- Recap: Lecture 2: 24th July 2015, 1530-1655 hrs.
  - Applications of gas turbine engines
  - Review of thermodynamics concepts
    - Energy, Enthalpy, Entropy
    - *T-s* diagram
    - Isentropic processes
    - *Tds* equations

Energy analysis of steady flow systems

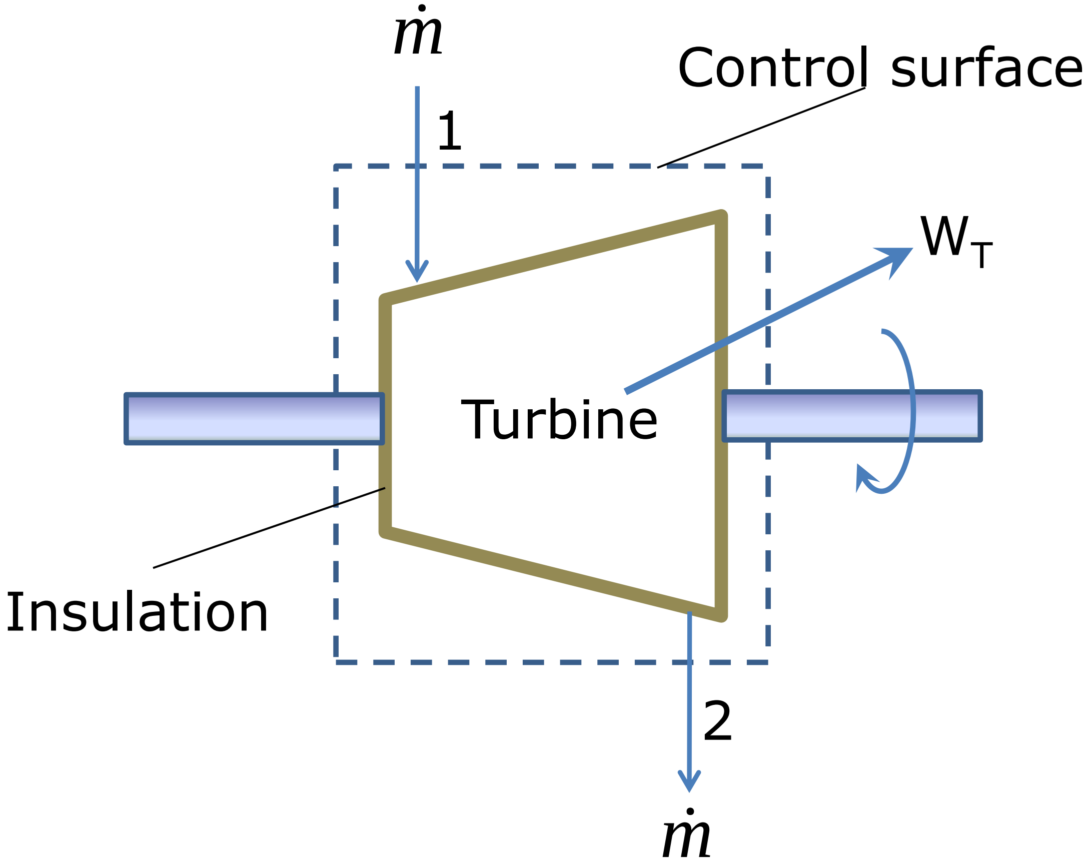
- For single entry and exit devices,

$$\dot{Q} - \dot{W} = \dot{m}\left[h_2 - h_1 + \frac{V_2^{\,2} - V_1^{\,2}}{2} + g(z_2 - z_1)\right]$$

or per unit mass,

$$\dot{q} - \dot{w} = h_2 - h_1 + \frac{V_2^{\,2} - V_1^{\,2}}{2} + g(z_2 - z_1)$$

Turbines and compressors



Turbines and compressors

- For a turbine for eg., the energy equation would be:

$$\dot{m}\left(h_1 + \frac{V_1^{\,2}}{2} + gz_1\right) = \dot{W}_{out} + \dot{m}\left(h_2 + \frac{V_2^{\,2}}{2} + gz_2\right)$$

