- Recap: Lecture 5: 4th July 2015, 1530-1655 hrs.
  - Velocity triangles
  - Work and compression
  - Design parameters
    - Flow coefficient
    - Stage loading
    - Degree of reaction

Design parameters

- The following design parameters are often used in the parametric analysis of axial compressors:
  - Flow coefficient,

    $$\phi = C_a / U$$

  - Stage loading or loading coefficient,

    $$\psi = \Delta h_0 / U^2 = \Delta C_w / U$$

  - Degree of reaction, R_x
  - Diffusion factor, D^*

Degree of reaction

$$R_x = \frac{\text{Static enthalpy rise in the rotor}}{\text{Stagnation enthalpy rise in the stage}}$$

$$= \frac{h_2 - h_1}{h_{03} - h_{01}} \approx \frac{h_2 - h_1}{h_{02} - h_{01}}$$

For a nearly incompressible, isentropic flow,

$$h_2 - h_1 \cong \frac{1}{\rho}\left(P_2 - P_1\right) \text{ for the rotor}$$

$$\text{and for the stage, } h_{03} - h_{01} \cong \frac{1}{\rho}\left(P_{03} - P_{01}\right)$$

$$\therefore R_x = \frac{h_2 - h_1}{h_{02} - h_{01}} \cong \frac{P_2 - P_1}{P_{02} - P_{01}}$$

Degree of reaction

From the steady flow energy equation,

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$\therefore R_x = \frac{h_2 - h_1}{h_{03} - h_{01}} = \frac{V_1^2 - V_2^2}{2U(C_{w2} - C_{w1})}$$

For constant axial velocity, $V_1^2 - V_2^2 = V_{w1}^2 - V_{w2}^2$

And, $V_{w1} - V_{w2} = C_{w1} - C_{w2}$

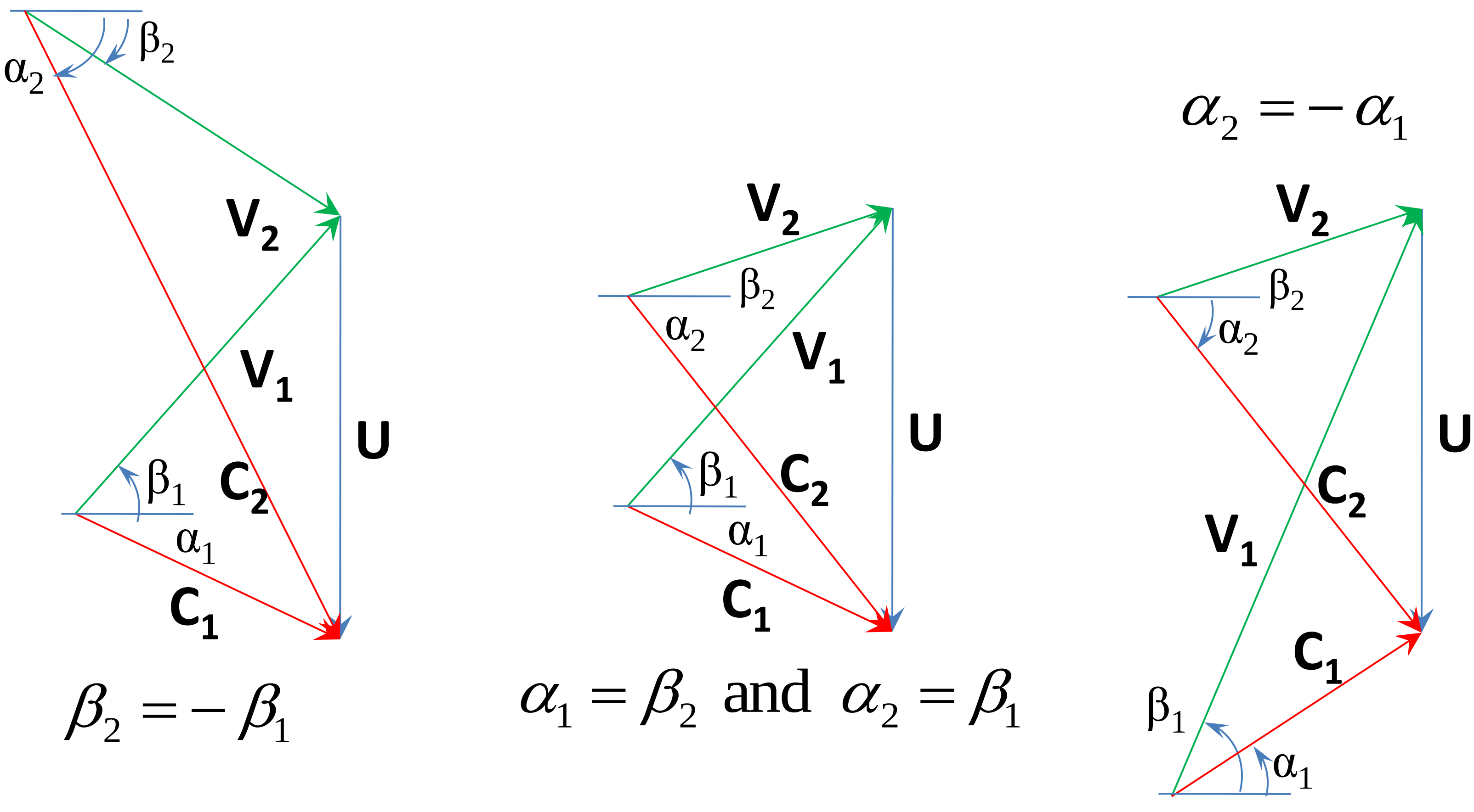
On simplification, $R_x = \frac{1}{2} - \frac{C_a}{2U}(\tan\alpha_1 - \tan\beta_2)$

