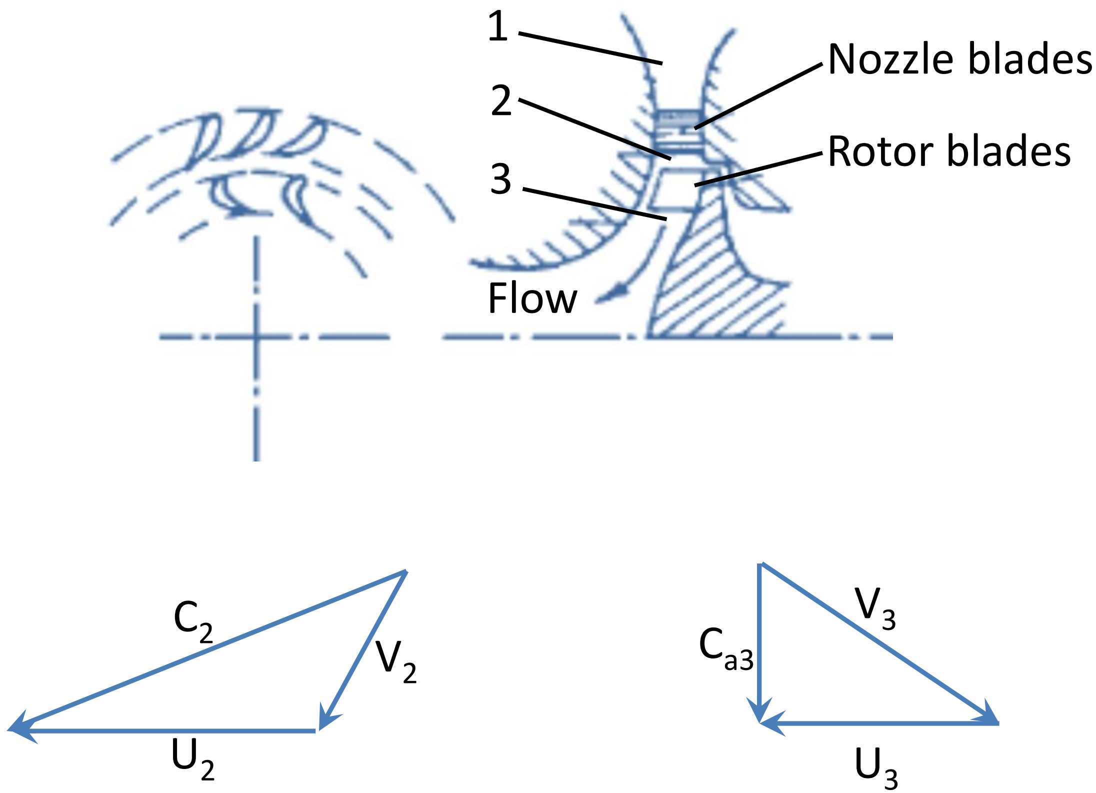
- Recap: Lecture 23, 23rd October 2015, 1530-1655 hrs.
  - Choking
    - Inlet, impeller and diffuser passages
  - Selection of turbomachines based on Specific speed and specific diameter
  - Tutorial #4: Centrifugal compressors
  - Radial Turbines
    - Introduction
    - Types of radial turbines
    - Thermodynamics of radial turbines

# Radial turbines

- There are two types of inward flow radial turbines
  - Cantilever turbine
  - $90^o$ IFR turbine
- Cantilever turbine
  - Similar to the impulse type turbine
  - Little change in relative velocity across the rotor
  - Aerodynamically very similar to the axial impulse turbine
  - Can be designed in a similar manner as axial turbines

# Radial turbines

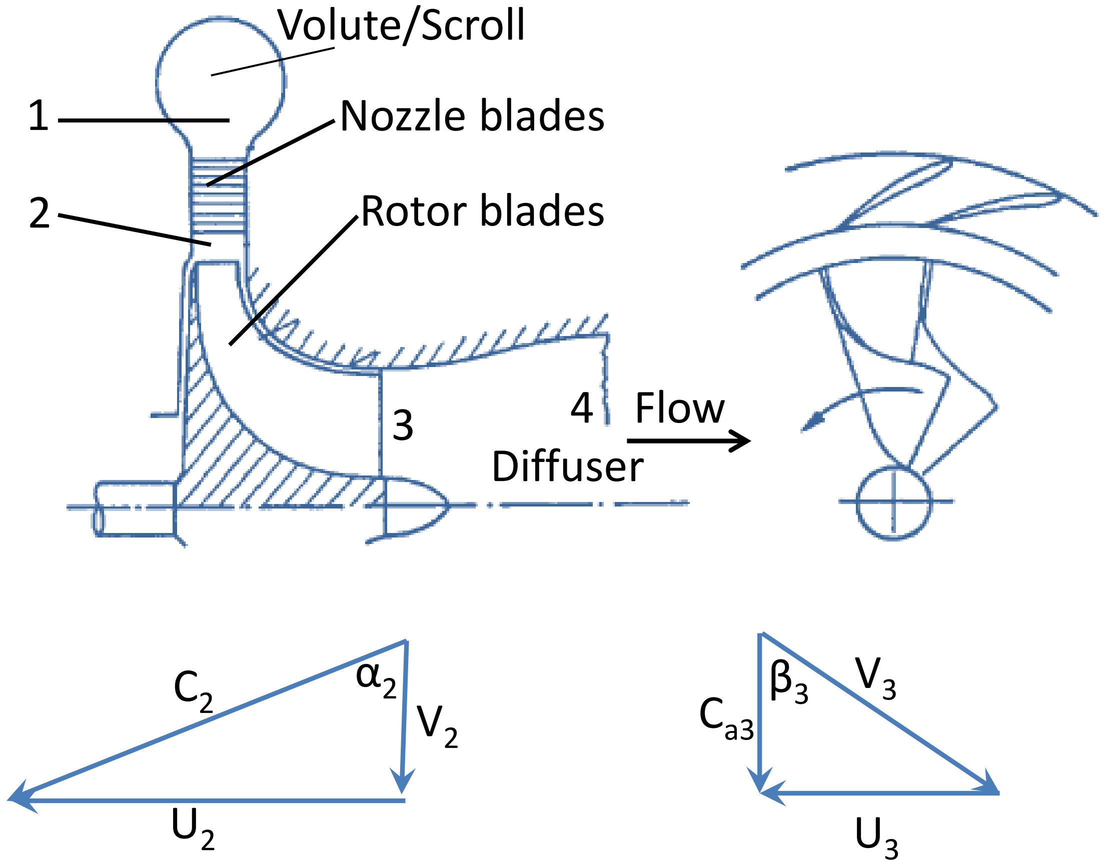



Cantilever turbine arrangement and velocity triangles

# Radial turbines

- $90^o$ IFR turbine
  - This turbine has a striking similarity with a centrifugal compressor.
  - The flow direction and blade motion are reversed.
  - The flow enters the turbine radially and exits the turbine axially.
  - Straight radial blades are generally preferred as curved blades would incur additional stresses.
  - The rotor or impeller ends with an exducer.
  - Usually the flow exiting the rotor passes through a diffuser to recover KE, which would otherwise be wasted.