If KE and PE are negligible,

$$\dot{W}_{out} = \dot{m}(h_1 - h_2)$$

Stagnation properties

- Enthalpy represents the total energy of a fluid in the absence of potential and kinetic energies.
- For high speed flows, though potential energy may be negligible, but not kinetic energy.
- Combination of enthalpy and KE is called stagnation enthalpy (or total enthalpy)

$$h_0 = h + V^2/2 \qquad \text{(kJ/kg)}$$

Stagnation enthalpy Static enthalpy Kinetic energy

Stagnation properties

- Consider a steady flow through a duct (no shaft work, heat transfer etc.).
- The steady flow energy equation for this is: $h_1 + V_1^2/2 = h_2 + V_2^2/2$

  or, $h_{01}=h_{02}$

- That is in the absence of any heat and work interactions, the stagnation enthalpy remains a constant during a steady flow process.

Stagnation properties

- If the fluid were brought to rest at state 2,

$$h_1 + V_1^2/2 = h_2 = h_{02}$$

- The stagnation enthalpy represents the enthalpy of a fluid when it is brought to rest adiabatically.
- During a stagnation process, the kinetic energy of a fluid is converted to enthalpy (internal energy + flow energy), which results in an increase in the fluid temperature and pressure.

Stagnation properties

- When the fluid is approximated as an ideal gas with constant specific heats,

$$c_p T_0 = c_p T + V^2/2$$

$$\text{or,} \quad T_0 = T + V^2/2c_p$$

- T_0 is called the stagnation temperature and represents the temperature an ideal gas attains when it is brought to rest adiabatically.
- The term $V^2/2c_p$ corresponds to the temperature rise during such a process and is called the dynamic temperature.

Stagnation properties

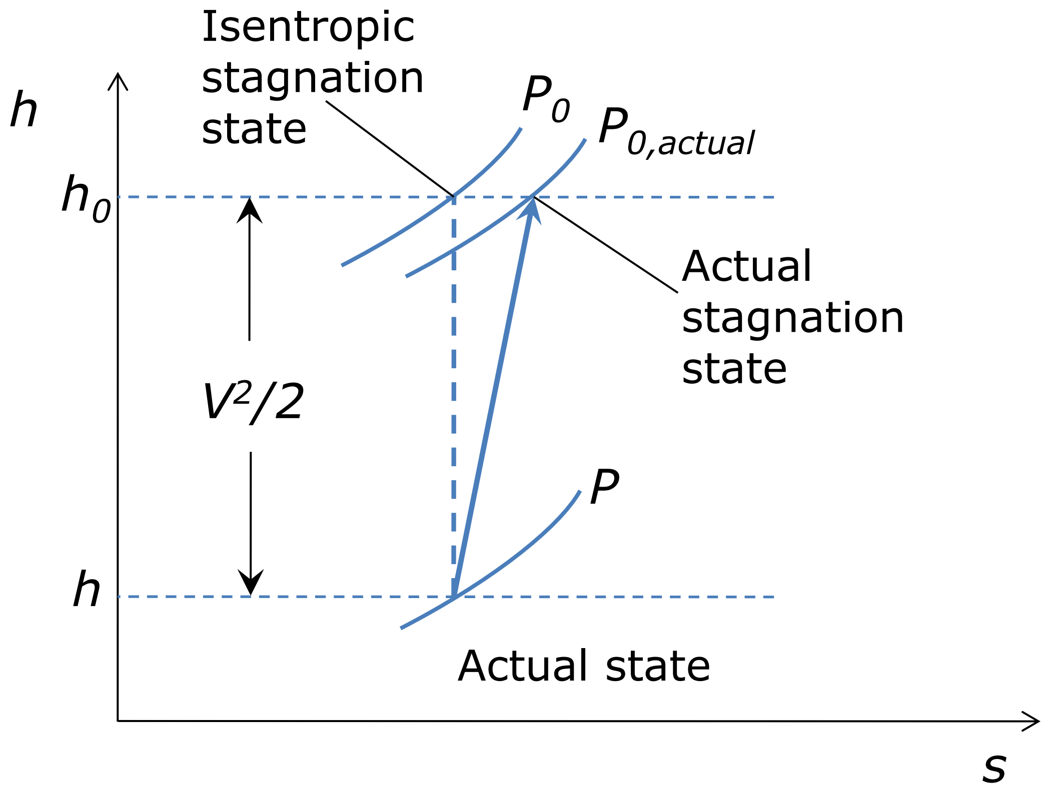
- The pressure a fluid attains when brought to rest isentropically is called the stagnation pressure, P_0.
- For ideal gases, from isentropic relations,

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)}$$

Similarly, for density we have,

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/(\gamma-1)}$$

Stagnation properties



The actual state, actual stagnation state, and isentropic stagnation state of a fluid on an *h-s* diagram.