or, $R_x = \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2)$

Degree of reaction

- Special cases of R_x
  - $R_x = 0, \beta_2 = -\beta_1$, There is no pressure rise in the rotor, the entire pressure rise is due to the stator, the rotor merely deflects the incoming flow: impulse blading
  - $R_x = 0.5$, gives $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$, the velocity triangles are symmetric, equal pressure rise in the rotor and the stator
  - $R_x = 1.0, \alpha_2 = -\alpha_1$, entire pressure rise takes place in the rotor while the stator has no contribution.

Degree of reaction



$$\beta_2 = -\beta_1$$

$$\alpha_1 = \beta_2 \text{ and } \alpha_2 = \beta_1$$

R_x=0.0 R_x=0.5 R_x=1.0

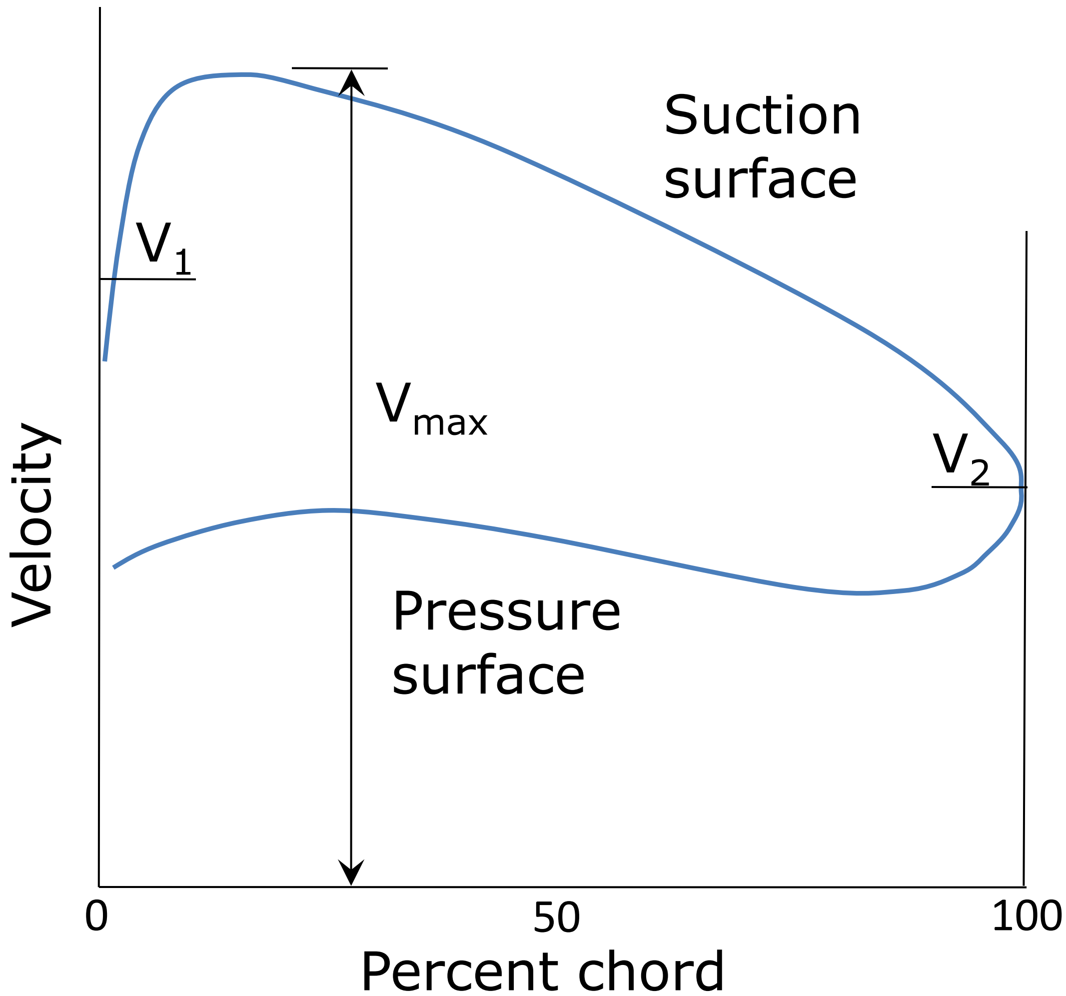
Diffusion factor

- Fluid deflection (β_2-β_1)is an important parameter that affects the stage pressure rise.
- Excessive deflection, which means high rate of diffusion, will lead to blade stall.
- Diffusion factor is a parameter that associates blade stall with deceleration on the suction surface of the airfoil section.
- Diffusion factor, D*, is defined as

$$D^* = \frac{V_{max} - V_2}{V_1}$$

Where, V_{max} is the ideal surface velocity at the minimum pressure point and V_2 is the ideal velocity at the trailing edge and V_1 is the velocity at the leading edge.

Diffusion factor



Diffusion factor

- Lieblein (1953) proposed an empirical parameter for diffusion factor.
  - It is expressed entirely in terms of known or measured quantities.
  - It depends strongly upon solidity (C/s).
  - It has been proven to be a dependable indicator of approach to separation for a variety of blade shapes.
  - D^* is usually kept around 0.5.

$$D^* = 1 - \frac{V_2}{V_1} + \frac{V_{w1} - V_{w2}}{2\left(\frac{C}{s}\right)V_1}$$

Where, C is the chord of the blade and s is the spacing between the blades.

