• Recap: Lecture 2: 24\textsuperscript{th} July 2015, 1530-1655 hrs.
  – Applications of gas turbine engines
  – Review of thermodynamics concepts
    • Energy, Enthalpy, Entropy
    • $T$-$s$ diagram
    • Isentropic processes
    • $Tds$ equations
Energy analysis of steady flow systems

- For single entry and exit devices,

\[ \dot{Q} - \dot{W} = m \left[ h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] \]

or per unit mass,

\[ \dot{q} - \dot{w} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \]
Turbines and compressors

\[ m_1 \rightarrow 1 \rightarrow \text{Turbine} \rightarrow W_T \rightarrow 2 \rightarrow m_2 \]

- Control surface
- Insulation

\[ \dot{m} \]

\[ W_T \]
Turbines and compressors

• For a turbine for eg., the energy equation would be:

\[ \dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) = \dot{W}_{out} + \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) \]

If KE and PE are negligible,

\[ \dot{W}_{out} = \dot{m}(h_1 - h_2) \]
Stagnation properties

• Enthalpy represents the total energy of a fluid in the absence of potential and kinetic energies.

• For high speed flows, though potential energy may be negligible, but not kinetic energy.

• Combination of enthalpy and KE is called **stagnation enthalpy** (or total enthalpy)

\[ h_0 = h + \frac{V^2}{2} \]  \hspace{2cm} (kJ/kg)

Stagnation enthalpy  Static enthalpy  Kinetic energy
Stagnation properties

• Consider a steady flow through a duct (no shaft work, heat transfer etc.).
• The steady flow energy equation for this is:
  \[ h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \]
  or, \[ h_{01} = h_{02} \]

• That is in the absence of any heat and work interactions, the stagnation enthalpy remains a constant during a steady flow process.
Stagnation properties

• If the fluid were brought to rest at state 2,
  \[ h_1 + \frac{V_1^2}{2} = h_2 = h_{02} \]

• The stagnation enthalpy represents the enthalpy of a fluid when it is brought to rest adiabatically.

• During a stagnation process, the kinetic energy of a fluid is converted to enthalpy (internal energy + flow energy), which results in an increase in the fluid temperature and pressure.
Stagnation properties

• When the fluid is approximated as an ideal gas with constant specific heats,
  
  \[ c_p T_0 = c_p T + \frac{V^2}{2} \]

  or,
  
  \[ T_0 = T + \frac{V^2}{2c_p} \]

• \( T_0 \) is called the **stagnation temperature** and represents the temperature an ideal gas attains when it is brought to rest adiabatically.

• The term \( \frac{V^2}{2c_p} \) corresponds to the temperature rise during such a process and is called the **dynamic temperature**.
Stagnation properties

• The pressure a fluid attains when brought to rest isentropically is called the stagnation pressure, \( P_0 \).

• For ideal gases, from isentropic relations, we have

\[
\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)}
\]

Similarly, for density we have,

\[
\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/(\gamma-1)}
\]
The actual state, actual stagnation state, and isentropic stagnation state of a fluid on an $h$-$s$ diagram.
Compressor/fan performance

• Compressors are to a high degree of approximation, adiabatic.
• Compressor performance can be evaluated using the isentropic efficiency, $\eta_c$

$$\eta_c = \frac{\text{Ideal work of compression for given pressure ratio}}{\text{Actual work of compression for given pressure ratio}}$$

$$= \frac{w_{ci}}{w_c} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}}$$
Compressor/fan performance

Actual and ideal compression processes

1: Compressor inlet
2: Compressor exit
Compressor/fan performance

\[ \eta_C = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}} \approx \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} \]

\[ = \frac{T_{02s} / T_{01} - 1}{T_{02} / T_{01} - 1} = \frac{(P_{02} / P_{01})^{(\gamma - 1)/\gamma} - 1}{\tau_C - 1} \]

\[ = \frac{(\pi_C)^{(\gamma - 1)/\gamma} - 1}{\tau_C - 1} \]

- The isentropic efficiency is thus a function of the total pressure ratio and the total temperature ratio.
Compressor/fan performance

- Besides isentropic efficiency, there are other efficiency definitions, stage efficiency and polytropic efficiency that are used in assessing the performance of multistage compressors.
- Stage efficiency will be discussed in detail during later lectures.
- The three efficiency terms can be related to one another.
Compressor/fan performance

- The polytropic efficiency, $\eta_{poly}$, is defined as

$$\eta_{poly} = \frac{\text{Ideal work of compression for a differential pressure change}}{\text{Actual work of compression for a differential pressure change}}$$

$$= \frac{\frac{dW_s}{dw}}{\frac{dh_0}{dh}} = \frac{\frac{dT_{0s}}{dT}}{\frac{dh_0}{dh}}$$

For an ideal compressor, the isentropic relation gives,

$$T_{0s} = P_0^{(\gamma-1)/\gamma} \times \text{constant.}$$

And, $dT_{0s} = T_0 \left[ \left( \frac{P_0 + dP_0}{P_0} \right)^{(\gamma-1)/\gamma} - 1 \right]$.

