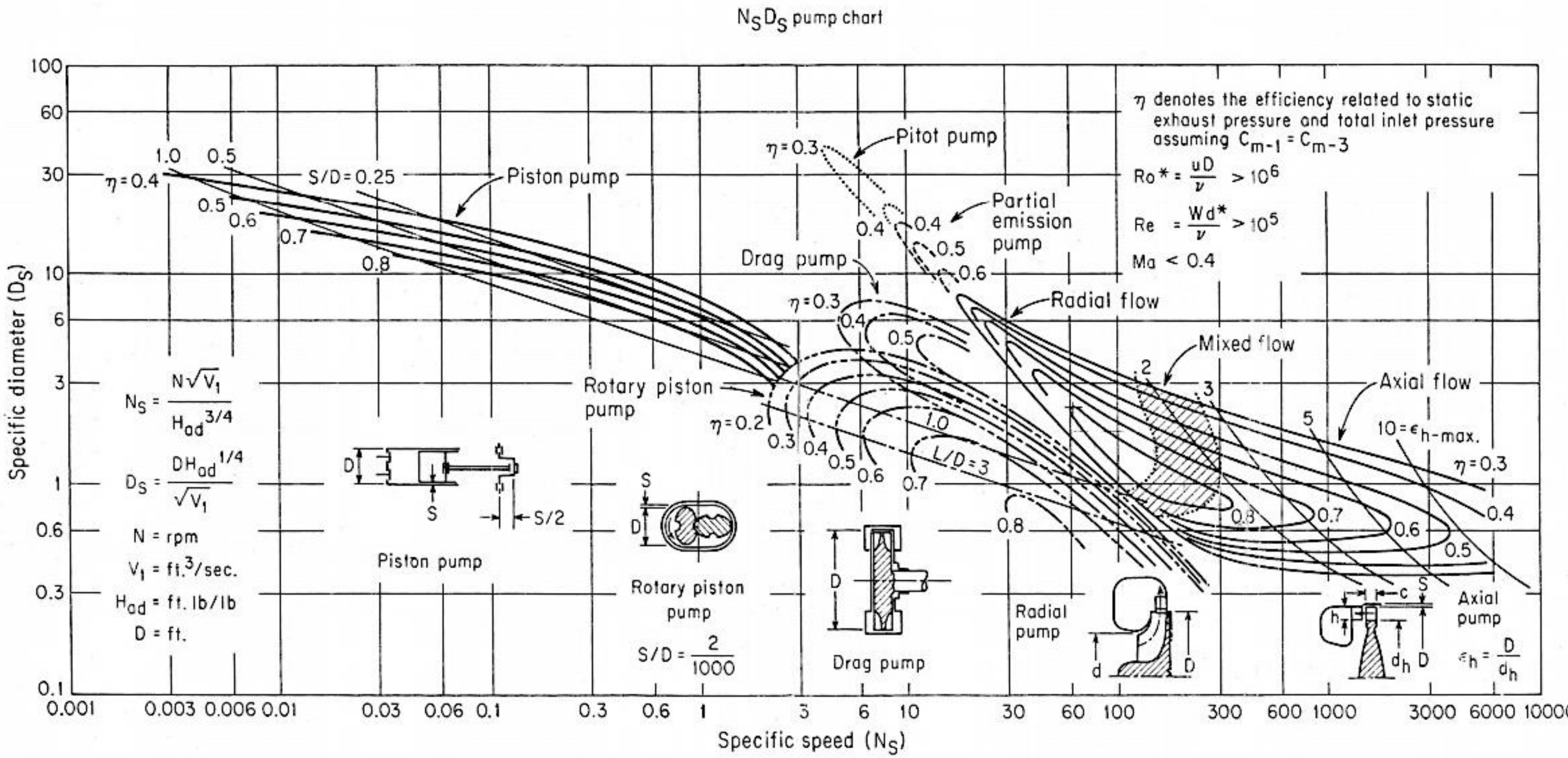
Specific Speed – Pumps/ Compressors





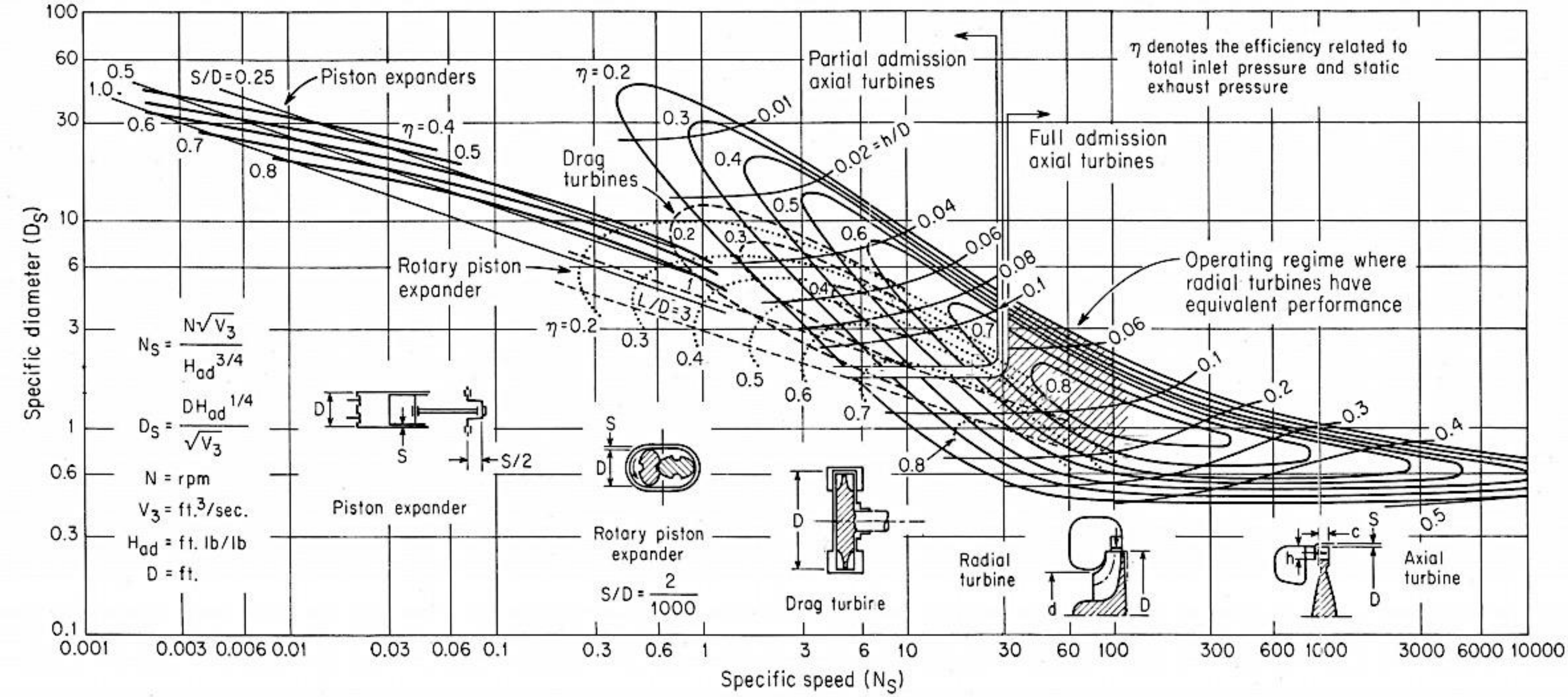
Specific Speed - Turbines



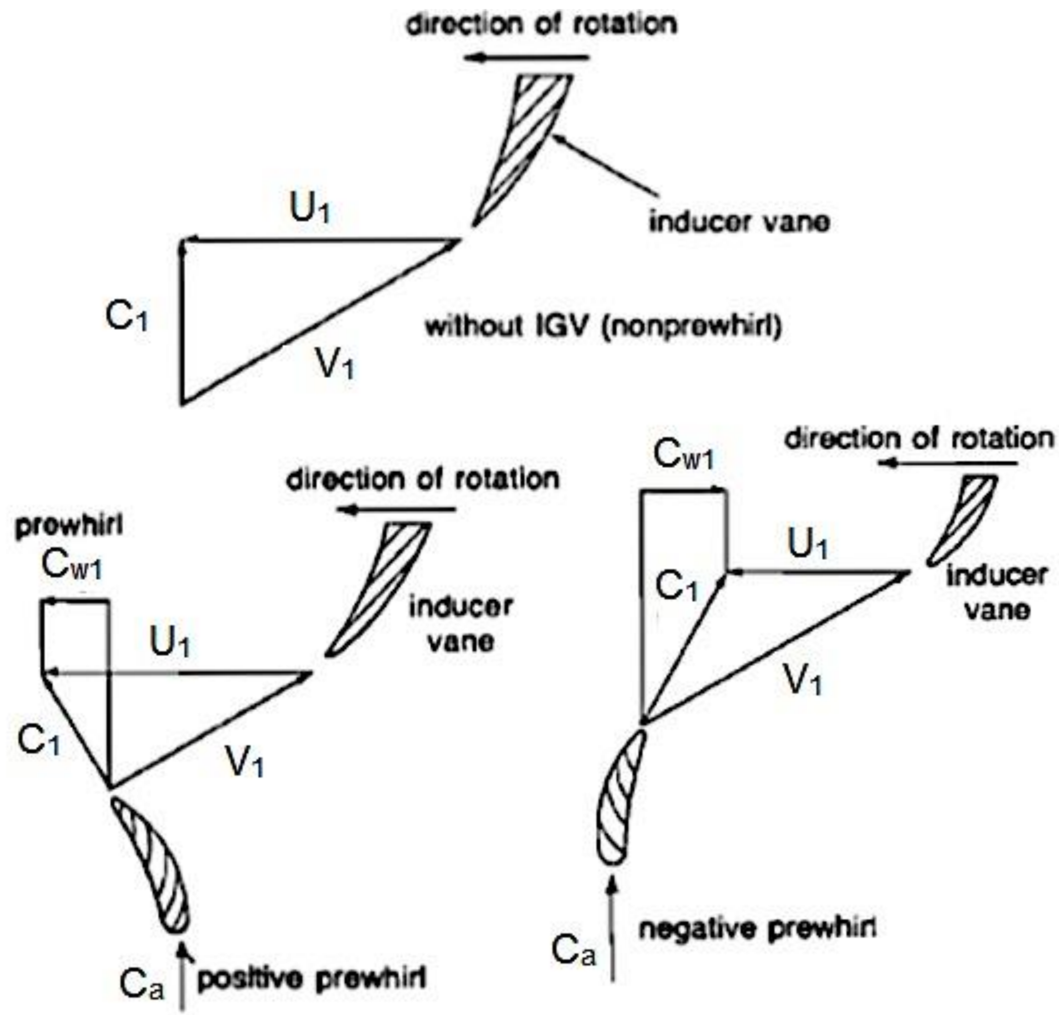


http://www.barber-nichols.com/sites/default/files/wysiwyg/images/how_to_select_turbomachinery_for_your_application.pdf

$N_S D_S$ turbine chart



http://www.barber-nichols.com/sites/default/files/wysiwyg/images/how_to_select_turbomachinery_for_your_application.pdf



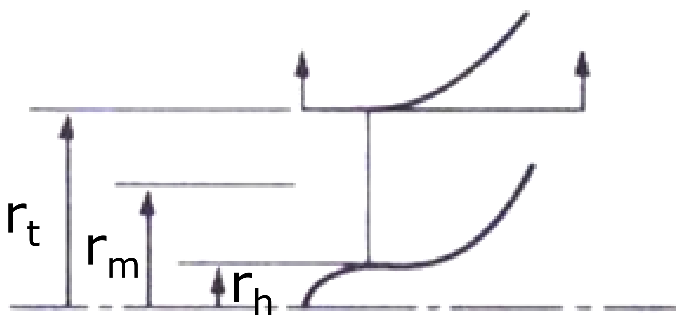
Inducer inlet velocity triangles and pre-whirl

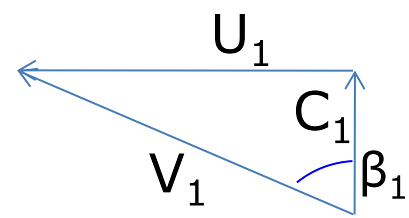
From: Prin. of operation and perf. estimation of centrifugal comps. --Boyce.

Problem # 1

- At the inlet of a centrifugal compressor eye, the relative Mach number is to be limited to 0.97. The hub-tip radius ratio of the inducer is 0.4. The eye tip diameter is 20 cm. If the inlet velocity is axial, determine, (a) the maximum mass flow rate for a rotational speed of 29160 rpm, (b) the blade angle at the inducer tip for this mass flow. The inlet conditions can be taken as 101.3 kPa and 288 K.