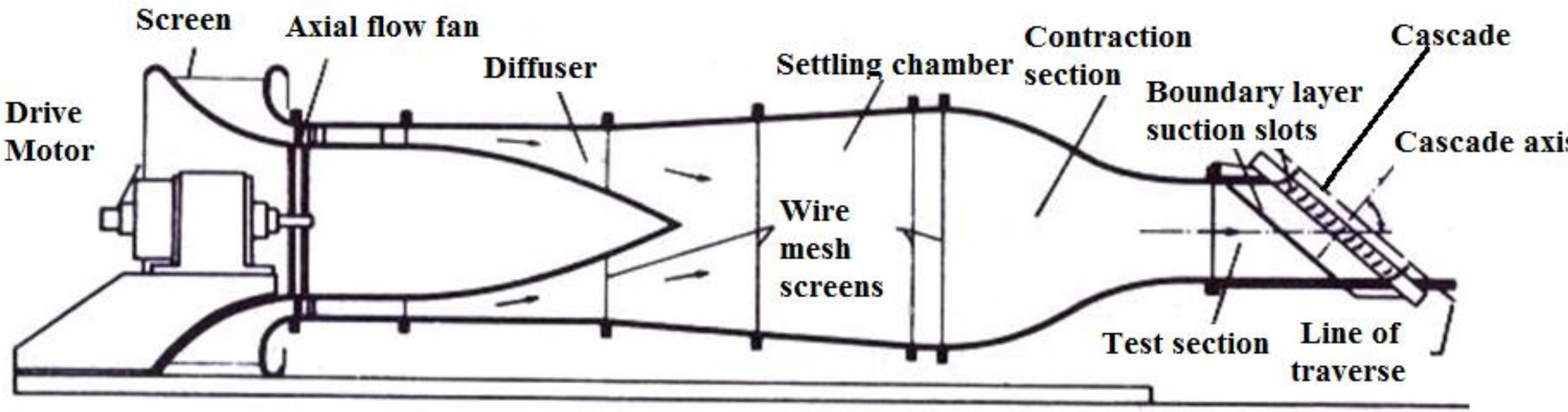
Cascade aerodynamics

- A cascade is a stationary array of blades.
- Cascade is constructed for measurement of performance similar to that used in axial compressors.
- Cascade usually has porous end-walls to remove boundary layer for a two-dimensional flow.
- Radial variations in the velocity field can therefore be excluded.
- Cascade analysis relates the fluid turning angles to blading geometry and measure losses in the stagnation pressure.

Cascade aerodynamics

- The cascade is mounted on a turntable so that its angular direction relative to the inlet can be set at different incidence angles.
- Measurement usually consist of pressures, velocities and flow angles downstream of the cascade.
- Probe traverse at the trailing edge of the blades for measurement.
- Blade surface static pressure using static pressure taps: c_p distribution.