# Radial turbines

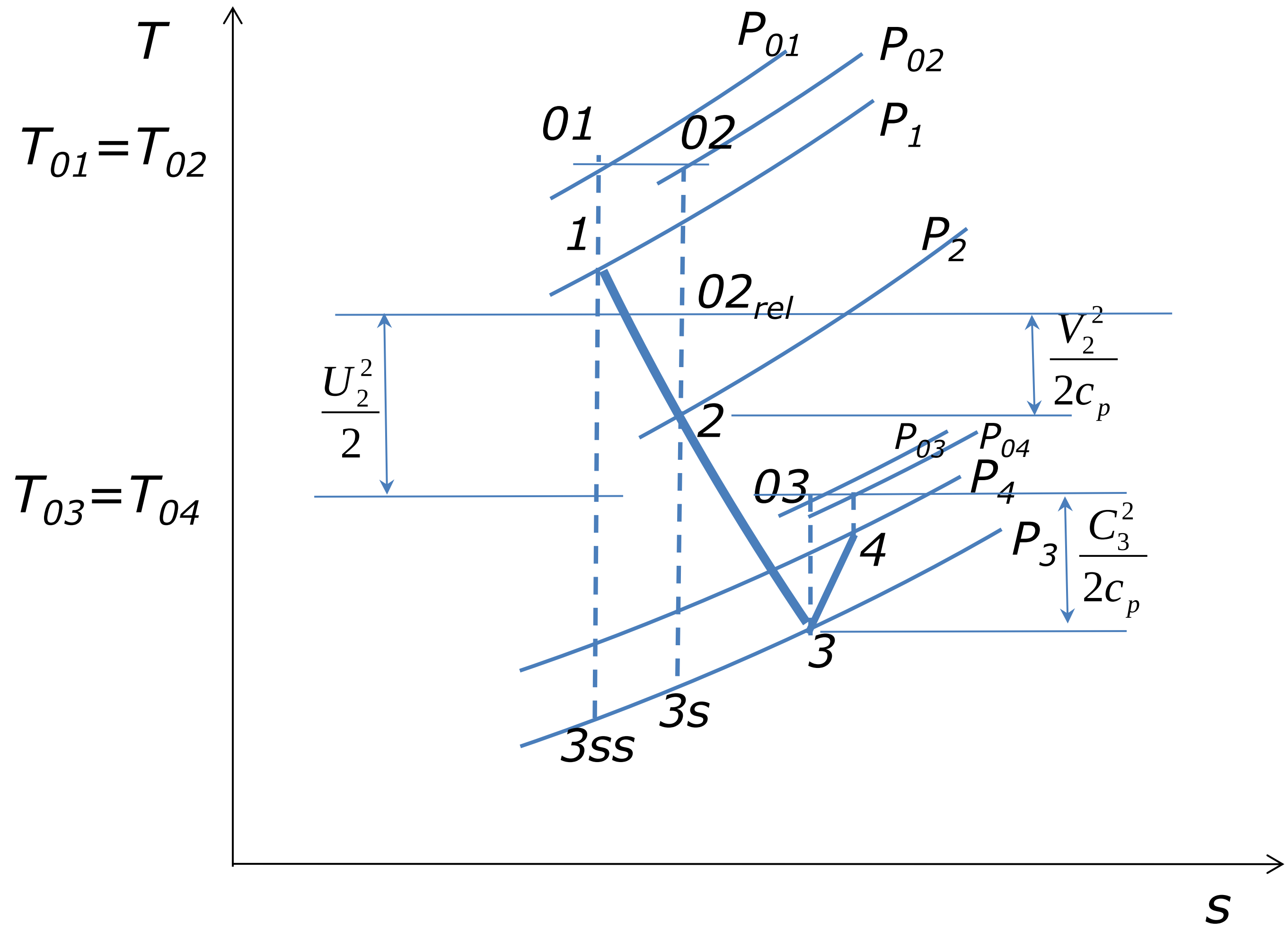



90$^{o}$ IFR turbine arrangement and velocity triangles

# Thermodynamics of radial turbines

- We shall consider a $90^o$ IFR turbine.
- Components include: nozzle, radial bladed rotor and diffuser.
- We shall assume complete adiabatic expansion in the turbine.
- Frictional processes cause the entropy to increase in all the components.
- There is no change in stagnation enthalpy/temperature across the nozzle and the diffuser.

# Thermodynamics of radial turbines




T-s diagram for an IFR turbine

# Thermodynamics of radial turbines

Across the nozzle, $h_{01} = h_{02}$
Therefore, the static enthalpy drop is,

$$h_1 - h_2 = \frac{1}{2}(C_2^2 - C_1^2)$$

In a radial flow machine, the rothalpy is conserved

$$I = h_{0rel} - \tfrac{1}{2}U^2$$

For the rotor, $h_{02rel} - \tfrac{1}{2}{U_2}^2 = h_{03rel} - \tfrac{1}{2}{U_3}^2$

$$\because h_{0rel} = h + \tfrac{1}{2}V^2$$

$$h_2 - h_3 = \frac{1}{2}[({U_2}^2 - {U_3}^2) - ({V_2}^2 - {V_3}^2)]$$

# Thermodynamics of radial turbines

The nozzle irreversibilities are lumped together with any friction losses occurring in the annular space between nozzle exit and rotor entry.

