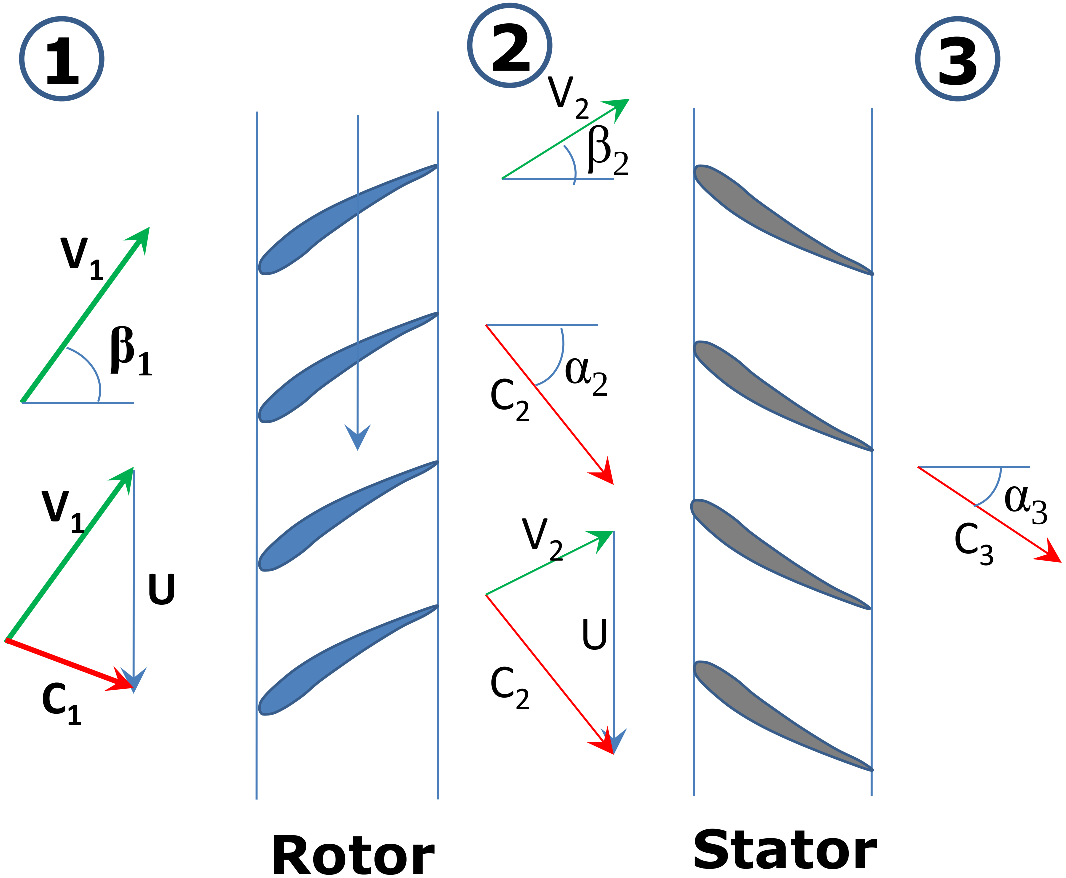
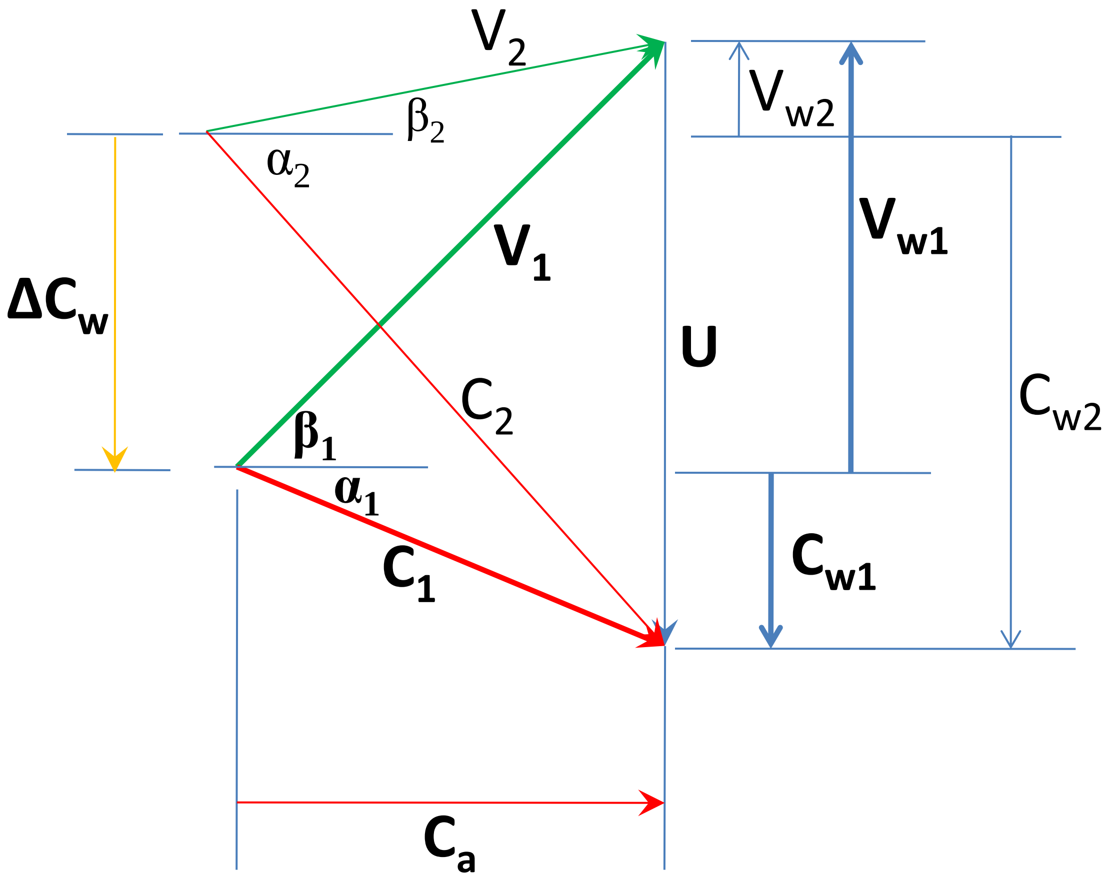
- Recap: Lecture 4: 1st July 2015, 1530-1655 hrs.
  - Thermodynamics of compression
    - Static and total pressure/temperature variations through a stage
  - Basic operation of axial compressors
  - Velocity triangles