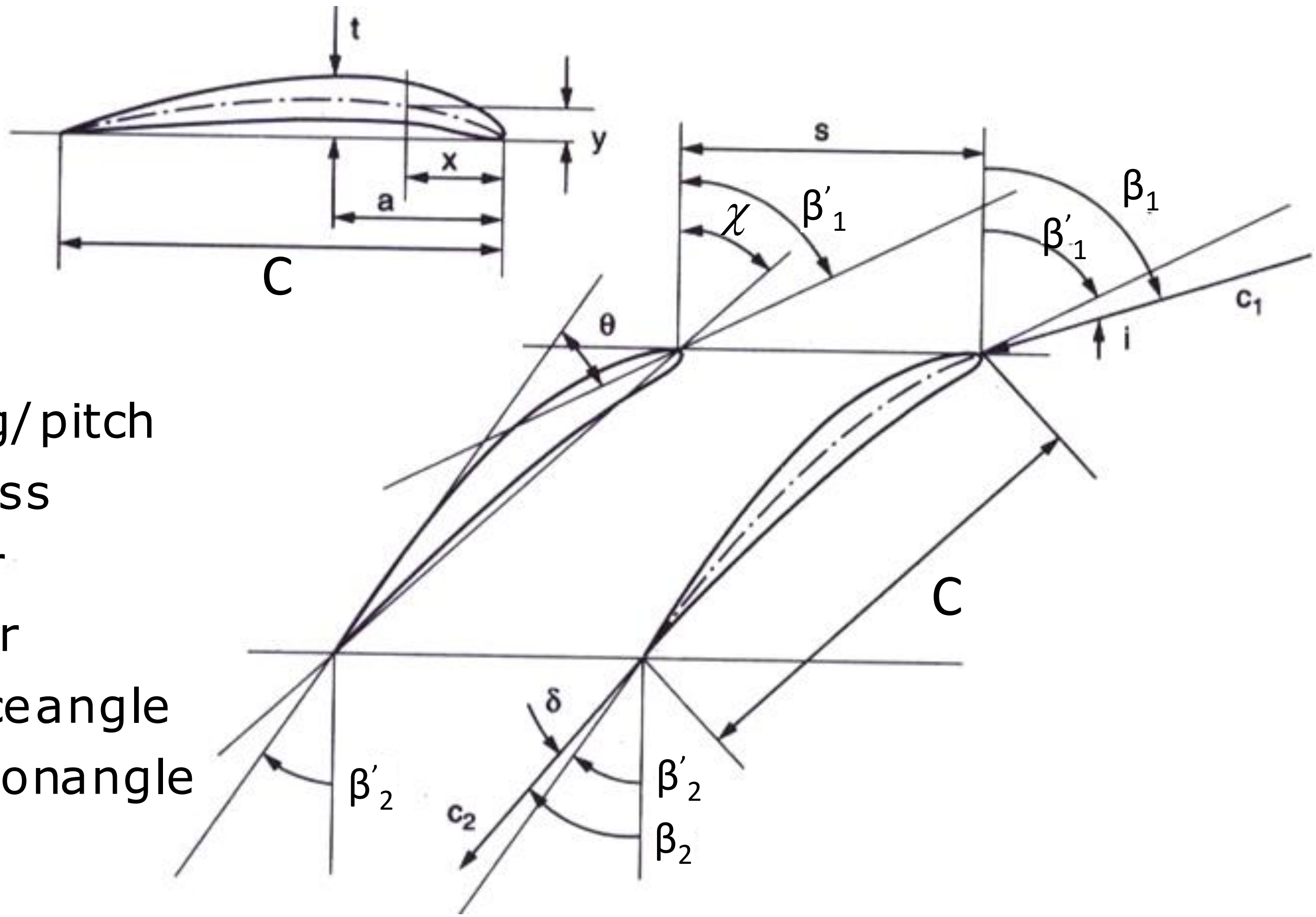
Cascade wind tunnel



Linear open circuit cascade wind tunnel

Low-speed Cascade Tunnel at the
Turbomachinery Research Lab, IITB

Cascade nomenclature



C = Chord

s = spacing/ pitch

t = thickness

θ = camber

χ = stagger

i = incidenceangle

δ = deflectionangle

Performance parameters

- Measurements from cascade: velocities, pressures, flow angles ...
- Loss in total pressure expressed as total pressure loss coefficient