Across the diffuser, the stagnation enthalpy remains a constant. Hence,

$$h_{03} = h_{04} \quad \text{and} \qquad h_4 - h_3 = \frac{1}{2}(C_3^2 - C_4^2)$$

The specific work done by the fluid on the rotor is

$\Delta W = h_{01} - h_{03} = U_2 C_{w2} - U_3 C_{w3}$

Since, $h_{01} = h_{02}$

$\Delta W = h_{01} - h_{03} = h_2 - h_3 + \frac{1}{2}(C_2^2 - C_3^2)$

$$= \frac{1}{2}[({U_2}^2 - {U_3}^2) - ({V_2}^2 - {V_3}^2) + (C_2^2 - C_3^2)]$$

# Thermodynamics of radial turbines

$\Delta W = \frac{1}{2}[({U_2}^2 - {U_3}^2) - ({V_2}^2 - {V_3}^2) + (C_2^2 - C_3^2)]$

Each term on the RHS of the above equation, contributes to the specific work done.

A significant contribution comes from the first term $\frac{1}{2}({U_2}^2 - {U_3}^2)$.
This is the main reason why inward radial turbines have an advantage over outward radial turbines.

For axial turbines $U_2 = U_3$ and this contribution will be zero.

# Thermodynamics of radial turbines

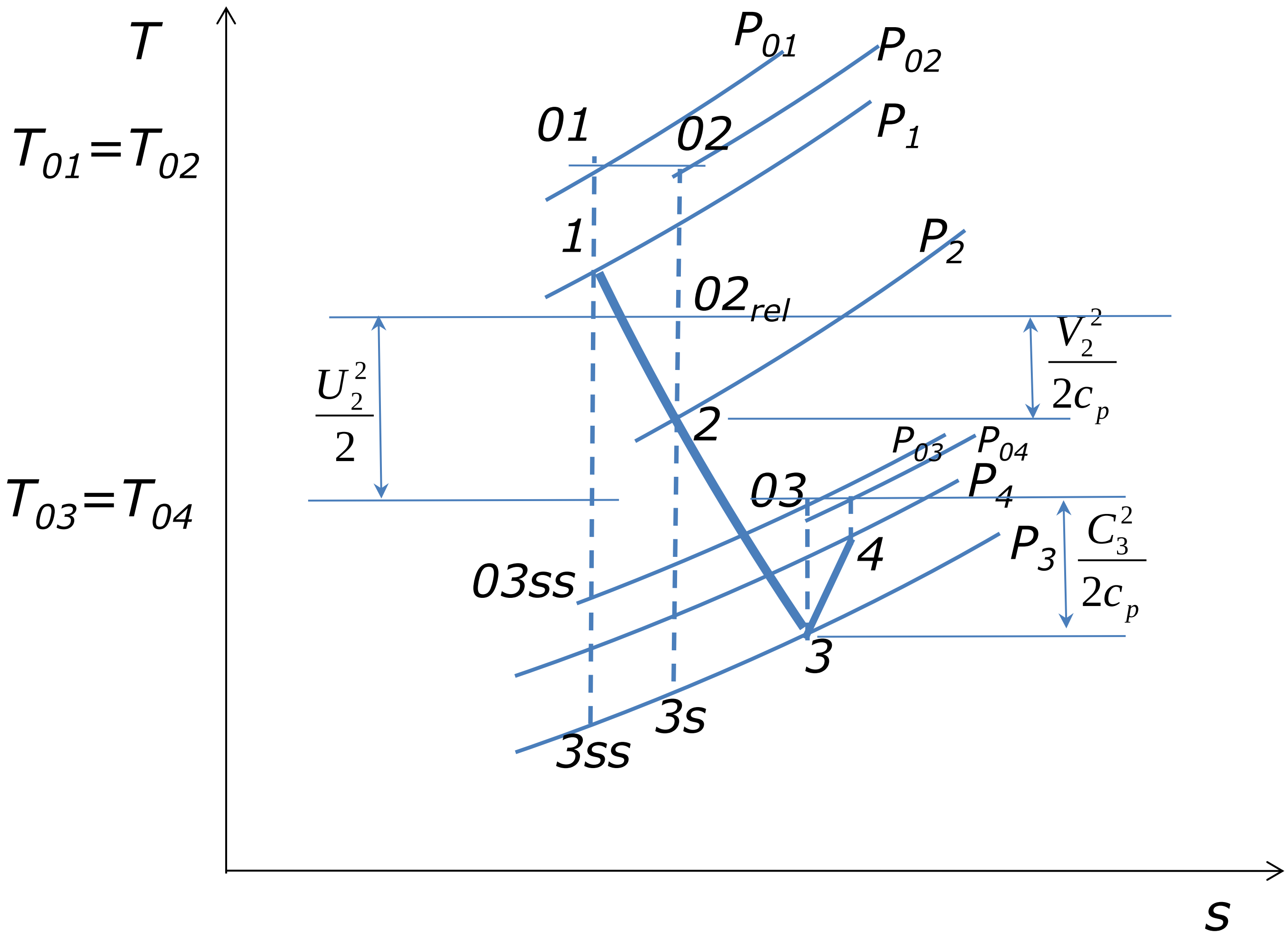
- Nominal design
  - Defined by a relative flow of zero incidence at the rotor inlet ($V_2=C_{r2}$).
  - An absolute flow at the rotor exit that is axial ($C_3=C_{a3}$).
  - Therefore, with $C_{w3}$=0 and $C_{w2}$=$U_2$, the specific work for nominal design is $\Delta W=U_2^2$.