Problem # 1





Solution: Problem # 1

The rotational speed at the inducer tip is

$$U_1 = \pi d N / 60 = \pi \times 0.2 \times 29160 / 60 = 305.36 m/s$$

From the velocity traingle, we can see that

$$M_{1rel} = \frac{V_1}{\sqrt{\gamma R T_1}} = \frac{\sqrt{C_1^2 + U_1^2}}{\sqrt{\gamma R T_1}}$$

$$T_1 = T_{01} - C_1^2 / 2c_P = 288 - C_1^2 / 2010$$

$$M_{1rel} = \frac{\sqrt{C_1^2 + U_1^2}}{\sqrt{\gamma R (288 - C_1^2 / 2010)}}$$

$$0.97^2 = \frac{C_1^2 + 305.63^2}{115718.4 - 0.2 C_1^2}$$

Simplifying, $C_1 = 114.62 m/s$

Solution: Problem # 1

$$T_1 = T_{01} - C_1^2 / 2c_P = 288 - C_1^2 / 2010 = 281.464K$$

$$\frac{P_{01}}{P_1} = \left(\frac{T_{01}}{T_1}\right)^{\gamma/(\gamma-1)}$$

$$\text{Substituting}, P_1 = 93.48kPa$$

$$\therefore \rho_1 = P_1 / RT_1 = 1.157kg/m^3$$

$$\text{Annulus area at the inlet}, A_1 = \frac{\pi}{4} d^2 (1 - r_h / r_t)$$

$$A_1 = 0.0264m^2$$

Solution: Problem # 1

Since the flow is axial,

$$C_{a1} = C_1$$

$$\therefore \dot{m} = \rho_1 A_1 C_1 = 1.157 \times 0.0264 \times 114.62 = 3.5 kg/s$$

The blade inlet angle at the tip is

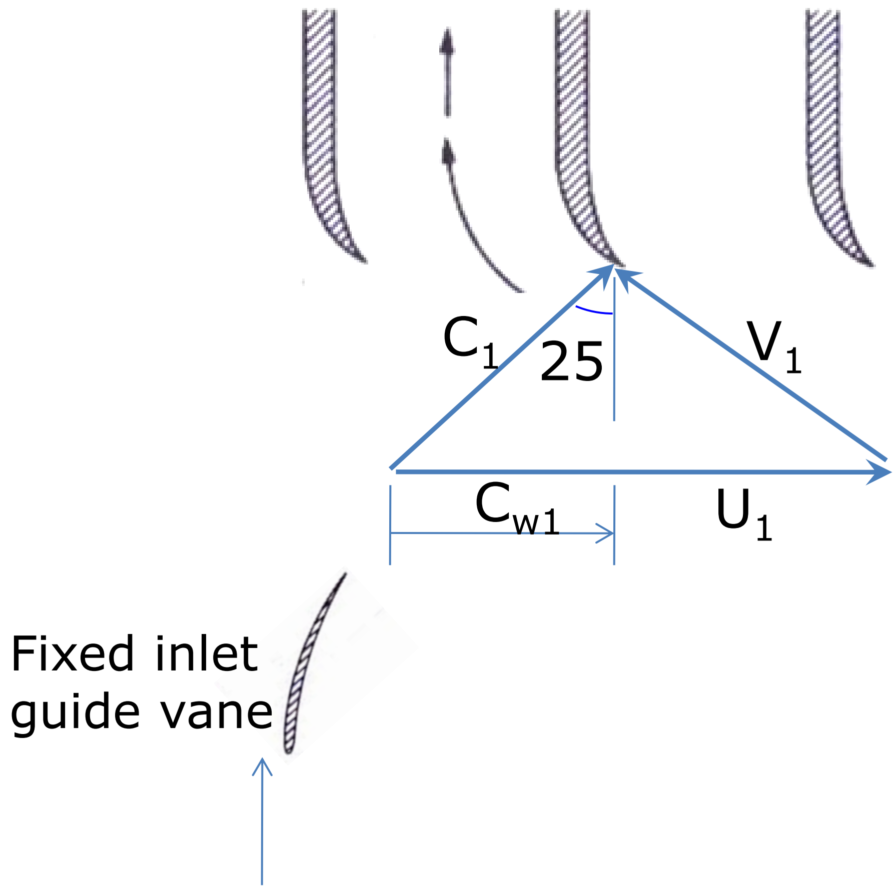
$$\tan \beta_1 = U_1 / C_1$$

$$\therefore \beta_1 = 69.42^o$$

Problem # 2

- A centrifugal compressor has a pressure ratio of 4:1 with an isentropic efficiency of 80% when running at 15000 rpm and inducing air at 293 K. Curved vanes at the inlet give the air a pre-whirl of 25^o to the axial direction at all radii. The tip diameter of the eye of the impeller is 250 mm. The absolute velocity at inlet is 150 m/s and the impeller diameter is 600 mm. Calculate the slip factor.

Problem # 2



Inlet velocity triangle

Solution: Problem # 2

Exit stagnation temperature is

$$T_{02} = T_{01}(\pi_c)^{(\gamma-1)/\gamma} = 293(4)^{(1.4-1)/1.4} = 435.56\text{K}$$

Therefore the isentropic temperature rise,

$$\Delta T_{0s} = 435.56 - 293 = 142.56\text{K}$$

The actual temperature rise, $\Delta T_0 = \Delta T_{0s} / \eta_c$

$$\Delta T_0 = 178.2\text{K}$$

Work done per unit mass is, $w = c_P \Delta T_0$

$$w = 1.005 \times 178.2 = 179\ \text{kJ/kg}$$

Solution: Problem # 2

Peripheral velocity at the tip of the eye,

$$U_1 = \pi dN/60 = \pi \times 0.25 \times 15000/60 = 196.25 m/s$$

$$C_{w1} = C_1 \sin 25 = 63.4 m/s$$

Peripheral velocity at the tip of the impeller,

$$U_2 = \pi DN/60 = \pi \times 0.60 \times 15000/60 = 471.2 m/s$$

Solution: Problem # 2

We know that power input is, $w = U_2 C_{w2} - U_1 C_{w1}$

$179 \times 10^3 = 471.24 \times C_{w2} - 196.35 \times 63.4$

or, $C_{w2} = 406.27 m/s$

Therefore, the slip factor is,

$\sigma_s = C_{w2} / U_2 = 0.862$

Problem # 3

2 kg/s of air at a stagnation temperature of 22°C enters the impeller of a centrifugal compressor in the axial direction. The rotor, which has 17 radial vanes, rotates at 15,000 rpm. The stagnation pressure ratio between diffuser outlet and impeller inlet is 4.2 and total-to-total efficiency is 83%. Determine the impeller tip radius. Assume the air density at impeller outlet is 2kg/m^3 and the axial width at entrance to the diffuser is 11mm, determine the absolute Mach number at that point. Assume that the slip factor $\sigma = 1 - 2/N$, where N is the number of vanes.