$$\varpi = \frac{P_{01} - P_{02}}{\frac{1}{2}\rho V_1^{\,2}}$$

- Total pressure loss is very sensitive to changes in the incidence angle.
- At very high incidences, flow is likely to separate from the blade surfaces, eventually leading to stalling of the blade.

Performance parameters

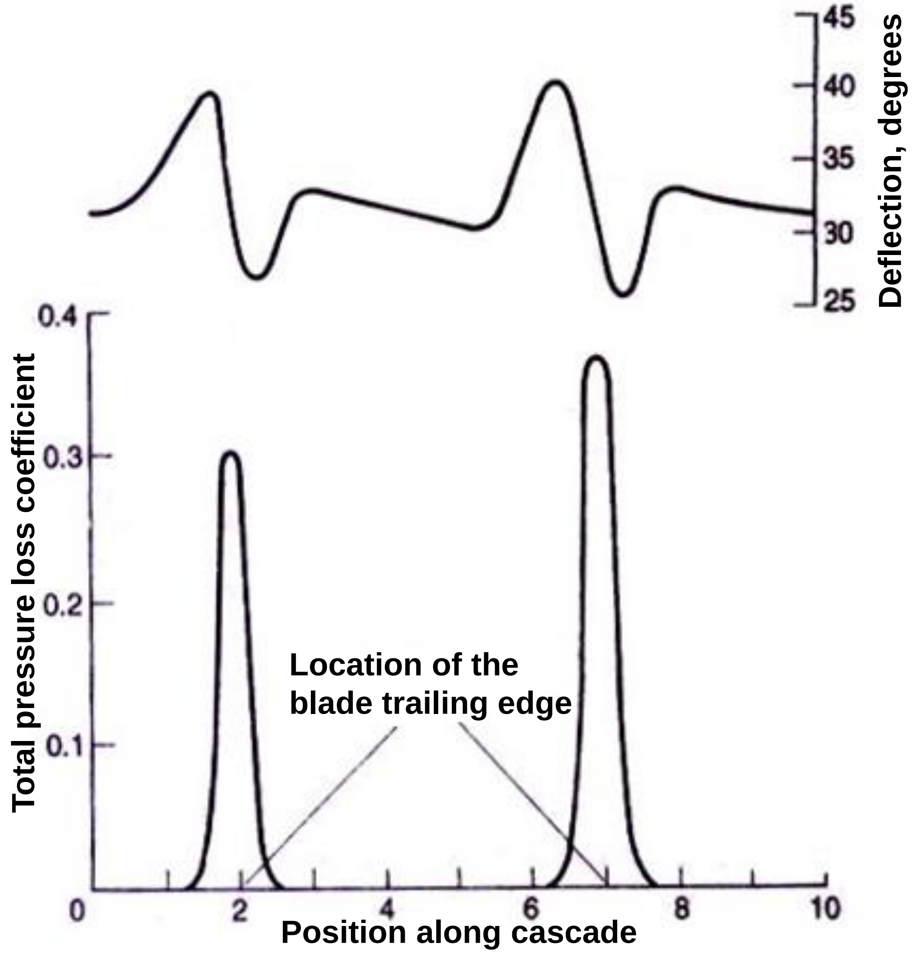
- Blade performance/loading can be assessed using static pressure coefficient:

$$C_P = \frac{P_{local} - P_{ref}}{\frac{1}{2}\rho V_1^2}$$

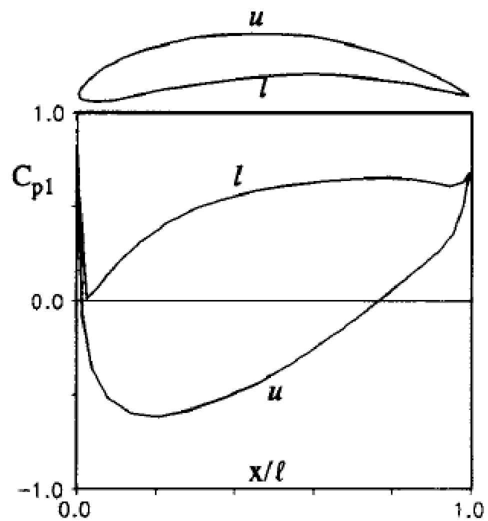
Where, P_{local} is the blade surface static pressure and P_{ref} is the reference static pressure (usually measured at the cascade inlet)

- The C_P distribution (usually plotted as C_P *vs. x/C*) gives an idea about the chordwise load distribution.

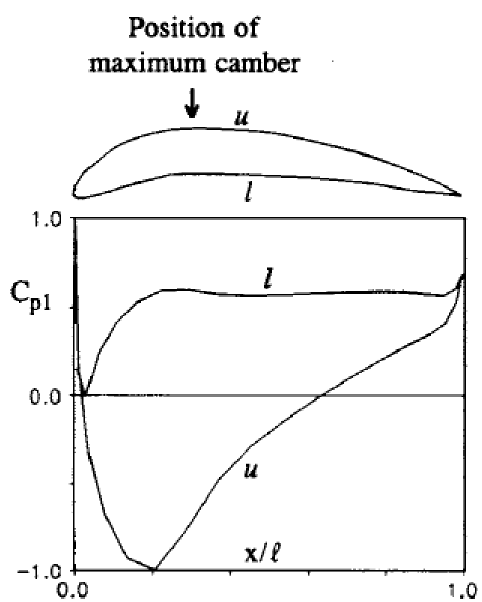
Performance parameters



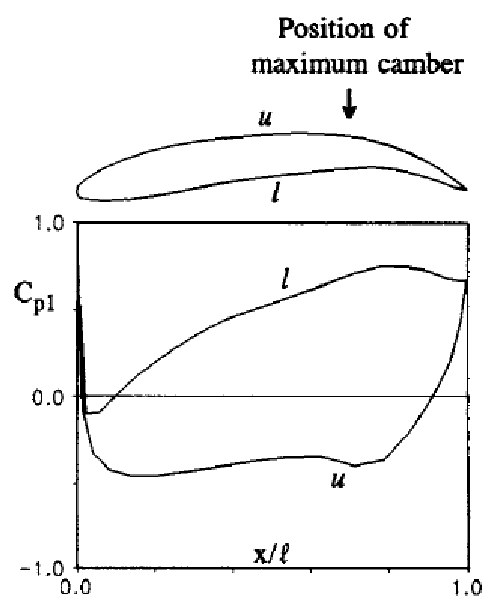
Performance parameters



(a) Circular arc camber

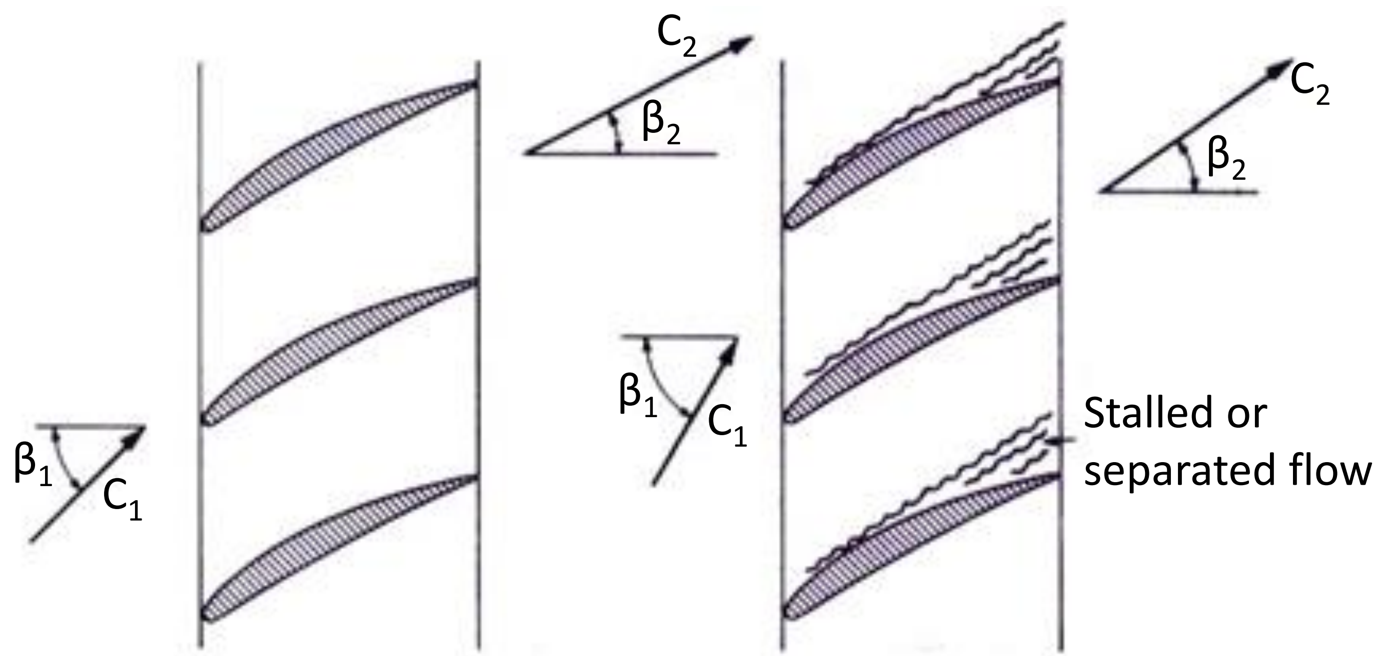


(b) Parabolic camber



(c) Parabolic camber

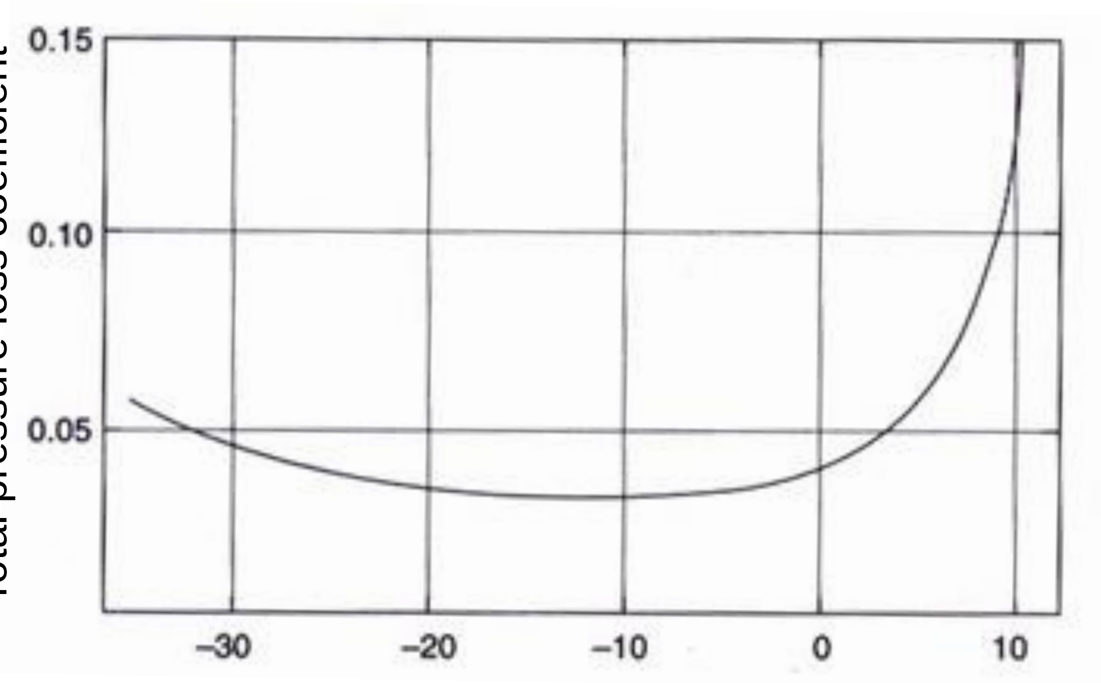
Performance parameters



(a) Normal operation (b) Stalled operation

Performance parameters

Total pressure loss coefficient



Incidence angle, degrees

Losses in a compressor blade

- Nature of losses in an axial compressor
  - Viscous losses