Velocity triangles



$$\vec{C} = \vec{U} + \vec{V}$$

Velocity triangles



Work and compression

- Assuming $C_a = C_{a1} = C_{a2}$, from the velocity triangles, we can see that

$$\frac{U}{C_a} = \tan\alpha_1 + \tan\beta_1 \quad \text{and} \quad \frac{U}{C_a} = \tan\alpha_2 + \tan\beta_2$$

- By considering the change in angular momentum of the air passing through the rotor, work done per unit mass flow is

$w = U\left(C_{w2} - C_{w1}\right)$, where C_{w1} and C_{w2} are the tangential components of the fluid velocity before and after the rotor, respectively.

Work and compression

The above equation can also be written as,

$$w = UC_a(\tan\alpha_2 - \tan\alpha_1)$$

Since, $(\tan\alpha_2 - \tan\alpha_1) = (\tan\beta_1 - \tan\beta_2)$

$$\therefore\ w = UC_a(\tan\beta_1 - \tan\beta_2)$$

In other words, $w = U\Delta C_w$

- The input energy will reveal itself in the form of rise in stagnation temperature of the air.
- The work done as given above will also be equal to the change in stagnation enthalpy across the stage.

Work and compression

$$h_{02} - h_{01} = U\Delta C_w$$

$$T_{02} - T_{01} = \frac{U\Delta C_w}{c_p} \Rightarrow \frac{\Delta T_0}{T_{01}} = \frac{U\Delta C_w}{c_p T_{01}}$$

Since the flow is adiabatic and no work is done as the fluid passes through the stator, $T_{03} = T_{02}$

Let us define stage efficiency, η_{st}, as

$$\eta_{st} = \frac{h_{03s} - h_{01}}{h_{03} - h_{01}}$$

This can be expressed as

$$\frac{T_{03s}}{T_{01}} = 1 + \eta_{st} \frac{\Delta T_0}{T_{01}}$$

Work and compression

In the above equation, $\Delta T_0 = T_{03} - T_{01}$

In terms of pressure ratio,

$$\frac{P_{03}}{P_{01}} = \left[1 + \eta_{st}\,\frac{\Delta T_0}{T_{01}}\right]^{\gamma/(\gamma-1)}$$

This can be combined with the earlier equation to give,

$$\frac{P_{03}}{P_{01}} = \left[1 + \eta_{st}\,\frac{U\Delta C_w}{c_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

Work and compression

- From the above equation that relates the per stage temperature rise to the pressure ratio, it can be seen that to obtain a high temperature ratio for a given overall pressure ratio (for minimizing number of stages),
  - High blade speed: limited by blades stresses
  - High axial velocity, high fluid deflection $(\beta_1 - \beta_2)$: Aerodynamic considerations and adverse pressure gradients limit the above.

Design parameters

- The following design parameters are often used in the parametric analysis of axial compressors:
  - Flow coefficient,

    $$\phi = C_a / U$$

  - Stage loading or loading coefficient,

    $$\psi = \Delta h_0 / U^2 = \Delta C_w / U$$

  - Degree of reaction, R_x
  - Diffusion factor, D^*

Degree of reaction

- Diffusion takes place in both rotor and the stator.
- Static pressure rises in the rotor as well as the stator.
- Degree of reaction provides a measure of the extent to which the rotor contributes to the overall pressure rise in the stage.