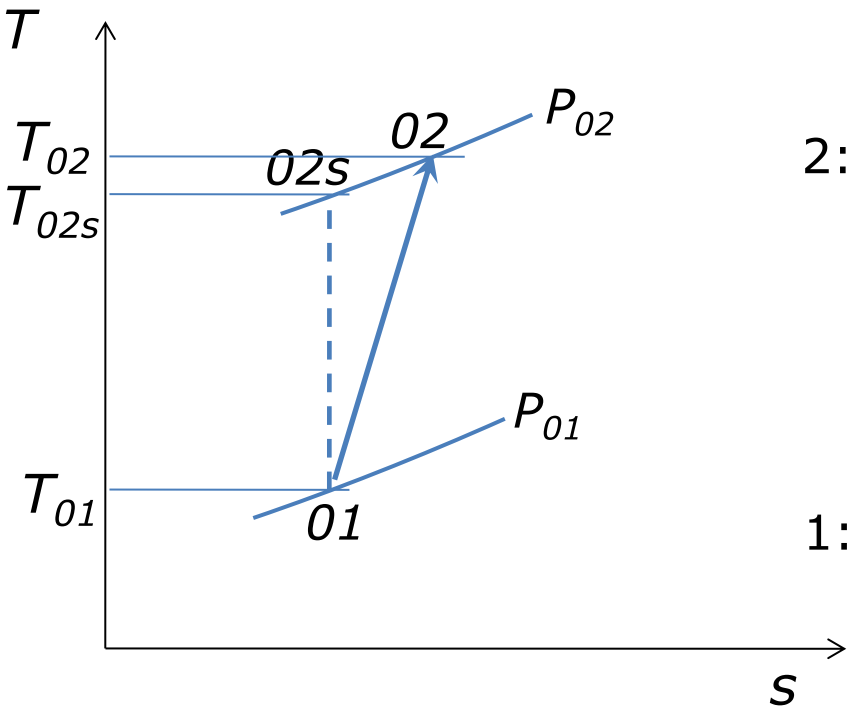
Compressor/fan performance

- Compressors are to a high degree of approximation, adiabatic.
- Compressor performance can be evaluated using the isentropic efficiency, η_c

$$\eta_C = \frac{\text{Ideal work of compression for given pressure ratio}}{\text{Actual work of compression for given pressure ratio}}$$

$$= \frac{w_{ci}}{w_c} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}}$$

Compressor/fan performance



2: Compressor exit

1: Compressor inlet

Actual and ideal compression processes

Compressor/fan performance

$$\eta_C = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}} \cong \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

$$= \frac{T_{02s}/T_{01} - 1}{T_{02}/T_{01} - 1} = \frac{\left(P_{02}/P_{01}\right)^{(\gamma-1)/\gamma} - 1}{\tau_C - 1}$$

$$= \frac{\left(\pi_C\right)^{(\gamma-1)/\gamma} - 1}{\tau_C - 1}$$

- The isentropic efficiency is thus a function of the total pressure ratio and the total temperature ratio.

Compressor/fan performance

- Besides isentropic efficiency, there are other efficiency definitions, stage efficiency and polytropic efficiency that are used in assessing the performance of multistage compressors.
- Stage efficiency will be discussed in detail during later lectures
- The three efficiency terms can be related to one another.

Compressor/fan performance

- The polytropic efficiency, η_{poly}, is defined as

$$\eta_{poly} = \frac{\text{Ideal work of compression for a differential pressure change}}{\text{Actual work of compression for a differential pressure change}}$$

$$= \frac{dw_s}{dw} = \frac{dh_{0s}}{dh_0} = \frac{dT_{0s}}{dT_0}$$

For an ideal compressor, the isentropic relation gives,

$$T_{0s} = P_0^{(\gamma-1)/\gamma} \times \text{constant.}$$

$$\text{And, } \mathrm{dT_{0s}} = T_0\left[\left(\frac{P_0 + dP_0}{P_0}\right)^{(\gamma-1)/\gamma} - 1\right]$$

