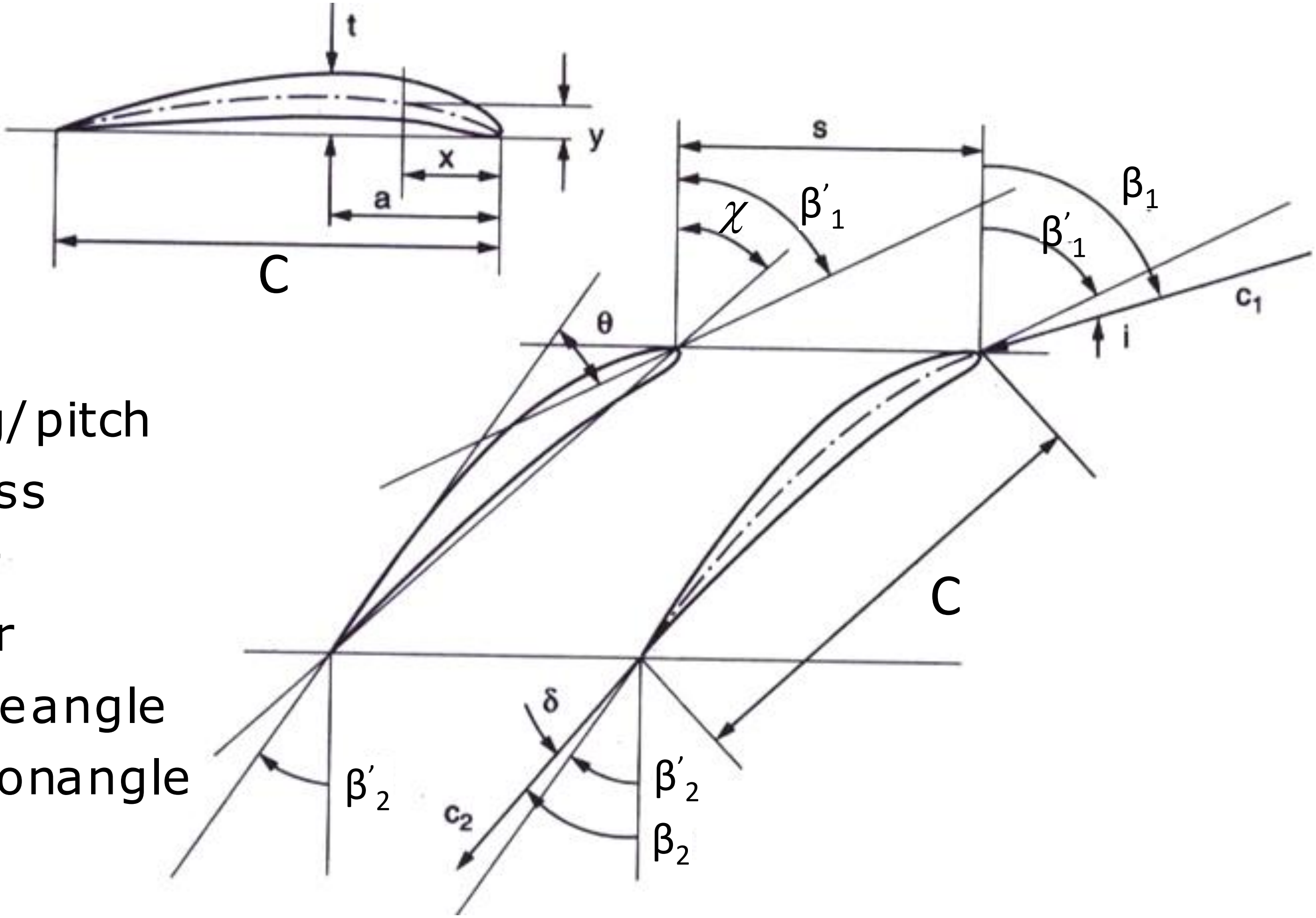
- Recap: Lecture 6: 7th July 2015, 1530-1655 hrs.
  - Design parameters
    - Degree of reaction
    - Diffusion factor
  - Cascade aerodynamics
    - Cascade tunnel
    - Need for cascade tests
    - Cascade nomenclature
    - Basic data from cascade tests: total pressure loss, blade static pressure distribution

- Note: Tutorial # 1: 2D analysis of axial compressors

Friday, 14th August 2015: 1530-1615 hrs.