Degree of reaction

$$R_x = \frac{\text{Static enthalpy rise in the rotor}}{\text{Stagnation enthalpy rise in the stage}}$$

$$= \frac{h_2 - h_1}{h_{03} - h_{01}} \approx \frac{h_2 - h_1}{h_{02} - h_{01}}$$

For a nearly incompressible, isentropic flow,

$$h_2 - h_1 \cong \frac{1}{\rho}\left(P_2 - P_1\right) \text{ for the rotor}$$

and for the stage, $h_{03} - h_{01} \cong \dfrac{1}{\rho}\left(P_{03} - P_{01}\right)$

$$\therefore R_x = \frac{h_2 - h_1}{h_{02} - h_{01}} \cong \frac{P_2 - P_1}{P_{02} - P_{01}}$$

Degree of reaction

From the steady flow energy equation,

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$\therefore R_x = \frac{h_2 - h_1}{h_{03} - h_{01}} = \frac{V_1^2 - V_2^2}{2U(C_{w2} - C_{w1})}$$

For constant axial velocity, $V_1^2 - V_2^2 = V_{w1}^2 - V_{w2}^2$

And, $V_{w1} - V_{w2} = C_{w1} - C_{w2}$

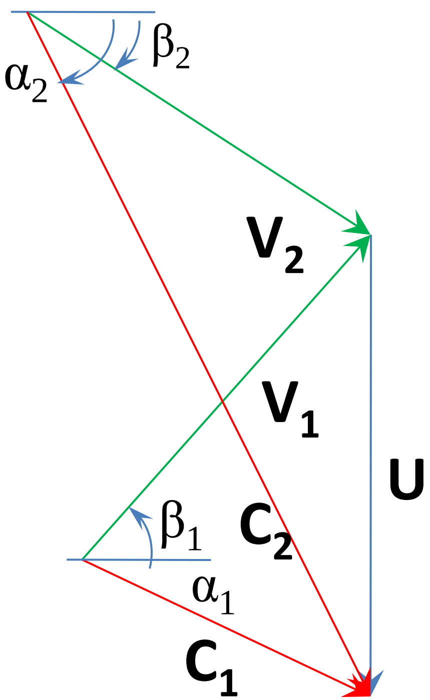
On simplification, $R_x = \frac{1}{2} - \frac{C_a}{2U}(\tan\alpha_1 - \tan\beta_2)$

or, $R_x = \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2)$

Degree of reaction

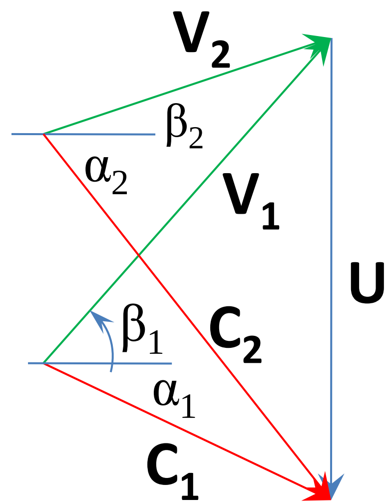
- Special cases of R_x
  - $R_x=0, \beta_2 = -\beta_1$, There is no pressure rise in the rotor, the entire pressure rise is due to the stator, the rotor merely deflects the incoming flow: impulse blading
  - $R_x=0.5$, gives $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$, the velocity triangles are symmetric, equal pressure rise in the rotor and the stator
  - $R_x=1.0, \alpha_2 = -\alpha_1$, entire pressure rise takes place in the rotor while the stator has no contribution.

Degree of reaction



$$\beta_2 = -\beta_1$$

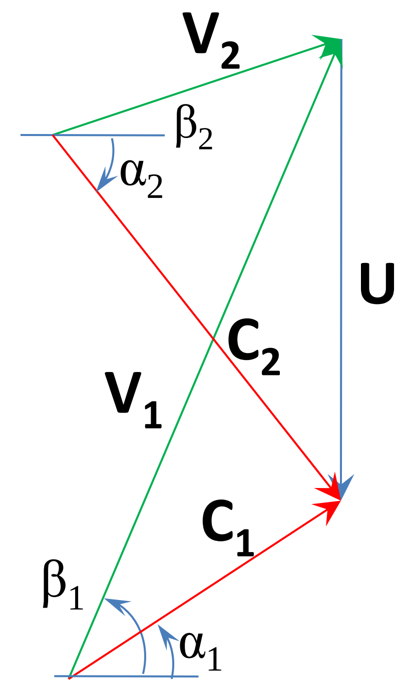
R_x=0.0



$$\alpha_1 = \beta_2 \text{ and } \alpha_2 = \beta_1$$

R_x=0.5

$$\alpha_2 = -\alpha_1$$



R_x=1.0