Using binomial expansion for $dP_0 / P_0 << 1$,

$$\left(1 + \frac{dP_0}{P_0}\right)^{(\gamma-1)/\gamma} = 1 + \frac{\gamma-1}{\gamma}\frac{dP_0}{P_0}$$

Compressor/fan performance

Therefore, $\dfrac{dT_{0s}}{T_0} = \dfrac{\gamma - 1}{\gamma}\dfrac{dP_0}{P_0}$

$$\eta_{poly} = \frac{dT_{0s}}{dT_0} = \frac{dT_{0s}/T_0}{dT_0/T_0} = \frac{\gamma-1}{\gamma}\frac{dP_0/P_0}{dT_0/T_0}$$

Rewriting the above equation,

$$\frac{dT_0}{T_0} = \frac{\gamma-1}{\gamma\eta_{poly}}\frac{dP_0}{P_0}$$

Integrating between states 01 and 02,

$$T_{02}/T_{01} = (P_{02}/P_{01})^{(\gamma-1)/(\gamma\eta_{poly})}$$

$$or, \eta_C = \frac{(\pi_C)^{(\gamma-1)/\gamma} - 1}{\tau_C - 1} = \frac{(\pi_C)^{(\gamma-1)/\gamma} - 1}{\pi_C^{(\gamma-1)/(\gamma\eta_{poly})} - 1}$$

The above equation relates the isentropic efficiency with the pressure ratio assuming a constant polytropic efficiency.

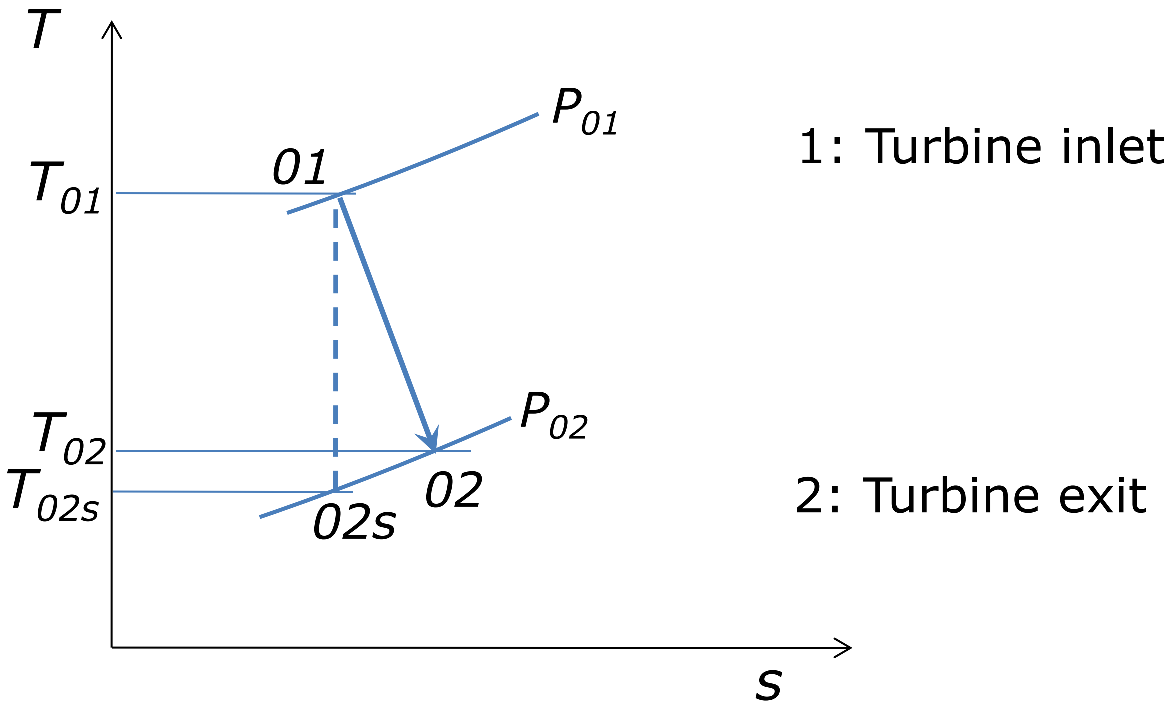
Turbine performance

- The flow in a turbine is also assumed to be adiabatic, though in actual engines there could be turbine blade cooling.
- Isentropic efficiency of the turbine is defined in a manner similar to that of the compressor.