Problem # 3

The specific work required is

$$w_c = U_2 C_{w2} - U_1 C_{w1}$$

Since $C_{w1} = 0$, $w = U_2 C_{w2} = \sigma U_2^2$

Expressing U_2 in terms of efficiency and pressure ratio,

$$U_2^2 = \frac{c_p T_{01}(\pi_c^{(\gamma-1)/\gamma} - 1)}{\sigma \eta_{c-tt}}$$

$$\sigma = 1 - 2/N = 1 - 2/17 = 0.8824$$

Substituting all other values, $U_2 = 452 \text{ m/s}$

Since, $\Omega = 15000 \times 2\pi / 60 = 1570 \text{ rad/s}$

Therefore, the impeller radius is

$$r_t = U_2 / \Omega = 0.288 \text{ m}$$

Problem # 3

Mach number, $M_2 = C_2 / a_2 = C_2 / \sqrt{\gamma R T_2}$

where, $C_2 = \sqrt{C_{w2}^2 + C_{r2}^2}$

$C_{r2} = \dot{m} / (\rho_2 2\pi r_t b_2) = 2 / (2 \times 2\pi \times 0.288 \times 0.011) = 50.3\, m/s$

$C_{w2} = \sigma U_2 = 400\ m/s$

$\therefore C_2 = \sqrt{50.3^2 + 400^2} = 402.5\ m/s$

We know that $h_{02} = h_{01} + w_c = h_{01} + \sigma U_2^2$

or, $h_2 = h_{01} + \sigma U_2^2 - \frac{1}{2} C_2^2$

or, $T_2 = T_{01} + (\sigma U_2^2 - \frac{1}{2} C_2^2) / c_p$

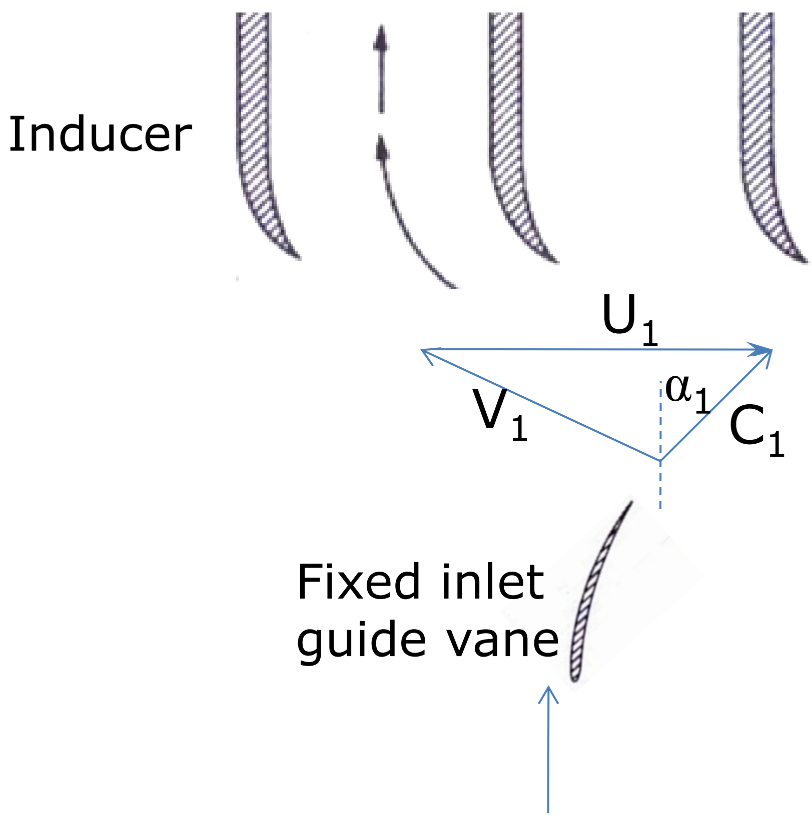
$= 394.5\ K$

Therefore, $M_2 = 402.5 / \sqrt{1.4 \times 287 \times 394.5} = 1.01$

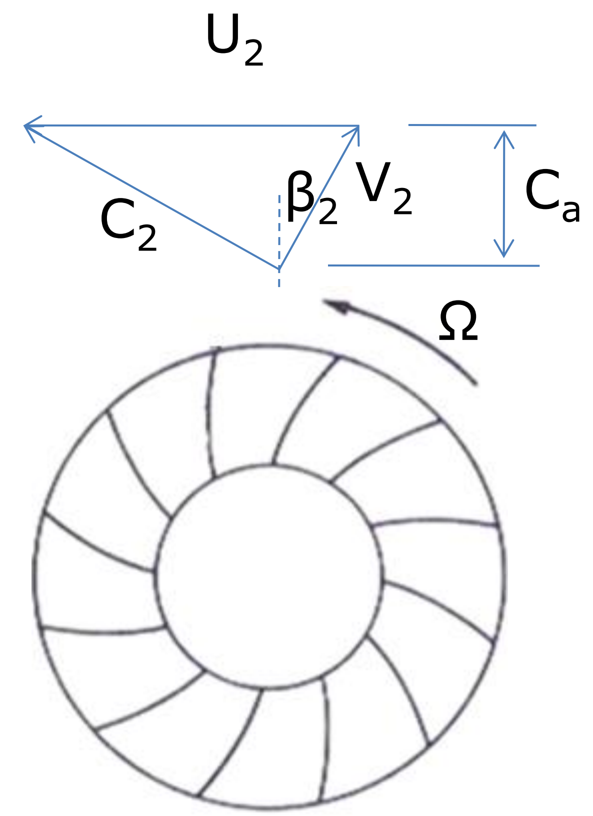
Problem # 4

A centrifugal compressor with backward leaning blades develops a pressure ratio of 5:1 with an isentropic efficiency of 83 percent. The compressor runs at 15000 rpm. Inducers are provided at the inlet of the compressor so that air enters at an absolute velocity of 120 m/s. The inlet stagnation temperature is 250 K and the inlet air is given a pre-whirl 22^o to the axial direction at all radii. The mean diameter of the eye of the impeller is 250 mm and the impeller tip diameter is 600 mm. Determine the slip factor and the relative Mach number at the impeller tip.