  - 3-D effects like tip leakage flows, secondary flows etc.
  - Shock losses
  - Mixing losses
- Estimating the losses crucial designing loss control mechanisms.
- However isolating these losses not easy and often done through empirical correlations.
- Total losses in a compressor is the sum of the above losses.

Losses in a compressor blade

- Viscous losses
  - Profile losses: on account of the profile or nature of the airfoil cross-sections
  - Annulus losses: growth of boundary layer along the axis
  - Endwall losses: boundary layer effects in the corner (junction between the blade surface and the casing/hub)
- 3-D effects:
  - Secondary flows: flow through curved blade passages
  - Tip leakage flows: flow from pressure surface to suction surface at the blade tip

Losses in a compressor blade

- The loss manifests itself in the form of stagnation pressure loss (or entropy increase).

$$\frac{\Delta s}{R} = -\ln\frac{P_{02}}{P_{01}} = -\ln\left[1 - \frac{(\Delta P_o)_{loss}}{P_{01}}\right]$$

Expandingthe aboveequationin aninfinite series,

$$\frac{\Delta s}{R} = \frac{(\Delta P_o)_{loss}}{P_{01}} + \frac{1}{2}\left(\frac{(\Delta P_o)_{loss}}{P_{01}}\right)^2 + \ldots$$

Neglectinghigher orderterms, $\frac{\Delta s}{R} = \frac{(\Delta P_o)_{loss}}{P_{01}}$

Since, $\omega = \frac{(\Delta P_o)_{loss}}{\frac{1}{2}\rho V_1^2} = \frac{\Delta s}{R}\frac{P_{01}}{\frac{1}{2}\rho V_1^2}$

or, $\frac{\Delta s}{R} = \left(\frac{\omega\rho V_1^2}{2P_{01}}\right)$

Losses in a compressor blade

- The overall losses in a turbomachinery can be summarised as:

$$\omega = \omega_P + \omega_m + \omega_{sh} + \omega_s + \omega_L + \omega_E + \ldots$$

Where, ω_P : profile losses

ω_m : mixing losses

ω_{sh} : shock losses

ω_s : secondary flow loss

ω_L : tip leakage loss

ω_E : Endwall losses