• Recap: Lecture 5: 4\textsuperscript{th} July 2015, 1530-1655 hrs.
  – Velocity triangles
  – Work and compression
  – Design parameters
    • Flow coefficient
    • Stage loading
    • Degree of reaction
Design parameters

- The following design parameters are often used in the parametric analysis of axial compressors:
  - Flow coefficient,
    \[ \phi = \frac{C_a}{U} \]
  - Stage loading or loading coefficient,
    \[ \psi = \frac{\Delta h_0}{U^2} = \frac{\Delta C_w}{U} \]
  - Degree of reaction, \( R_x \)
  - Diffusion factor, \( D^* \)
Degree of reaction

\[ R_x = \frac{\text{Static enthalpy rise in the rotor}}{\text{Stagnation enthalpy rise in the stage}} \]

\[ = \frac{h_2 - h_1}{h_{03} - h_{01}} \approx \frac{h_2 - h_1}{h_{02} - h_{01}} \]

For a nearly incompressible, isentropic flow,

\[ h_2 - h_1 \approx \frac{1}{\rho} (P_2 - P_1) \text{ for the rotor} \]

and for the stage, \[ h_{03} - h_{01} \approx \frac{1}{\rho} (P_{03} - P_{01}) \]

\[ \therefore R_x = \frac{h_2 - h_1}{h_{02} - h_{01}} \approx \frac{P_2 - P_1}{P_{02} - P_{01}} \]
Degree of reaction

From the steadyflow energy equation,

\[ h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \]

\[ \therefore R_x = \frac{h_2 - h_1}{h_{03} - h_{01}} = \frac{V_1^2 - V_2^2}{2U(C_{w2} - C_{w1})} \]

For constant axial velocity, \( V_1^2 - V_2^2 = V_{w1}^2 - V_{w2}^2 \)
And, \( V_{w1} - V_{w2} = C_{w1} - C_{w2} \)

On simplification, \( R_x = \frac{1}{2} - \frac{C_a}{2U} (\tan \alpha_1 - \tan \beta_2) \)

or, \( R_x = \frac{C_a}{2U} (\tan \beta_1 + \tan \beta_2) \)
Degree of reaction

• Special cases of $R_x$
  - $R_x=0, \beta_2 = -\beta_1$, There is no pressure rise in the rotor, the entire pressure rise is due to the stator, the rotor merely deflects the incoming flow: impulse blading
  - $R_x=0.5$, gives $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$, the velocity triangles are symmetric, equal pressure rise in the rotor and the stator
  - $R_x=1.0, \alpha_2 = -\alpha_1$, entire pressure rise takes place in the rotor while the stator has no contribution.
Degree of reaction

\[ \beta_2 = - \beta_1 \]

\[ \alpha_1 = \beta_2 \text{ and } \alpha_2 = \beta_1 \]

\[ R_x = 0.0 \]

\[ R_x = 0.5 \]

\[ R_x = 1.0 \]
Diffusion factor

• Fluid deflection ($\beta_2 - \beta_1$) is an important parameter that affects the stage pressure rise.
• Excessive deflection, which means high rate of diffusion, will lead to blade stall.
• Diffusion factor is a parameter that associates blade stall with deceleration on the suction surface of the airfoil section.
• Diffusion factor, $D^*$, is defined as

$$D^* = \frac{V_{\text{max}} - V_2}{V_1}$$

Where, $V_{\text{max}}$ is the ideal surface velocity at the minimum pressure point and $V_2$ is the ideal velocity at the trailing edge and $V_1$ is the velocity at the leading edge.
Diffusion factor

- **Velocity**
- **Percent chord**
- **Suction surface**
- **Pressure surface**

- $V_1$
- $V_{\text{max}}$
- $V_2$
Diffusion factor

• Lieblein (1953) proposed an empirical parameter for diffusion factor.
  – It is expressed entirely in terms of known or measured quantities.
  – It depends strongly upon solidity (C/s).
  – It has been proven to be a dependable indicator of approach to separation for a variety of blade shapes.
  – $D^*$ is usually kept around 0.5.

$$D^* = 1 - \frac{V_2}{V_1} + \frac{V_{w1} - V_{w2}}{2\left(\frac{C}{s}\right) V_1}$$

Where, $C$ is the chord of the blade and $s$ is the spacing between the blades.
Cascade aerodynamics

- A cascade is a stationary array of blades.
- Cascade is constructed for measurement of performance similar to that used in axial compressors.
- Cascade usually has porous end-walls to remove boundary layer for a two-dimensional flow.
- Radial variations in the velocity field can therefore be excluded.
- Cascade analysis relates the fluid turning angles to blading geometry and measure losses in the stagnation pressure.
Cascade aerodynamics

• The cascade is mounted on a turntable so that its angular direction relative to the inlet can be set at different incidence angles.