Problem # 4



Inlet velocity triangle



Exit velocity triangle

Solution: Problem # 4

$$T_{01} = 300\ \text{K}$$

$$T_{02s} = T_{01}\left(\pi_c\right)^{(\gamma-1)/\gamma} = 250\,(5)^{0.4/1.4} = 395.95\ \text{K}$$

$$\Delta T_{0s} = 395.95 - 300 = 95.95\ \text{K}$$

Actual temperature rise, $\Delta T_{0actual} = \Delta T_{0s} / \eta_c = 95.95 / 0.83$

$$= 115.6\,\text{K}$$

The specific work required, $w_c = c_p \Delta T_{0actual} = 1005 \times 115.6$

$$= 116.186\ \text{kJ/kg}$$

Given that $C_1 = 150\,\text{m/s}$, $\therefore\ C_{w1} = C_1 \sin\alpha_1 = 150 \sin 22$

$$= 56.2\,\text{m/s}$$

Solution: Problem # 4

$U_1 = \pi d_m N / 60 = \pi \times 0.25 \times 15000 / 60 = 196.3\,m/s$

and $U_2 = \pi d_t N / 60 = \pi \times 0.6 \times 15000 / 60 = 471.24\ m/s$

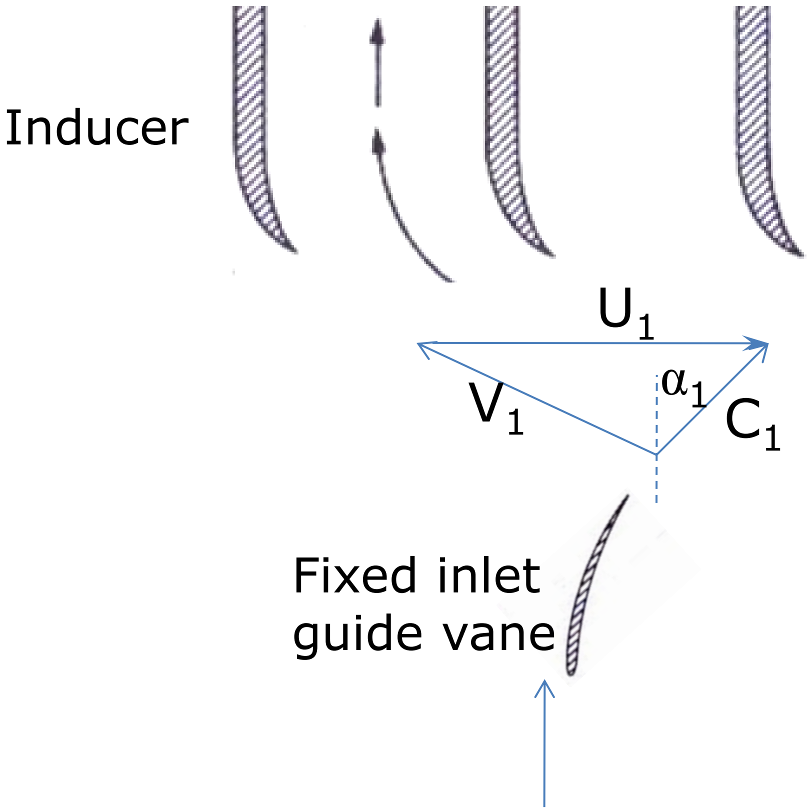
Since, $w_c = U_2 C_{w2} - U_1 C_{w1}$

$116.186 \times 10^3 = 471.24 \times C_{w2} - 196.3 \times 56.2$

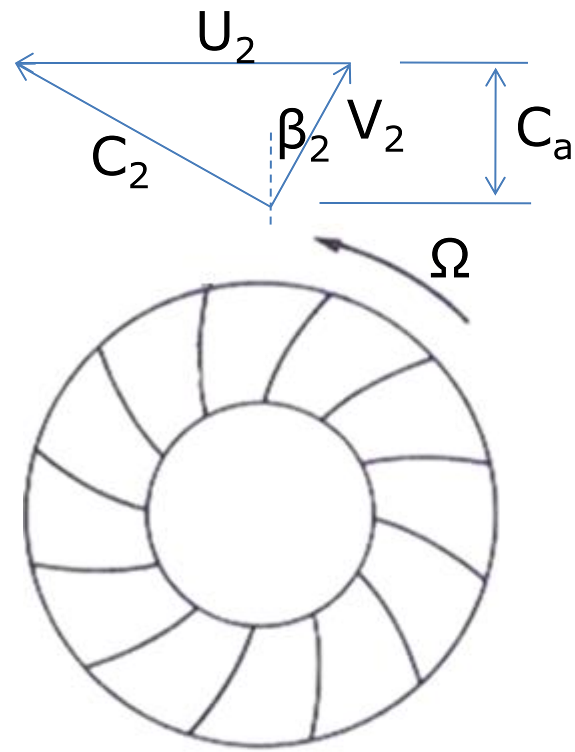
$\therefore C_{w2} = 269.96\ m/s$

The slip factor, $\sigma = C_{w2} / U_2 = 269.96 / 471.24 = 0.573$

Problem # 4



Inlet velocity triangle



Exit velocity triangle

Solution: Problem # 4

From the impeller exit velocity triangle, assuming $C_a = C_r$

$$V_2 = \sqrt{C_r^2 + (U_2 - C_{w2})^2} = \sqrt{(C_1 \cos\alpha_1)^2 + (U_2 - C_{w2})^2}$$

$$= 222.9\ m/s$$

$$M_{rel} = V_2 / \sqrt{\gamma R T_2}$$

$$T_2 = T_{02} - C_2^2 / 2c_p$$

$$T_{02} = T_{01} + \frac{T_{02s} - T_{01}}{\eta_c} = 415.6\,K$$

$$and\ \ C_2 = \sqrt{C_{w2}^2 + C_r^2} = \sqrt{269.9^2 + 139.08^2} = 303.68\ m/s$$

$$\therefore T_2 = 365.61 - 303.68^2 / 2 \times 1005 = 319.73\,K$$

The relative Mach number at the impeller tip is

$$M_{rel} = 222.9 / \sqrt{(1.4 \times 287 \times 319.73)} = 0.62$$

Exercise Problem # 1

- The design mass flow rate of a centrifugal compressor is 7.5 kg/s with inlet stagnation temperature of 300 K and pressure of 100 kPa. The compressor has straight radial blades at the outlet. The blade angle at the inducer inlet tip is 50^o and the inlet hub-tip ratio is 0.5. The impeller is designed to have a relative Mach number of 0.9 at the inducer inlet tip. If the tip speed is 450 m/s, determine (a) the air density at inducer inlet, (b) the inducer inlet diameter, (c) the rotor rpm (d) the impeller outlet diameter.
- Ans: 0.988 kg/m3, 0.258 m, 17100 rpm, 0.502 m

Exercise Problem # 2

- A centrifugal compressor runs at 10000 rpm and delivers 600 m^3/min of air at a pressure of 4:1. The isentropic efficiency of the compressor is 0.82. The outer radius of the impeller is twice the inner radius. The axial velocity is 60m/s. If the ambient conditions are 1 bar and 293 K, determine (a) the impeller diameter at inlet and outlet (b) the power input (c) the impeller and diffuser angles at inlet.
- Ans: 0.92, 0.461, 2044 kW, 13.9^o, 7.1^o

Exercise Problem # 3

- 30 kg of air per second is compressed in a centrifugal compressor at a rotational speed of 15000 rpm. The air enters the compressor axially. The compressor has a tip radius of 30 cm. The air leaves the tip with a relative velocity of 100 m/s at an angle of 80^o. Assuming an inlet stagnation pressure and temperature of 1 bar and 300 K, respectively, find (a) the torque required to drive the compressor, (b) the power required (c) the compressor delivery pressure
- Ans: 4085 Nm, 6.417 MW, 6.531 bar

Exercise Problem # 4

A centrifugal compressor has an impeller tip speed of 366 m/s. Determine the absolute Mach number of the flow leaving the radial vanes of the impeller when the radial component of velocity at impeller exit is 30.5 m/s and the slip factor is 0.90. Given that the flow area at impeller exit is $0.1 m^2$ and the total-to-total efficiency of the impeller is 90%, determine the mass flow rate.