# Thermodynamics of radial turbines

- Spouting velocity
  - The velocity which has an associated KE equal to the isentropic enthalpy drop across the turbine.
  - This can be defined based on total or static conditions and depending upon whether a diffuser is used or not.
  - $\frac{1}{2}C_0^2 = h_{01} - h_{03ss}$ for total condition
  - $\frac{1}{2}C_0^2 = h_{01} - h_{3ss}$ for static condition
  - In an ideal (frictionless) radial turbine with complete recovery of the exhaust KE and with $C_{w2} = U_2$ (Nominal design)

$$\Delta W = U_2^2 = \frac{1}{2}C_0^2$$

# Thermodynamics of radial turbines




T-s diagram for an IFR turbine

# Thermodynamics of radial turbines

- Nominal design point efficiency
  - The total-to-static efficiency in the absence of a diffuser is
  - $\eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{3ss}} = \frac{\Delta W}{\Delta W + \frac{1}{2}C_3^2 + (h_3 - h_{3s}) + (h_{3s} - h_{3ss})}$
  - The passage enthalpy losses can be expressed as a fraction $(\zeta)$of the exit KE relative to the nozzle and rotor row.
  - $h_3 - h_{3s} = \frac{1}{2}V_3^2\zeta_R$ and $h_3 - h_{3ss} = \frac{1}{2}C_2^2\zeta_N(\frac{T_3}{T_2})$

# Thermodynamics of radial turbines

- Substituting the above, we can write the efficiency as

$$\eta_{ts} = \left[1 + \tfrac{1}{2}C_3^2 + \left(\tfrac{1}{2}V_3^2\zeta_R\right) + \left(\tfrac{1}{2}C_2^2\zeta_N\left(\tfrac{T_3}{T_2}\right)\right) / \Delta W\right]^{-1}$$

- From the velocity triangles,

$$C_2 = U_2 \operatorname{cosec}\alpha_2 \text{ , } V_3 = U_3 \operatorname{cosec}\beta_3 \text{ ,}$$

$$C_3 = U_3 \cot\beta_3 \text{ , } \Delta W = U_2^2 \text{ and } U_3 = U_2 r_3 / r_2$$

- $\eta_{ts} = \left[1 + \frac{1}{2}\left\{\zeta_N \frac{T_3}{T_2}\operatorname{cosec}^2\alpha_2 + \left(\frac{r_3}{r_2}\right)^2 (\zeta_R \operatorname{cosec}^2\beta_3 + \cot^2\beta_3)\right\}\right]^{-1}$
- This equation is used in a variety of forms with appropriate assumptions.

# Thermodynamics of radial turbines

- The temperature ratio ($T_3/T_2$) can also be related to $U_2$, $r_3/r_2$ and the flow angles.
- The total-to-static efficiency is also expressed as

$$\eta_{ts} = 1 - (C_3^2 + \zeta_N C_2^2 + \zeta_R V_3^2)/C_o^2$$

where, $C_0$ is the spouting velocity.

- The total-to-static efficiency is related to the total-to-total efficiency.

$$\frac{1}{\eta_{tt}} = \frac{1}{\eta_{ts}} - \frac{C_3^2}{2\Delta W}$$

# Losses in radial turbines

- There are several ways of representing losses in an IFR turbine.
- Nozzle loss coefficients:
  - The enthalpy loss coefficient defined earlier is $\zeta_N = (h_2 - h_{2s})/\frac{1}{2}c_2^2$
  - Also in use is the velocity coefficient,

    $\phi_N = C_2/C_{2s}$
  - The stagnation pressure loss coefficient,

    $\varpi_N = (P_{01} - P_{02})/(P_{02} - P_2)$

# Losses in radial turbines

- The stagnation pressure loss coefficient can be related to enthalpy loss coefficient as

$$\varpi_N \simeq \zeta_N(1 + \tfrac{1}{2}\gamma M_2^2)$$

- Since, $h_{01} = h_2 + \frac{1}{2}c_2^2 = h_{2s} + \frac{1}{2}c_{2s}^2$ ,

  Then, $h_2 - h_{2s} = \frac{1}{2}(c_{2s}^2 - c_2^2)$

- Therefore, $\zeta_N = \dfrac{1}{\phi_N^2} - 1$
- For a well designed nozzle row, during normal operation, $0.90 \leq \phi_N \leq 0.97$

# Losses in radial turbines

- Rotor loss coefficients:
  - The enthalpy loss coefficient defined earlier is $\zeta_R = (h_3 - h_{3s})/\frac{1}{2}V_3^2$
  - Also in use is the velocity coefficient,

    $\phi_R = V_3/V_{3s}$
  - This is related to $\zeta_R$ by

$$\zeta_R = \frac{1}{\phi_R^2} - 1$$

  - For a well designed rotor, during normal operation, $0.70 \leq \phi_R \leq 0.85$