• Measurement usually consist of pressures, velocities and flow angles downstream of the cascade.

• Probe traverse at the trailing edge of the blades for measurement.

• Blade surface static pressure using static pressure taps: $c_p$ distribution.
Cascade wind tunnel

Linear open circuit cascade wind tunnel
Low-speed Cascade Tunnel at the Turbomachinery Research Lab, IITB
Cascade nomenclature

C = Chord
s = spacing/pitch
t = thickness
θ = camber
χ = stagger
i = incidence angle
δ = deflection angle

Performance parameters

- Measurements from cascade: velocities, pressures, flow angles …
- Loss in total pressure expressed as total pressure loss coefficient
  \[ \bar{\omega} = \frac{P_{01} - P_{02}}{\frac{1}{2} \rho V_1^2} \]
- Total pressure loss is very sensitive to changes in the incidence angle.
- At very high incidences, flow is likely to separate from the blade surfaces, eventually leading to stalling of the blade.
Performance parameters

• Blade performance/loading can be assessed using static pressure coefficient:

\[ C_P = \frac{P_{\text{local}} - P_{\text{ref}}}{\frac{1}{2} \rho V_1^2} \]

Where, \( P_{\text{local}} \) is the blade surface static pressure and \( P_{\text{ref}} \) is the reference static pressure (usually measured at the cascade inlet).

• The \( C_P \) distribution (usually plotted as \( C_P \) vs. \( x/C \)) gives an idea about the chordwise load distribution.
Performance parameters

- Deflection, degrees
- Total pressure loss coefficient
- Position along cascade
- Location of the blade trailing edge
Performance parameters

(a) Normal operation  (b) Stalled operation
Performance parameters

![Graph showing Total pressure loss coefficient vs. Incidence angle, degrees]
Losses in a compressor blade

• Nature of losses in an axial compressor
  – Viscous losses
  – 3-D effects like tip leakage flows, secondary flows etc.
  – Shock losses
  – Mixing losses

• Estimating the losses crucial designing loss control mechanisms.

• However isolating these losses not easy and often done through empirical correlations.

• Total losses in a compressor is the sum of the above losses.
Losses in a compressor blade

• Viscous losses
  – Profile losses: on account of the profile or nature of the airfoil cross-sections
  – Annulus losses: growth of boundary layer along the axis
  – Endwall losses: boundary layer effects in the corner (junction between the blade surface and the casing/hub)

• 3-D effects:
  – Secondary flows: flow through curved blade passages
  – Tip leakage flows: flow from pressure surface to suction surface at the blade tip
Losses in a compressor blade

- The loss manifests itself in the form of stagnation pressure loss (or entropy increase).

\[
\frac{\Delta s}{R} = -\ln \frac{P_{02}}{P_{01}} = -\ln \left[1 - \frac{(\Delta P_o)_{\text{loss}}}{P_{01}}\right]
\]

Expanding the above equation in an infinite series,

\[
\frac{\Delta s}{R} = \frac{(\Delta P_o)_{\text{loss}}}{P_{01}} + \frac{1}{2} \left(\frac{(\Delta P_o)_{\text{loss}}}{P_{01}}\right)^2 + \ldots
\]

Neglecting higher order terms, \( \frac{\Delta s}{R} = \frac{(\Delta P_o)_{\text{loss}}}{P_{01}} \)

Since, \( \omega = \frac{(\Delta P_o)_{\text{loss}}}{\frac{1}{2} \rho V_1^2} = \frac{\Delta s}{R} \frac{P_{01}}{\frac{1}{2} \rho V_1^2} \)

or, \( \frac{\Delta s}{R} = \left(\frac{\omega \rho V_1^2}{2P_{01}}\right) \)
Losses in a compressor blade

- The overall losses in a turbomachinery can be summarised as:

\[ \omega = \omega_P + \omega_m + \omega_{sh} + \omega_s + \omega_L + \omega_E + \ldots \]

Where, \( \omega_P \) : profile losses
\( \omega_m \) : mixing losses
\( \omega_{sh} \) : shock losses
\( \omega_s \) : secondary flow loss
\( \omega_L \) : tip leakage loss
\( \omega_E \) : Endwall losses