$$\eta_t = \frac{\text{Actual work of expansion for given pressure ratio}}{\text{Ideal work of expansion for given pressure ratio}}$$

$$= \frac{w_t}{w_{ts}} = \frac{h_{01} - h_{02}}{h_{01} - h_{02s}} = \frac{1 - \tau_t}{1 - \pi_t^{(\gamma-1)/\gamma}}$$

Turbine performance



Actual and ideal turbine processes

Turbine performance

- The polytropic efficiency, η_{poly}, is defined as

$$\eta_{poly} = \frac{\text{Actual turbine work for a differential pressure change}}{\text{Ideal turbine work for a differential pressure change}}$$

$$= \frac{dw}{dw_s} = \frac{dh_0}{dh_{0s}} = \frac{dT_0}{dT_{0s}}$$

For an ideal turbine, the isentropic relation gives,

$T_{0s} = P_0^{(\gamma-1)/\gamma} \times \text{constant}$. Therefore,

$$\frac{dT_{0s}}{T_0} = \frac{\gamma-1}{\gamma}\frac{dP_0}{P_0}$$

$$\eta_{poly} = \frac{dT_0}{dT_{0s}} = \frac{dT_0/T_0}{dT_{0s}/T_0} = \frac{dT_0/T_0}{\left[(\gamma-1)/\gamma\right]dP_0/P_0}$$

Turbine performance

Integrating between states 01 and 02,

$$\pi_{\mathrm{t}} = \tau_t^{\gamma/\left[(\gamma-1)\eta_{poly}\right]}$$

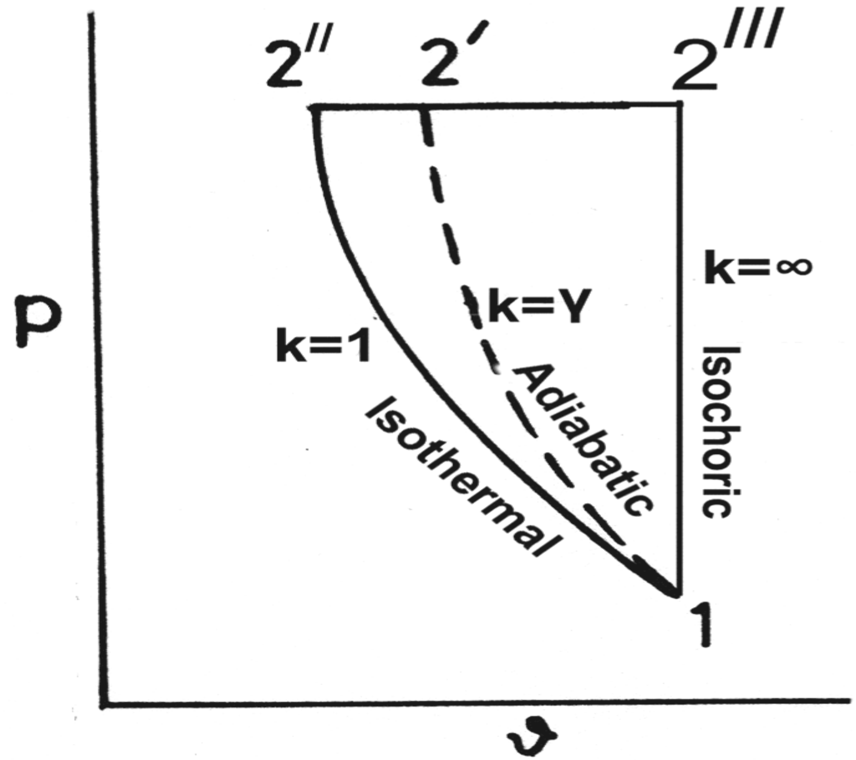
$$or, \eta_t = \frac{1-\tau_t}{1-\tau_t^{1/\eta_{poly}}} = \frac{1-\left(\pi_t\right)^{(\gamma-1)\eta_{poly}/\gamma}}{1-\left(\pi_t\right)^{(\gamma-1)/\gamma}}$$

The above equation relates the isentropic efficiency with the pressure ratio assuming a constant polytropic efficiency.