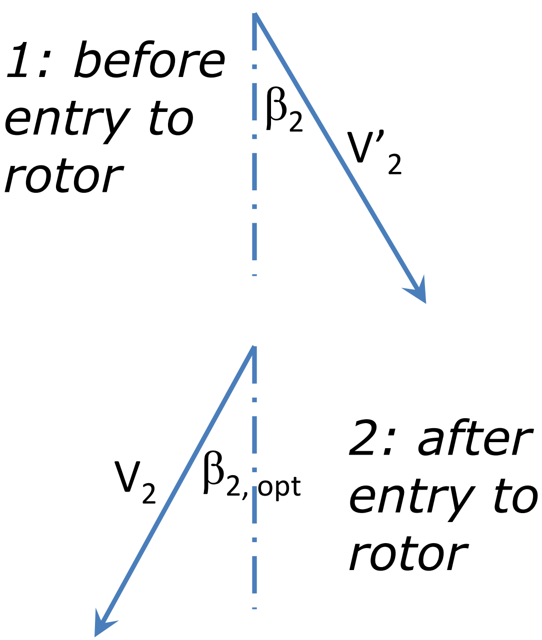
# Losses in radial turbines

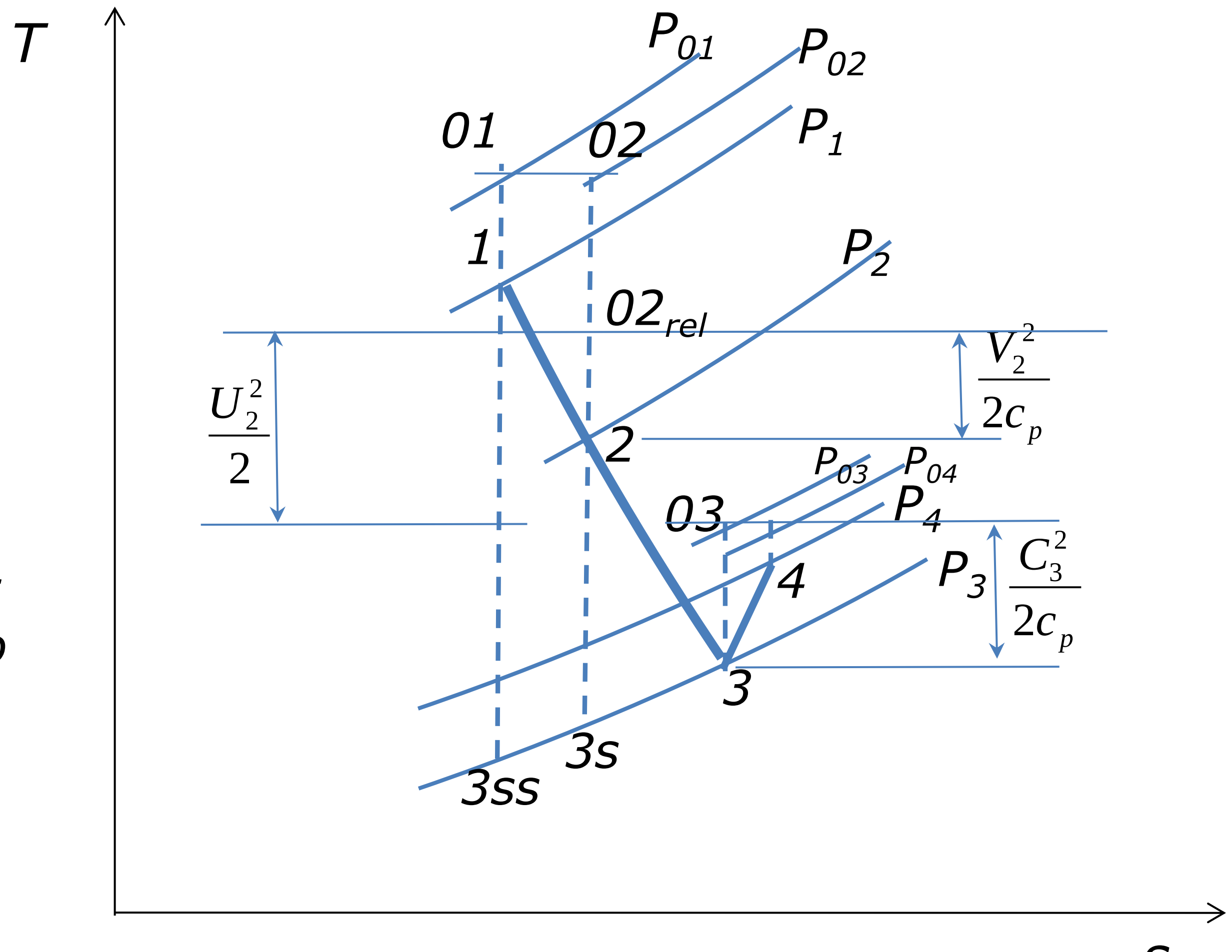
- In general, losses in a IFR turbine can be classified as:
  - nozzle blade row boundary layers,
  - rotor passage boundary layers,
  - rotor blade tip clearance,
  - disc windage (on the back surface of the rotor),
  - kinetic energy loss at exit.
- The above sources of losses are of significance for determining the optimum design geometry.

# Losses in radial turbines

- Incidence losses
  - At off-design conditions, the fluid is likely to enter the rotor at a relative flow angle different from the optimum angle.
  - This leads to an additional loss component due to incidence angles.
  - Often defined as equal to the kinetic energy corresponding to the component of velocity normal to the rotor vane at inlet.
  - There is an increase in entropy and hence a corresponding loss in enthalpy due to incidence.

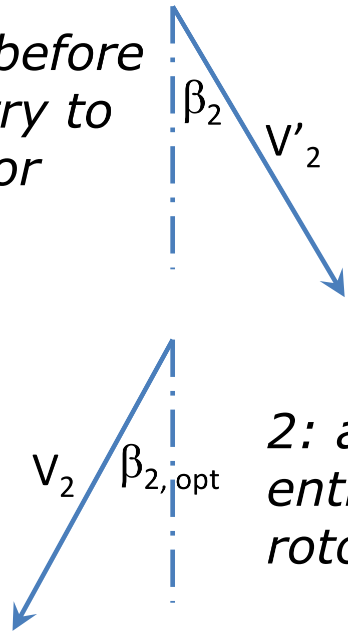
# Losses in radial turbines



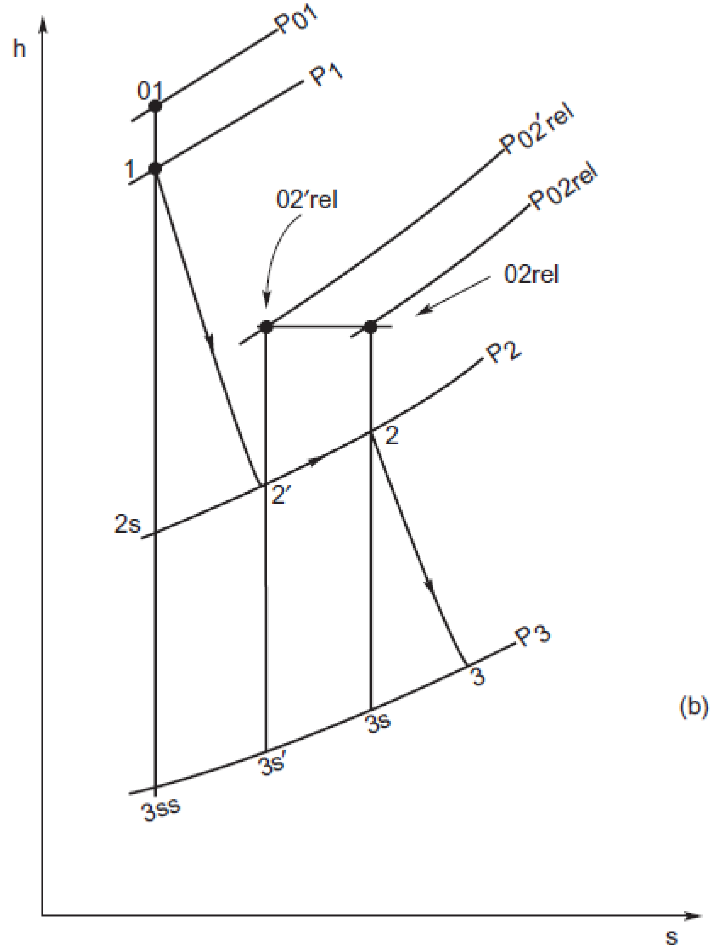




T-s diagram for an IFR turbine

*1: before entry to rotor*




*2: after entry to rotor*




Entropy increase, $(s_{3s} - s_{3s'})$ and enthalpy "loss", $(h_2 - h_{2'})$, as a constant pressure process resulting from non-optimum flow incidence