Ans: 0.875, 5.61 kg/s

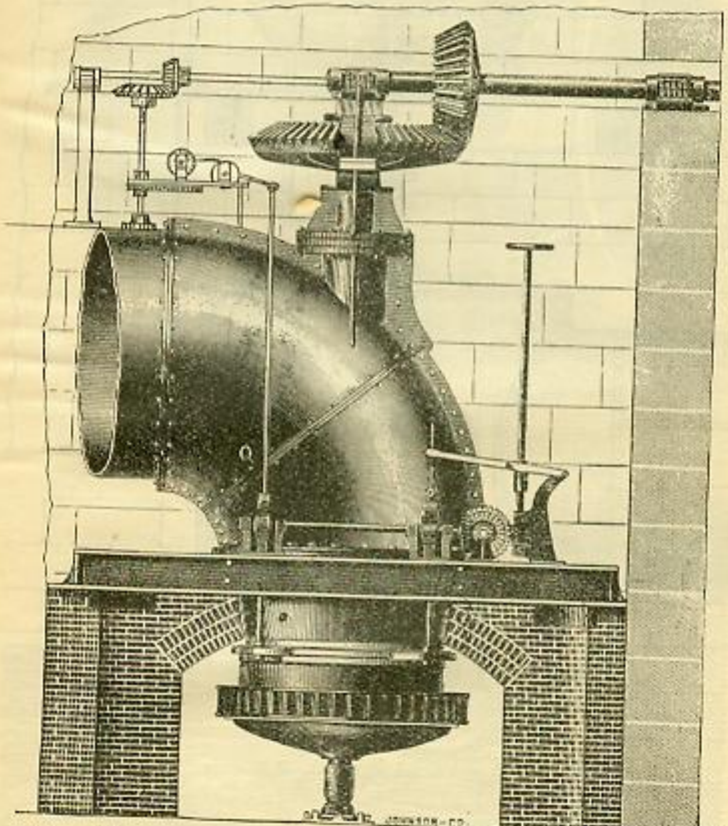
Radial turbines

- Development of radial flow turbines dates back to 1830’s by Fourneyron, who developed the radial outflow type turbine.
- Later on Francis and Boyden developed the radial inflow type turbine.
- The inward-flow radial (IFR) turbine covers tremendous ranges of power, rates of mass flow and rotational speeds.
- IFR turbines are used in a variety of applications ranging from hydroelectric power plants to small gas turbines.
  - Power output ranges from a few kW to several MW

BOYDEN'S

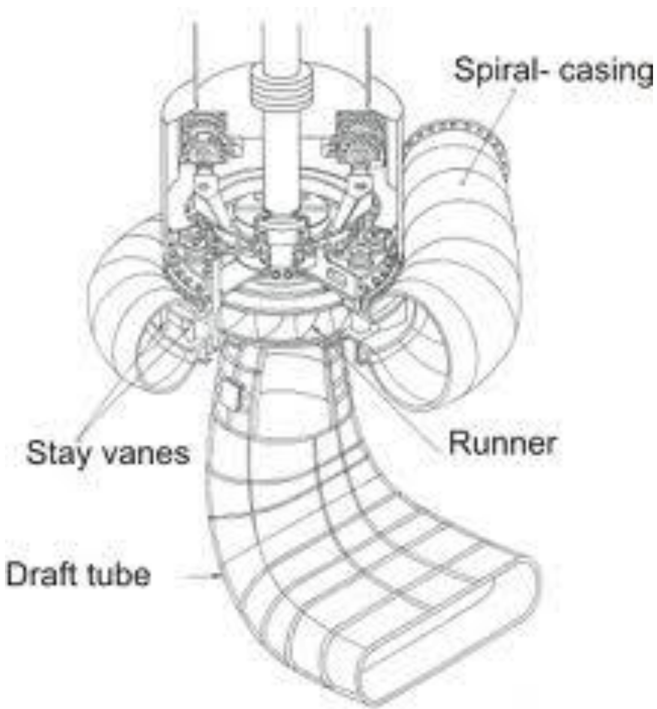
IMPROVED FOURNEYRON TURBINE.

WITH EMERSON'S MEASURING, GATE REGISTER.