Thermodynamics of compressors

- Simplified aero-thermodynamic analysis
  - Optimised cycle design to precede the detailed component design
  - Prediction of work requirements
  - Efficiency of the compressor
  - Enables faster design modifications

Thermodynamics of compression

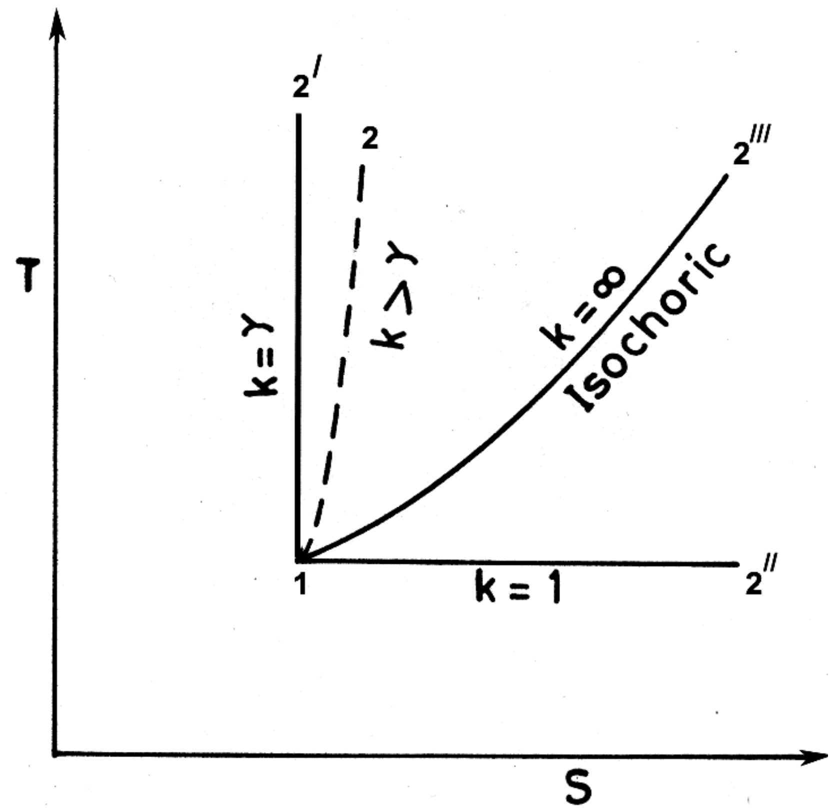


(i) Adiabatic (process 1-2$'$) , $Pv^{\gamma}=c$

(ii) Isothermal process (1-2$''$), $Pv=c$

(iii) Isochoric (Process 1-2$'''$), $Pv^{\infty}=c$

Thermodynamics of compressors



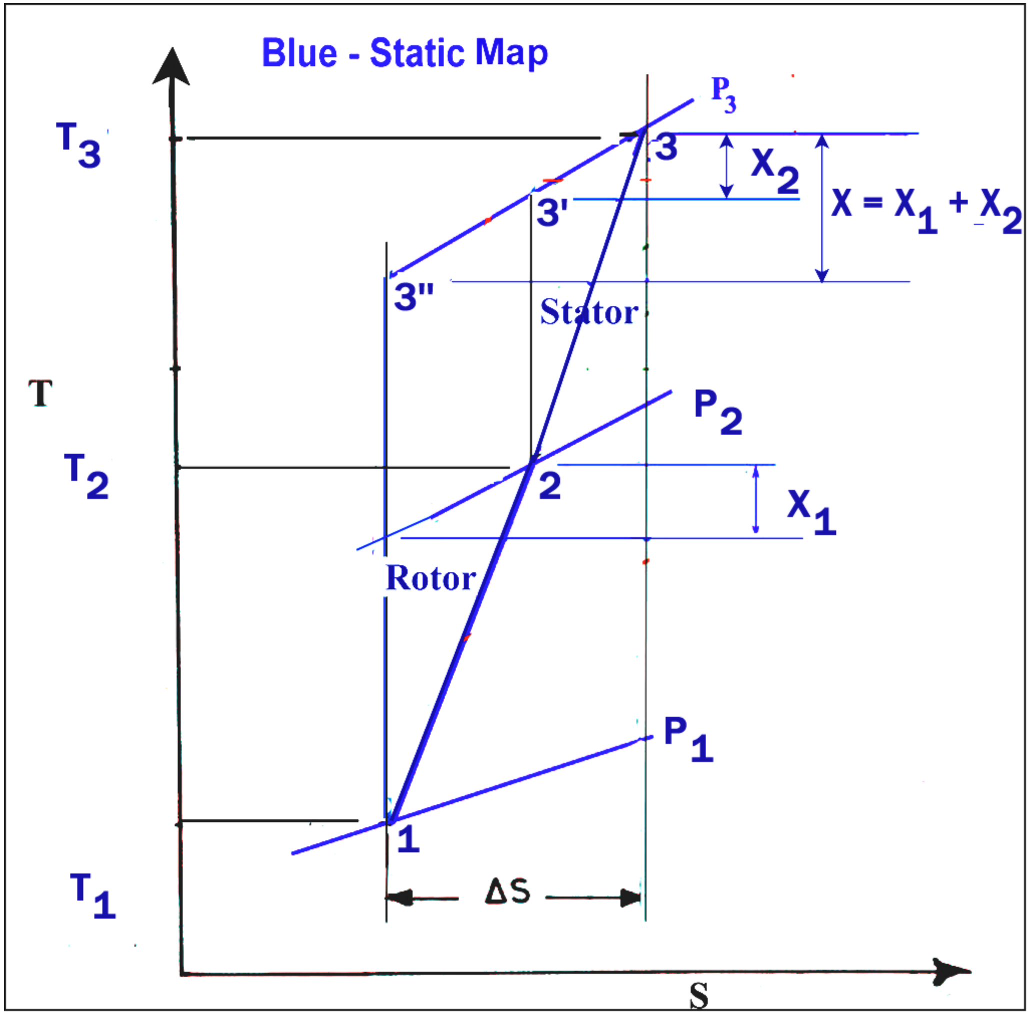
i) Isentropic process (1-2′)

ii) Polytropic process (1-2)

iii) Isothermal process (1-2″)

iv) Isochoric Process (1-2‴)

