  – Design parameters
    • Degree of reaction
    • Diffusion factor
  – Cascade aerodynamics
    • Cascade tunnel
    • Need for cascade tests
    • Cascade nomenclature
    • Basic data from cascade tests: total pressure loss, blade static pressure distribution

• Note: Tutorial # 1: 2D analysis of axial compressors
  Friday, 14th August 2015: 1530-1615 hrs.
Cascade nomenclature

C = Chord
s = spacing/pitch
t = thickness
θ = camber
χ = stagger
i = incidence angle
δ = deflection angle
Losses in a compressor blade

• Nature of losses in an axial compressor
  – Viscous losses
  – 3-D effects like tip leakage flows, secondary flows etc.
  – Shock losses
  – Mixing losses

• Estimating the losses crucial designing loss control mechanisms.

• However isolating these losses not easy and often done through empirical correlations.

• Total losses in a compressor is the sum of the above losses.
Losses in a compressor blade

• The overall losses in a turbomachinery can be summarised as:

\[ \omega = \omega_P + \omega_m + \omega_{sh} + \omega_s + \omega_L + \omega_E + \ldots \]

Where, \( \omega_P \) : profile losses
\( \omega_m \) : mixing losses
\( \omega_{sh} \) : shock losses
\( \omega_s \) : secondary flow loss
\( \omega_L \) : tip leakage loss
\( \omega_E \) : Endwall losses
2-D Losses in a compressor blade

- 2-D losses are relevant only to axial flow turbomachines.
- These are mainly associated with blade boundary layers, shock-boundary layer interactions, separated flows and wakes.
- The mixing of the wake downstream produces additional losses called mixing losses.
- The maximum losses occur near the blade surface and minimum loss occurs near the edge of the boundary layer.
2-D Losses in a compressor blade

• 2-D losses can be classified as:
  • Profile loss due to boundary layer, including laminar and/or turbulent separation.
  • Wake mixing losses
  • Shock losses
  • Trailing edge loss due to the blade.
2-D Losses in a compressor blade

• The profile loss depends upon:
  • Flow parameters like Reynolds number, Mach number, longitudinal curvature of the blade, inlet turbulence, free-stream unsteadiness and the resulting unsteady boundary layers, pressure gradient, and shock strength
  • Blade parameters like: thickness, camber, solidity, sweep, skewness of the blade, stagger angle and blade roughness.
2-D Losses in a compressor blade

- The mixing losses arise as a result of the mixing of the wake with the freestream.
- This depends upon, in addition to the parameters mentioned in the previous slide, the distance downstream.
- The physical mechanism is the exchange of momentum and energy between the wake and the freestream.
- This transfer of energy results in the decay of the free shear layer, increased wake centre line velocity and increased wake width.
2-D Losses in a compressor blade

- At far downstream, the flow becomes uniform.
- Theoretically, the difference between the stagnation pressure far downstream and the trailing edge represents the mixing loss.
- Most loss correlations are based on measurements downstream of the trailing edge (1/2 to 1 chord length) and therefore do not include all the mixing losses.
- If there is flow separation, the losses would include losses due to this zone and at its eventual mixing downstream.
2-D Losses in a compressor blade

The profile and mixing losses along a streamline can be written as:

\[ \bar{\omega}_{p+m} = \frac{2(P_{ot} - P_{02})}{\rho V_1^2} \]

To determine the above, it is necessary to relate the static pressure difference and velocities to the displacement and momentum thickness of the blade boundary layer at the trailing edge.
2-D Losses in a compressor blade

Detailed derivation of these correlations are given in Lakshminarayana's book (Chapter 6).

\[
\bar{\omega}_{p+m} = \frac{2(P_{0t} - P_{02})}{\rho V_1^2} = \frac{2(p_t - p_2)}{\rho V_1^2} + \frac{V_t^2 - V_2^2}{V_1^2}
\]

This is further expressed as:

\[
\bar{\omega}_{p+m} \sec^2 \alpha_1 = \left[ \frac{2\Theta + \Delta^2}{(1 - \Delta)^2} + \tan^2 \alpha_2 \left\{ \frac{(1 - \Delta)^2}{(1 - \Theta - \Delta)^2} - 1 \right\} \right]
\]

Neglecting higher order terms,

\[
\bar{\omega}_{p+m} \sec^2 \alpha_1 = 2(\Theta + \Theta \tan^2 \alpha_2)
\]