Cascade nomenclature



$C =$ Chord

$s =$ spacing/ pitch

$t =$ thickness

$\theta =$ camber

$\chi =$ stagger

$i =$ incidenceangle

$\delta =$ deflectionangle

Losses in a compressor blade

- Nature of losses in an axial compressor
  - Viscous losses
  - 3-D effects like tip leakage flows, secondary flows etc.
  - Shock losses
  - Mixing losses
- Estimating the losses crucial designing loss control mechanisms.
- However isolating these losses not easy and often done through empirical correlations.
- Total losses in a compressor is the sum of the above losses.

Losses in a compressor blade

- The overall losses in a turbomachinery can be summarised as:

$$\omega = \omega_P + \omega_m + \omega_{sh} + \omega_s + \omega_L + \omega_E + \dots$$

Where, ω_P : profile losses

ω_m : mixing losses

ω_{sh} : shock losses

ω_s : secondary flow loss

ω_L : tip leakage loss

ω_E : Endwall losses

2-D Losses in a compressor blade

- 2-D losses are relevant only to axial flow turbomachines.
- These are mainly associated with blade boundary layers, shock-boundary layer interactions, separated flows and wakes.
- The mixing of the wake downstream produces additional losses called mixing losses.
- The maximum losses occur near the blade surface and minimum loss occurs near the edge of the boundary layer.

2-D Losses in a compressor blade

- 2-D losses can be classified as:
  - Profile loss due to boundary layer, including laminar and/or turbulent separation.
  - Wake mixing losses
  - Shock losses
  - Trailing edge loss due to the blade.

2-D Losses in a compressor blade

- The profile loss depends upon:
  - Flow parameters like Reynolds number, Mach number, longitudinal curvature of the blade, inlet turbulence, free-stream unsteadiness and the resulting unsteady boundary layers, pressure gradient, and shock strength
  - Blade parameters like: thickness, camber, solidity, sweep, skewness of the blade, stagger angle and blade roughness.

2-D Losses in a compressor blade

- The mixing losses arise as a result of the mixing of the wake with the freestream.
- This depends upon, in addition to the parameters mentioned in the previous slide, the distance downstream.
- The physical mechanism is the exchange of momentum and energy between the wake and the freestream.
- This transfer of energy results in the decay of the free shear layer, increased wake centre line velocity and increased wake width.

2-D Losses in a compressor blade

- At far downstream, the flow becomes uniform.
- Theoretically, the difference between the stagnation pressure far downstream and the trailing edge represents the mixing loss.
- Most loss correlations are based on measurements downstream of the trailing edge (1/2 to 1 chord length) and therefore do not include all the mixing losses.
- If there is flow separation, the losses would include losses due to this zone and at its eventual mixing downstream.

2-D Losses in a compressor blade

The profile and mixing losses along a streamline can be written as:

$$\overline{\omega}_{p+m} = \frac{2(P_{0t} - P_{02})}{\rho V_1^2}$$

To determine the above, it is necessary to relate the static pressure difference and velocities to the displacement and momentum thickness of the blade boundary layer at the trailing edge.

2-D Losses in a compressor blade

Detailed derivation of these correlations are given in Lakshminarayana's book (Chapter 6).

$$\overline{\omega}_{p+m} = \frac{2(P_{0t} - P_{02})}{\rho V_1^2} = \frac{2(p_t - p_2)}{\rho V_1^2} + \frac{V_t^2 - V_2^2}{V_1^2}$$

This is further expressed as:

$$\overline{\omega}_{p+m} \sec^2 \alpha_1 = \left[\frac{2\Theta + \Delta^2}{(1-\Delta)^2} + \tan^2 \alpha_2 \left\{ \frac{(1-\Delta)^2}{(1-\Theta-\Delta)^2} - 1 \right\} \right]$$

Neglecting higher order terms,

$$\overline{\omega}_{p+m} \sec^2 \alpha_1 = 2(\Theta + \Theta \tan^2 \alpha_2)$$

Where, Δ is the blockage (related to displacement thickness) and Θ is the momentum thicknesss.