# Minimum number of blades

- Whitfield equation:

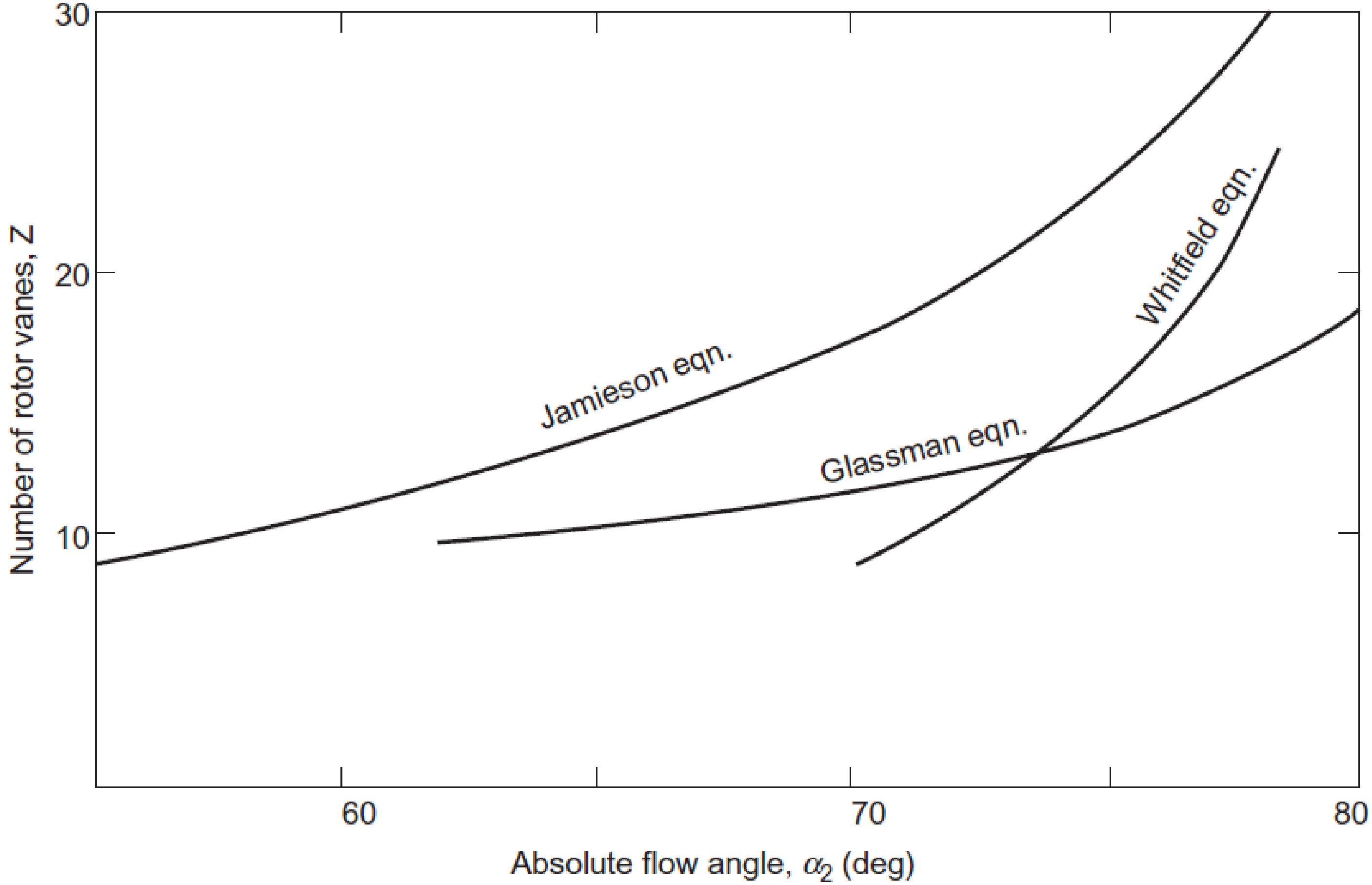
$$\cos^2 \alpha_2 = (1 - \cos \beta_2)/2 = 1/Z$$