Where, \(\Delta\) is the blockage (related to displacement thickness) and \(\Theta\) is the momentum thicknesss
2-D Losses in a compressor blade

• Thus, in a simplified manner, we see that the profile loss can be estimated based on the momentum thickness.
• The above loss correlation includes both profile and wake mixing loss.
• If flow separation occurs, additional losses are incurred. This is because the pressure distribution is drastically altered beyond the separation point.
• The losses increase due to increase in boundary layer displacement and momentum thicknesses.
2-D Losses in a compressor blade

• In addition to the losses discussed above, boundary layer growth and subsequent decay of the wake causes deviation in the outlet angle.

• An estimate of this is given as:

\[ \tan \alpha_2 \approx (1 - \Theta - \Delta) \tan \alpha_t \]

• Hence, viscous effect in a turbomachine always leads to decrease in the turning angle.

• The values of displacement and momentum thicknesses, depend upon, variation of freestream velocity, Mach number, skin friction, pressure gradient, turbulence intensity and Reynolds number.
2-D Losses in a compressor blade

• In general, the loss estimation may be carried out using one of the following methods:
  • Separate calculation of the potential or inviscid flow and the displacement and momentum thicknesses. Subsequently, use the equation discussed previously.
  • Using a Navier-Stokes based computational code. Here the local and the integrated losses can be computed directly.
Mach number and shock losses

• The static pressure rise in a compressor increases with Mach number.
• Thus the pressure gradient increases with increase in Mach number.
• This means that the momentum thickness and hence the losses increase with Mach number.
• Increasing Mach numbers also lead to increase in shock losses.
• At transonic speeds, the shock losses are very sensitive to leading and trailing edge geometries.
Mach number and shock losses

• An estimate of the 2-D shock losses for a compressor must include:
  • The losses due to the leading edge bluntness with supersonic upstream Mach number.
  • The location of the passage shock can be determined from inviscid theories. If the shock strength is known, the losses can be estimated.
  • The losses due to boundary layer growth and the shock-boundary layer interaction are most difficult to estimate. The contribution however is small for weak shocks.
Mach number and shock losses

• One of the empirical correlations for the shock loss was given by Freeman and Cumpsty (1989).

\[
\omega_{sh} = \frac{(\Delta P_0)_{loss}}{P_{01} - p_1} = \left[ \frac{(\Delta P_0)_{loss}}{P_{01} - p_1} \right]_{\text{normal shock}} + \left[ 2.6 + 0.18(\alpha_1' - 65^o) \right] 10^{-2} (\alpha_1 - \alpha_1')
\]

where \( \alpha_1' \) is the blade inlet angle

• This is valid for an incidence angle upto 5\(^o\).

• These empirical correlations are however, derived using the 2-D assumption.

• Actual flows are seldom 2-D in nature.
3-D flow in axial compressors
3-D flow in axial compressors

• Flow in axial compressors considered so far was 2-D: no radial component of velocity.
• Three dimensionality is caused by inviscid and viscous effects.
• Some of the inviscid effects are due to
  – Compressibility and radial pressure gradients
  – Radial variation in blade geometry
  – Tip leakage flow
  – Presence of shock
  – Secondary flows
3-D flow in axial compressors

- These inviscid effects can be analysed using the inviscid governing equations.
- The most dominant effect is the radial variation in velocity.
- The viscous and inviscid effects compliment each other.
- For eg. Tip leakage flow is essentially an inviscid effect, but its propagation and formation of leakage vortex is controlled by viscous effects.
3-D flow in axial compressors

• The equations of motion for 3-D analysis of flow through turbomachines are highly non-linear.

• Analytical solutions exist for simple flow fields.

• Depending upon the analysis, one may take up an axisymmetric analysis or a non-axisymmetric analysis.

• Axisymmetric analysis
  – Simple radial equilibrium analysis
  – Actuator disc theories
  – Passage averaged equations
3-D flow in axial compressors

• Non-axisymmetric analysis
  – Lifting line and lifting surface approach
  – Quasi-3-D approach
  – Numerical solution of exact equations
    (Euler or Navier-Stokes)

• Axisymmetric analysis is used to predict the radial or spanwise variation of properties far downstream of the blade.

• In the blade passage, cascade theories can be used to determine variation in properties at a given spanwise location.
Axisymmetric analysis

From Lakshminarayana, Chap 4, P 264