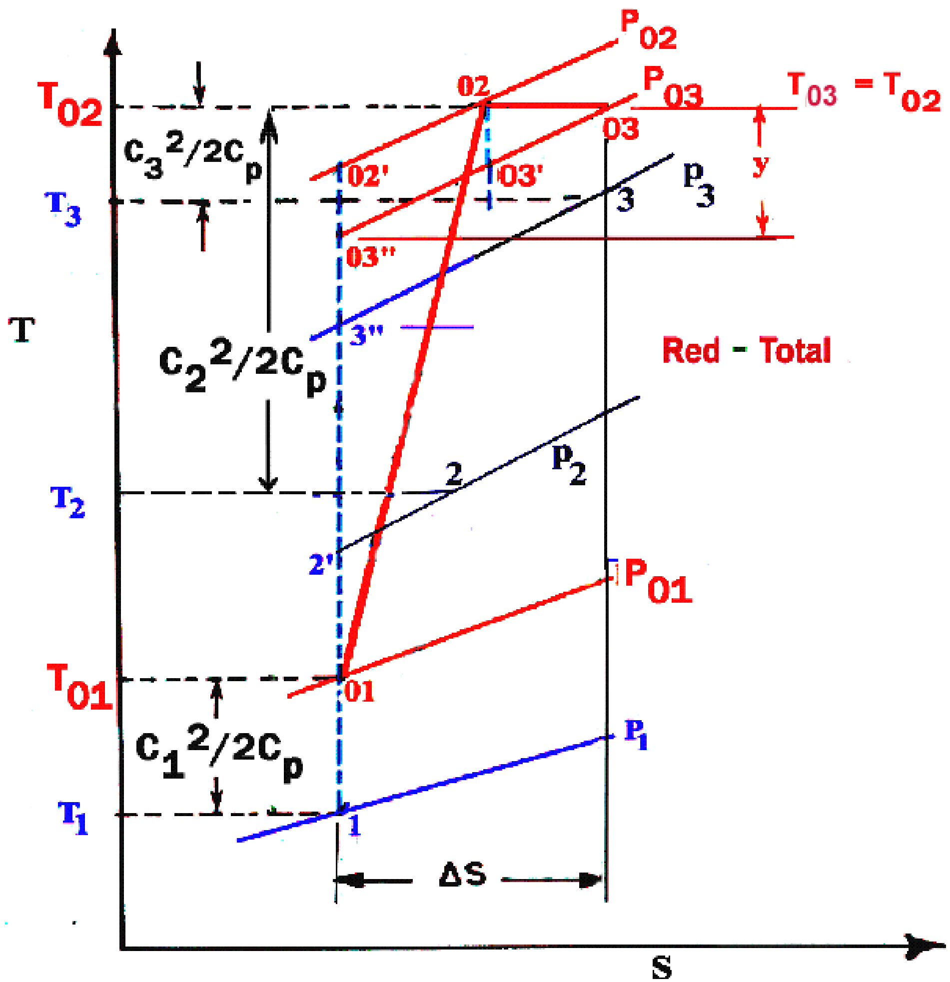
Thermodynamics of compressors



X_1 , X_2 are the losses in the rotor and the stator respectively

Compression in terms of static parameters

Thermodynamics of compressors

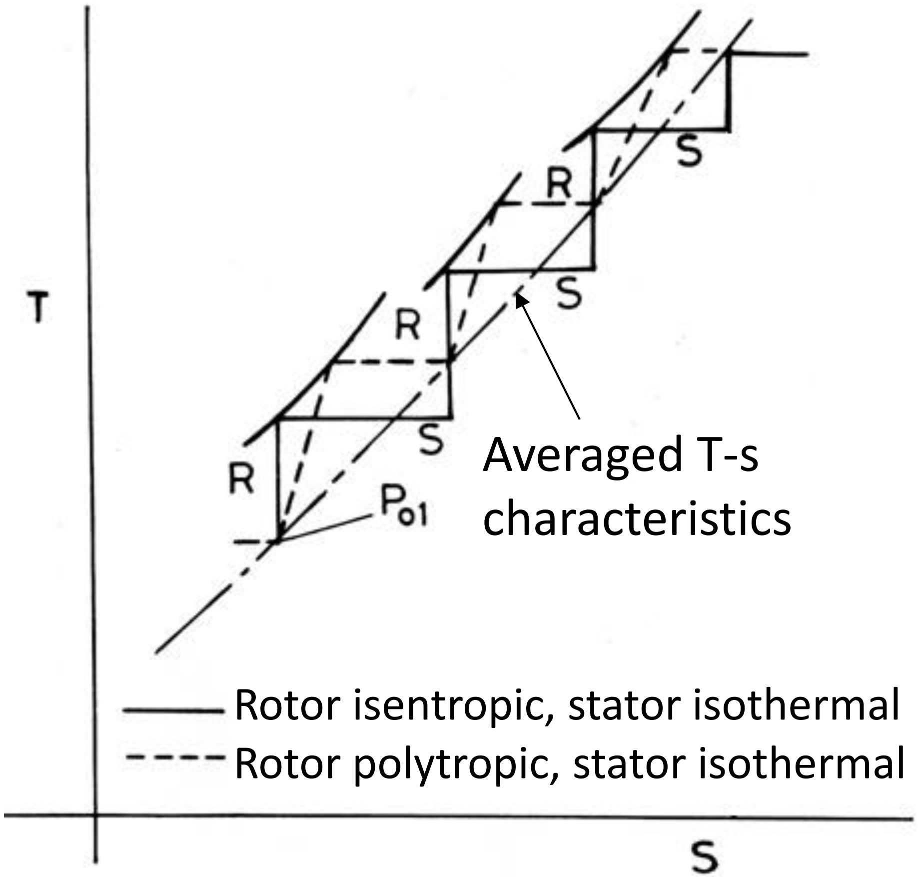


Compression in terms of total parameters

Basic operation of axial compressors

- Axial flow compressors usually consists of a series of stages.
- Each stage comprises of a row of rotor blades followed by a row of stator blades.
- The working fluid is initially accelerated by the rotor blades and then decelerated in the stator passages.
- In the stator, the kinetic energy transferred in the rotor is converted to static pressure.
- This process is repeated in several stages to yield the necessary overall pressure ratio.

Thermodynamics of multi-stage compressors



- The flow at the rotor exit with high kinetic energy is still to be converted to static pressure through diffusion.
- The exit kinetic energy of a compressor is of the same order as the entry kinetic energy and the entire work input is expected to be converted to pressure.