- Jamieson's equation

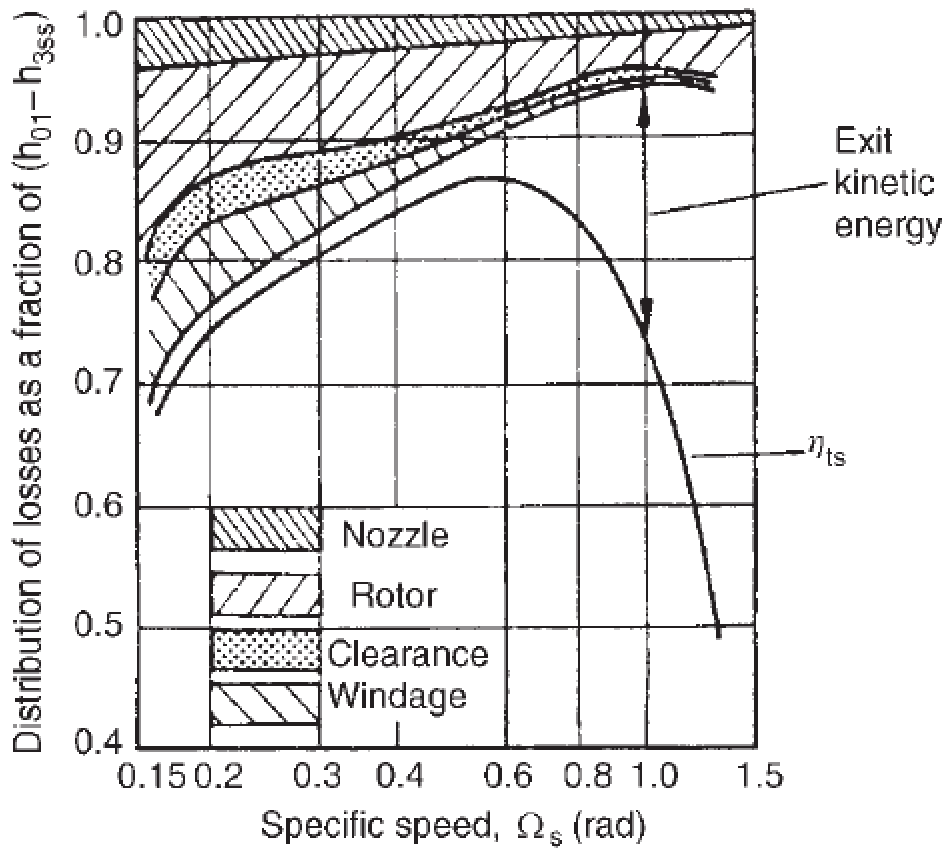
$$Z_{\min} = 2\pi \tan \alpha$$

- Glassman's equation

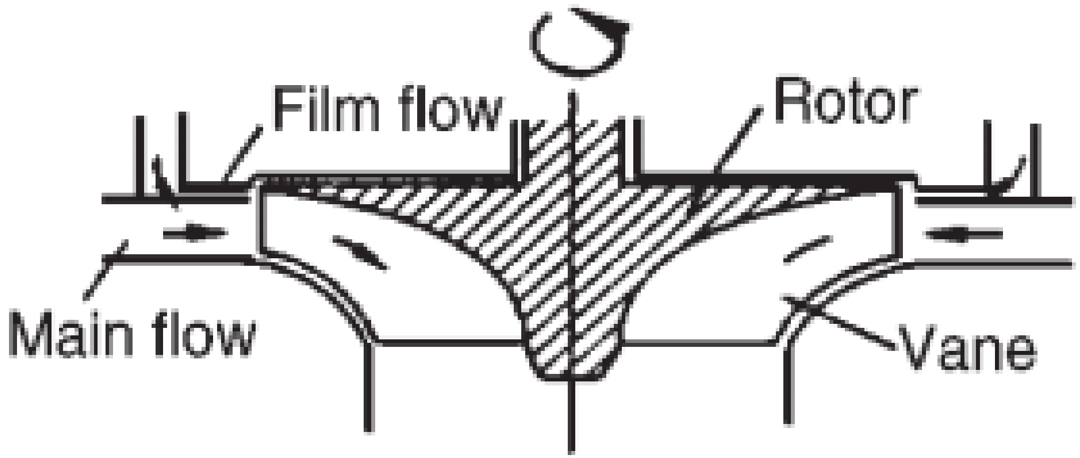
$$Z = \frac{\pi}{30}(110 - \alpha_2) \tan \alpha_2$$