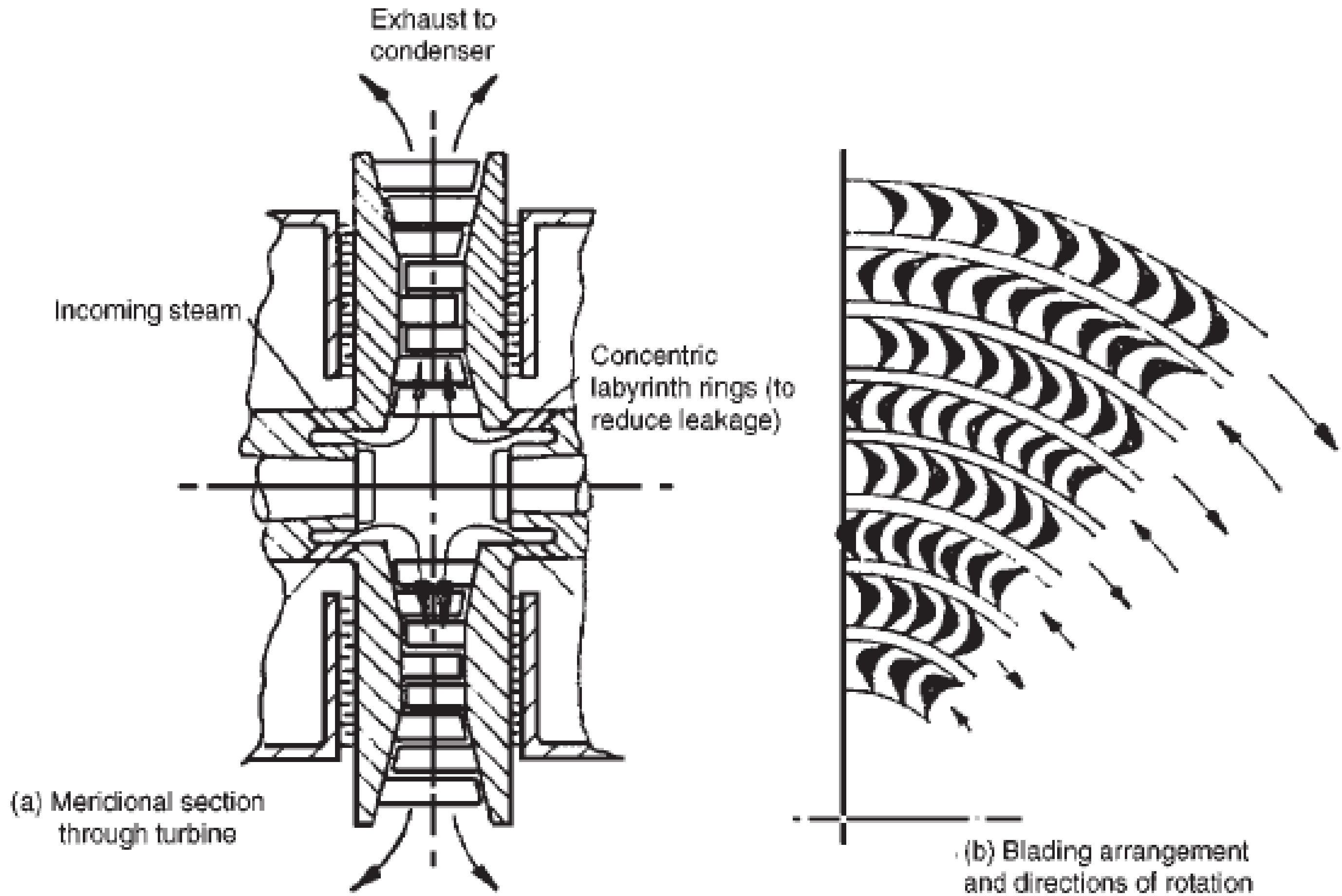


The White Elephant of the Lowell Corporations.

Radial turbine impellers



Francis turbine (hydraulic)



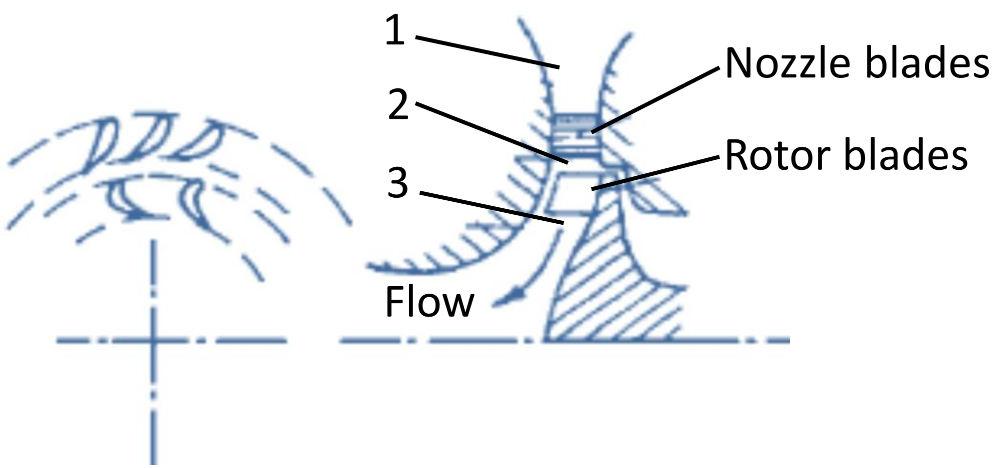
Ljungström steam turbine (1950s)

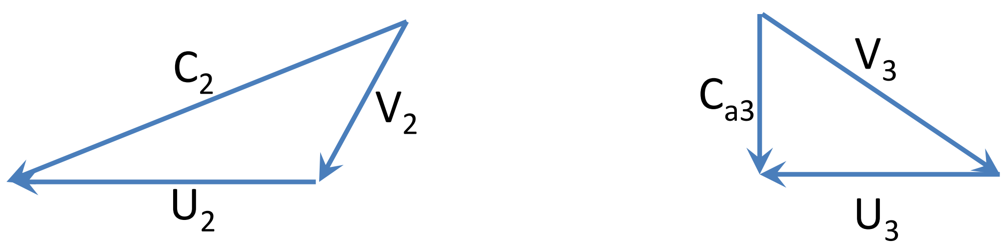
- tremendous increase in specific volume of steam, makes the radial-outflow efficient
- A unique feature of the Ljungström turbine is that it does not have any stationary blade rows.
- The two rows of blades comprising each of the stages rotate in opposite directions so that they can both be regarded as rotors.

Radial turbines

- There are two types of inward flow radial turbines
  - Cantilever turbine
  - 90^o IFR turbine
- Cantilever turbine
  - Similar to the impulse type turbine
  - Little change in relative velocity across the rotor
  - Aerodynamically very similar to the axial impulse turbine
  - Can be designed in a similar manner as axial turbines