2-D Losses in a compressor blade

- Thus, in a simplified manner, we see that the profile loss can be estimated based on the momentum thickness.
- The above loss correlation includes both profile and wake mixing loss.
- If flow separation occurs, additional losses are incurred. This is because the pressure distribution is drastically altered beyond the separation point.
- The losses increase due to increase in boundary layer displacement and momentum thicknesses.

2-D Losses in a compressor blade

- In addition to the losses discussed above, boundary layer growth and subsequent decay of the wake causes deviation in the outlet angle.
- An estimate of this is given as:

$$\tan\alpha_2 \approx (1 - \Theta - \Delta)\tan\alpha_t$$

- Hence, viscous effect in a turbomachine always leads to decrease in the turning angle.
- The values of displacement and momentum thicknesses, depend upon, variation of freestream velocity, Mach number, skin friction, pressure gradient, turbulence intensity and Reynolds number.

2-D Losses in a compressor blade

- In general, the loss estimation may be carried out using one of the following methods:
  - Separate calculation of the potential or inviscid flow and the displacement and momentum thicknesses. Subsequently, use the equation discussed previously.
  - Using a Navier-Stokes based computational code. Here the local and the integrated losses can be computed directly.

Mach number and shock losses

- The static pressure rise in a compressor increases with Mach number.
- Thus the pressure gradient increases with increase in Mach number.
- This means that the momentum thickness and hence the losses increase with Mach number.
- Increasing Mach numbers also lead to increase in shock losses.
- At transonic speeds, the shock losses are very sensitive to leading and trailing edge geometries.

Mach number and shock losses

- An estimate of the 2-D shock losses for a compressor must include:
    - The losses due to the leading edge bluntness with supersonic upstream Mach number.
    - The location of the passage shock can be determined from inviscid theories. If the shock strength is known, the losses can be estimated.
    - The losses due to boundary layer growth and the shock-boundary layer interaction are most difficult to estimate. The contribution however is small for weak shocks.