**Flow angle at rotor inlet as a function of the number of rotor vanes**

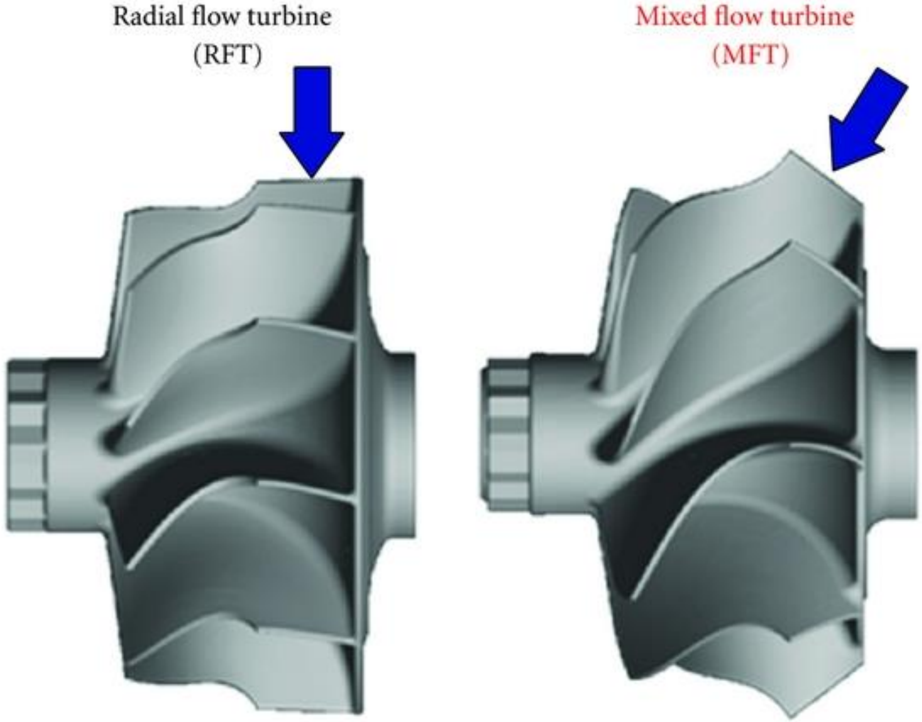



**Distribution of losses along envelope of maximum total-to-static efficiency**

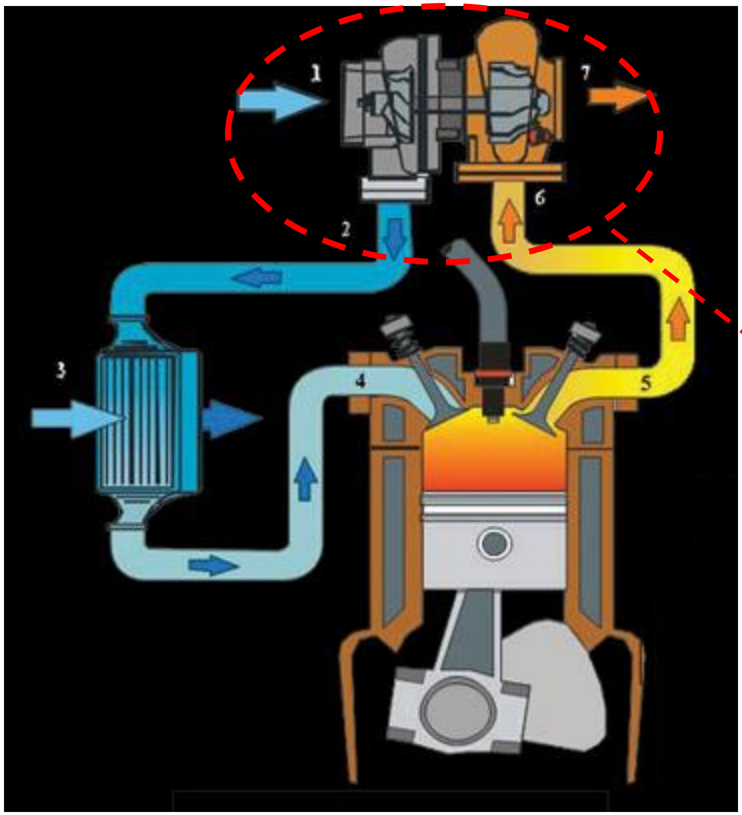


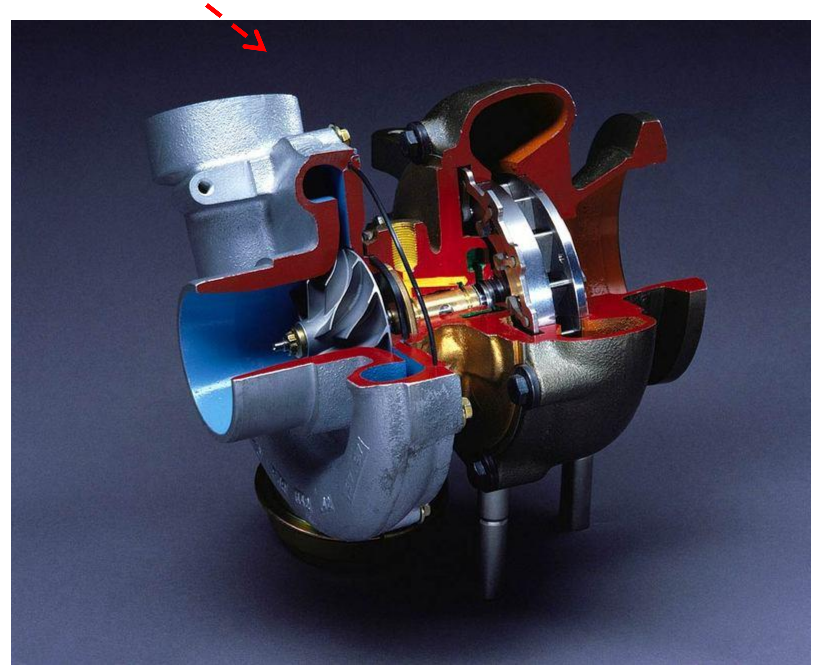

**Film cooling in a radial turbine impeller**

- Cooling technologies in radial turbine not as extensive as axial turbines
- Effectiveness of many of these methods are very low

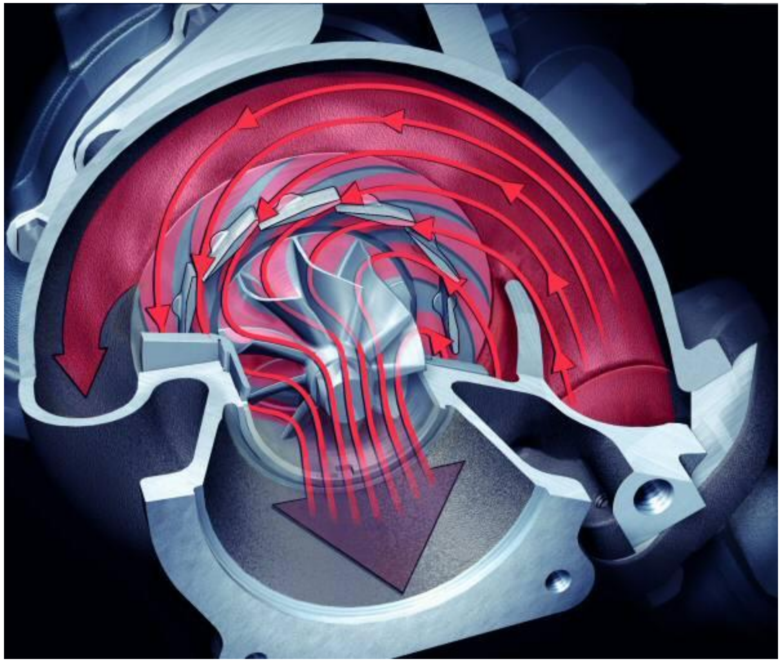



**Radial vs. mixed flow turbines**

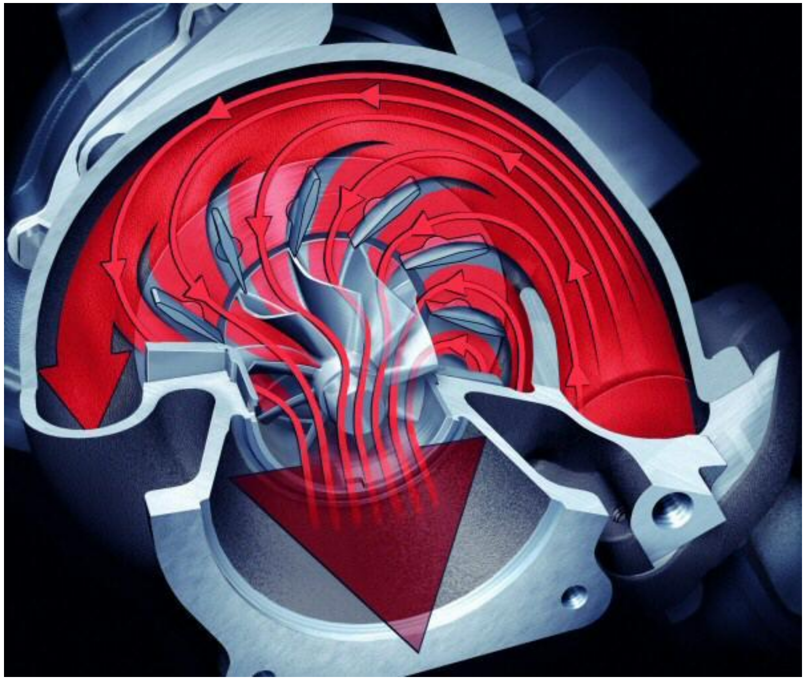


**Turbocharger of a typical high power car engine**



**Low Exhaust Flow VGT Position**
**VGT (Variable Geometry Turbine)**



**High Exhaust Flow VGT Position**