Using binomial expansion for $dP_0 / P_0 << 1$,

$$\left( 1 + \frac{dP_0}{P_0} \right)^{(\gamma-1)/\gamma} = 1 + \frac{\gamma - 1}{\gamma} \frac{dP_0}{P_0}$$
Compressor/fan performance

Therefore, \( \frac{dT_{0s}}{T_0} = \frac{\gamma - 1}{\gamma} \frac{dP_0}{P_0} \)

\( \eta_{\text{poly}} = \frac{dT_{0s}}{dT_0} = \frac{dT_{0s}}{P_0/T_0} = \frac{\gamma - 1}{\gamma} \frac{dP_0}{P_0} \)

Rewriting the above equation,

\( \frac{dT_0}{T_0} = \frac{\gamma - 1}{\gamma \eta_{\text{poly}}} \frac{dP_0}{P_0} \)

Integrating between states 01 and 02,

\( \frac{T_{02}}{T_{01}} = \left( \frac{P_{02}}{P_{01}} \right)^{(\gamma - 1)/(\gamma \eta_{\text{poly}})} \)

or, \( \eta_C = \frac{(\pi_C)^{(\gamma - 1)/\gamma} - 1}{\tau_C - 1} = \frac{(\pi_C)^{(\gamma - 1)/\gamma} - 1}{\pi_C^{(\gamma - 1)/(\gamma \eta_{\text{poly}})} - 1} \)

The above equation relates the isentropic efficiency with the pressure ratio assuming a constant polytropic efficiency.
Turbine performance

• The flow in a turbine is also assumed to be adiabatic, though in actual engines there could be turbine blade cooling.

• Isentropic efficiency of the turbine is defined in a manner similar to that of the compressor.

\[
\eta_t = \frac{\text{Actual work of expansion for given pressure ratio}}{\text{Ideal work of expansion for given pressure ratio}}
\]

\[
\eta_t = \frac{w_t}{w_{ts}} = \frac{h_{01} - h_{02}}{h_{01} - h_{02s}} = \frac{1 - \tau_t}{1 - \pi_t^{(\gamma-1)/\gamma}}
\]
Turbine performance

Actual and ideal turbine processes
Turbine performance

• The polytropic efficiency, $\eta_{poly}$, is defined as

$$\eta_{poly} = \frac{\text{Actual turbine work for a differential pressure change}}{\text{Ideal turbine work for a differential pressure change}}$$

$$= \frac{dw}{dw_s} = \frac{dh_0}{dh_{0s}} = \frac{dT_0}{dT_{0s}}$$

For an ideal turbine, the isentropic relation gives,

$$T_{0s} = P_{0}^{(\gamma - 1)/\gamma} \times \text{constant. Therefore,}$$

$$\frac{dT_{0s}}{T_0} = \frac{\gamma - 1}{\gamma} \frac{dP_0}{P_0}$$

$$\eta_{poly} = \frac{dT_0}{dT_{0s}} = \frac{dT_0 / T_0}{dT_{0s} / T_0} = \frac{dT_0 / T_0}{[(\gamma - 1) / \gamma]} \frac{dP_0}{P_0}$$
Turbine performance

Integrating between states 01 and 02,

\[ \pi_t = \tau_t^{\gamma/[(\gamma-1)\eta_{poly}]} \]

or,

\[ \eta_t = \frac{1 - \tau_t}{1 - \tau_t^{1/\eta_{poly}}} = \frac{1 - (\pi_t)^{(\gamma-1)/\gamma}}{1 - (\pi_t)^{(\gamma-1)/\gamma}} \]

The above equation relates the isentropic efficiency with the pressure ratio assuming a constant polytropic efficiency.
Thermodynamics of compressors

- Simplified aero-thermodynamic analysis
  - Optimised cycle design to precede the detailed component design
  - Prediction of work requirements
  - Efficiency of the compressor
  - Enables faster design modifications
Thermodynamics of compression

(i) Adiabatic (process 1-2'), \( P v^\gamma = c \)
(ii) Isothermal process (1-2''), \( P v = c \)
(iii) Isochoric (Process 1-2'''), \( P v = c^\infty \)
Thermodynamics of compressors

i) Isentropic process (1-2')

ii) Polytropic process (1-2)

iii) Isothermal process (1-2'')

iv) Isochoric Process (1-2''')
Thermodynamics of compressors

$X_1$, $X_2$ are the losses in the rotor and the stator respectively.

Compression in terms of static parameters
Thermodynamics of compressors

Compression in terms of total parameters
Basic operation of axial compressors

- Axial flow compressors usually consist of a series of stages.
- Each stage comprises of a row of rotor blades followed by a row of stator blades.
- The working fluid is initially accelerated by the rotor blades and then decelerated in the stator passages.
- In the stator, the kinetic energy transferred in the rotor is converted to static pressure.
- This process is repeated in several stages to yield the necessary overall pressure ratio.
Thermodynamics of multi-stage compressors

- The flow at the rotor exit with high kinetic energy is still to be converted to static pressure through diffusion.
- The exit kinetic energy of a compressor is of the same order as the entry kinetic energy and the entire work input is expected to be converted to pressure.