Mach number and shock losses

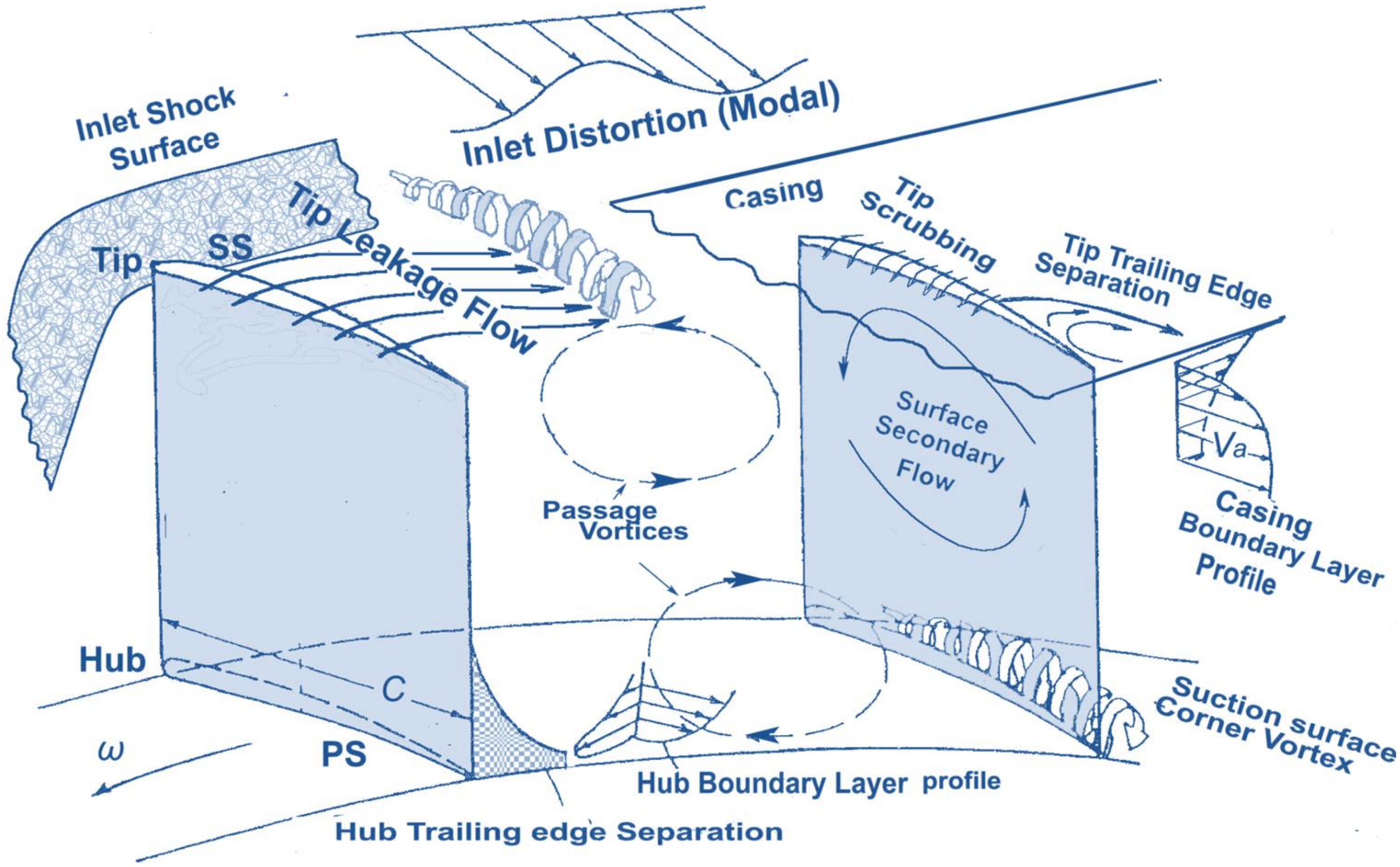
- One of the empirical correlations for the shock loss was given by Freeman and Cumpsty (1989).

$$\omega_{sh} = \frac{(\Delta P_0)_{loss}}{P_{01} - p_1} = \left[\frac{(\Delta P_0)_{loss}}{P_{01} - p_1}\right]_{normal\ shock} + \left[2.6 + 0.18(\alpha_1' - 65^0)\right]10^{-2}(\alpha_1 - \alpha_1')$$

where, α_1' is the blade inlet angle.

- This is valid for an incidence angle upto 5°.
- These empirical correlations are however, derived using the 2-D assumption.
- Actual flows are seldom 2-D in nature.

3-D flow in axial compressors



3-D flow in axial compressors

- Flow in axial compressors considered so far was 2-D: no radial component of velocity.
- Three dimensionality is caused by inviscid and viscous effects.
- Some of the inviscid effects are due to
  - Compressibility and radial pressure gradients
  - Radial variation in blade geometry
  - Tip leakage flow
  - Presence of shock
  - Secondary flows