Radial turbines



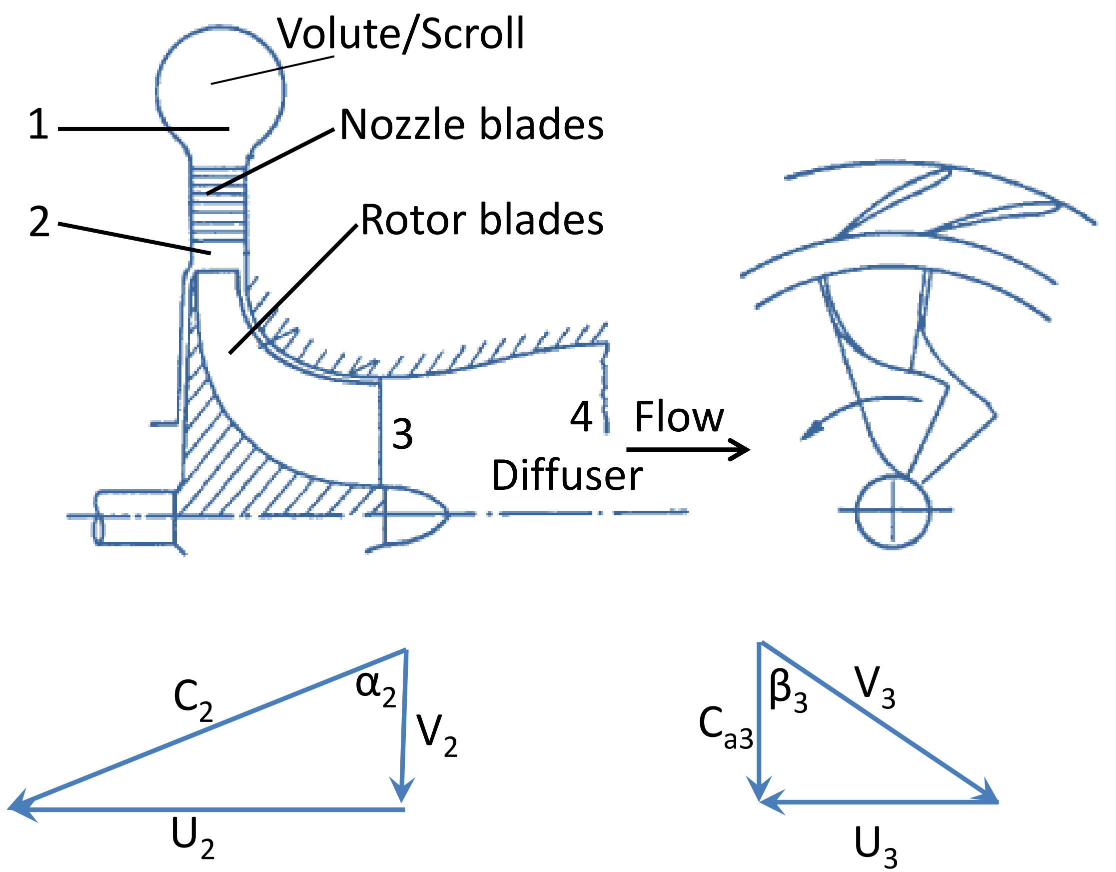


Cantilever turbine arrangement and velocity triangles

Radial turbines

- 90^{o} IFR turbine
  - This turbine has a striking similarity with a centrifugal compressor.
  - The flow direction and blade motion are reversed.
  - The flow enters the turbine radially and exits the turbine axially.
  - Straight radial blades are generally preferred as curved blades would incur additional stresses.
  - The rotor or impeller ends with an exducer.
  - Usually the flow exiting the rotor passes through a diffuser to recover KE, which would otherwise be wasted.

Radial turbines

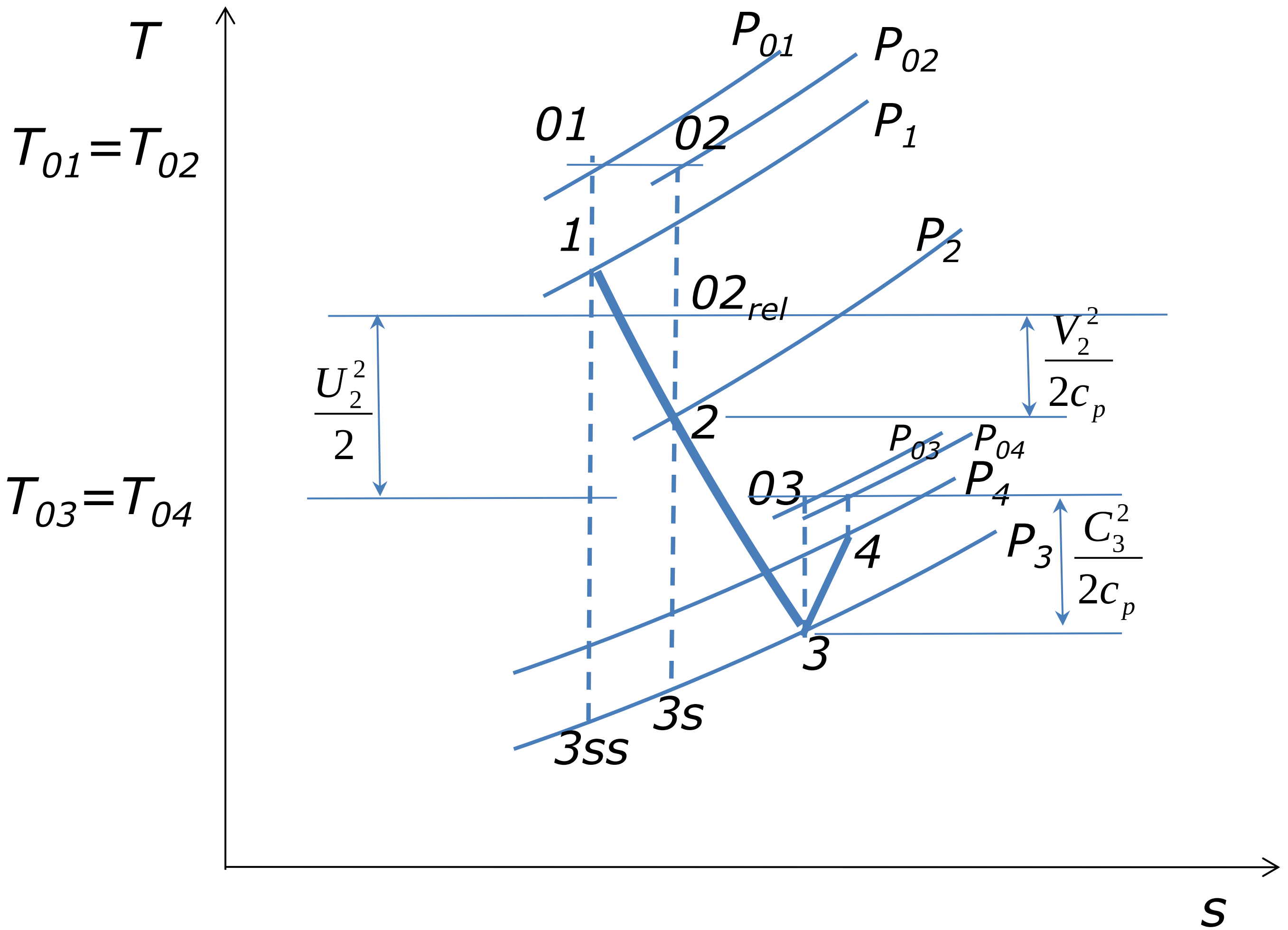


90° IFR turbine arrangement and velocity triangles

Thermodynamics of radial turbines

- We shall consider a 90^o IFR turbine.
- Components include: nozzle, radial bladed rotor and diffuser.
- We shall assume complete adiabatic expansion in the turbine.
- Frictional processes cause the entropy to increase in all the components.
- There is no change in stagnation enthalpy/temperature across the nozzle and the diffuser.

Thermodynamics of radial turbines



T-s diagram for an IFR turbine