3-D flow in axial compressors

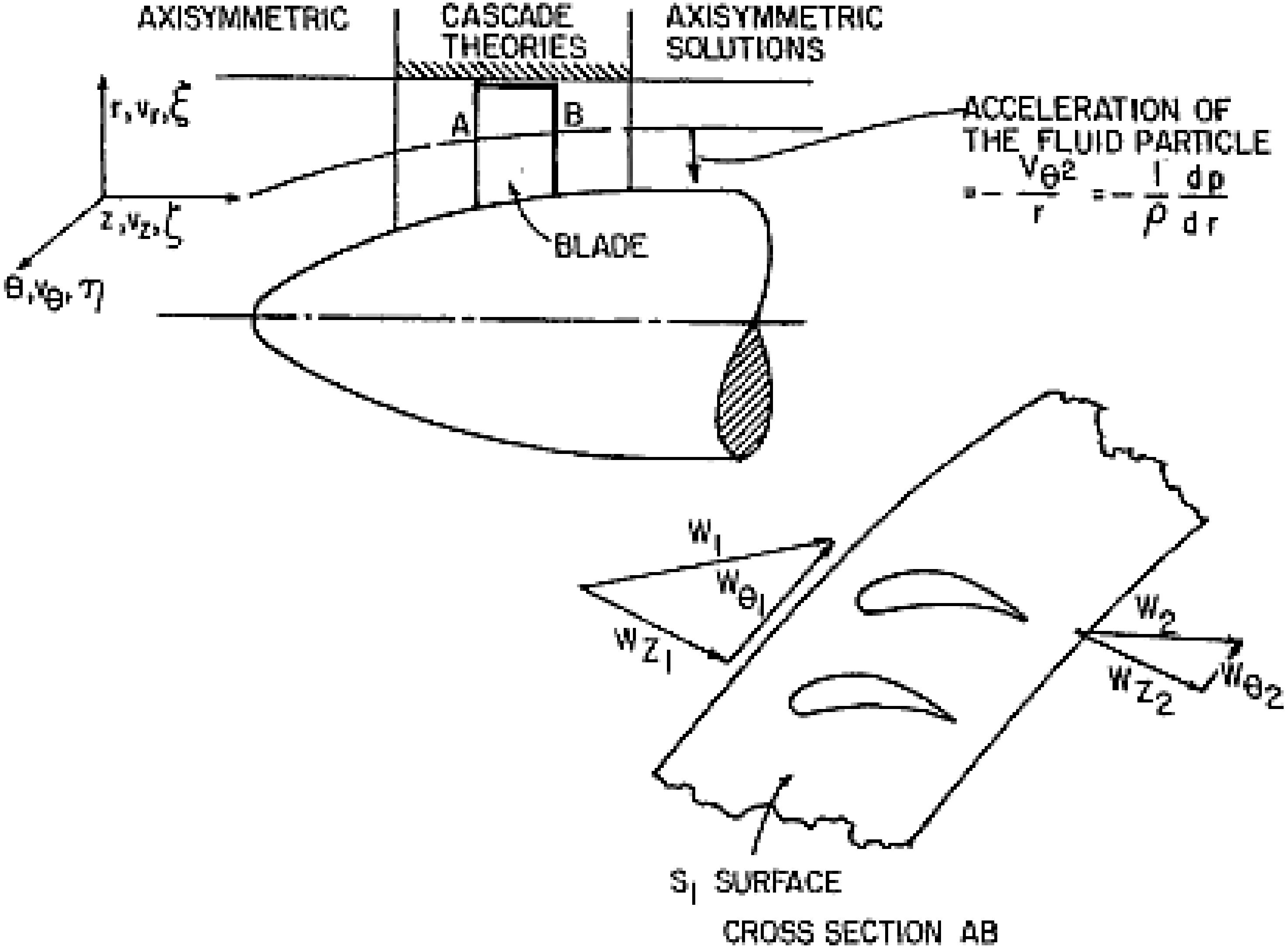
- These inviscid effects can be analysed using the inviscid governing equations.
- The most dominant effect is the radial variation in velocity.
- The viscous and inviscid effects compliment each other.
- For eg. Tip leakage flow is essentially an inviscid effect, but its propagation and formation of leakage vortex is controlled by viscous effects.

3-D flow in axial compressors

- The equations of motion for 3-D analysis of flow through turbomachines are highly non-linear.
- Analytical solutions exist for simple flow fields.
- Depending upon the analysis, one may take up an axisymmetric analysis or a non-axisymmetric analysis.
- Axisymmetric analysis
  - Simple radial equilibrium analysis
  - Actuator disc theories
  - Passage averaged equations

3-D flow in axial compressors

- Non-axisymmetric analysis
  - Lifting line and lifting surface approach
  - Quasi-3-D approach
  - Numerical solution of exact equations (Euler or Navier-Stokes)
- Axisymmetric analysis is used to predict the radial or spanwise variation of properties far downstream of the blade.
- In the blade passage, cascade theories can be used to determine variation in properties at a given spanwise location.



From Lakshminarayana, Chap 4, P 264

Axisymmetric analysis