Department of Aerospace Engineering
Indian Institute of Technology Bombay

AE 651: Aerodynamics of Compressors and Turbines

Mid-Semester Examination 10^{th} **September 2015**

Maximum Marks: 50 **Time: 1100-1300 hrs.**

Please note:
1. The answers must be brief and to the point.
2. Assume C_p=1.005 kJ/kgK and γ=1.4.
3. One A-4 size sheet containing formulae, equations etc. is permitted.

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1. Answer the following questions in brief (not more than half a page each):

a. Show the compression process in a stage of an axial compressor using a T-s diagram. Show the static as well as stagnation temperature and pressure variations.

b. How will the velocity triangles of an axial compressor look like when the operating flow coefficient is (a) greater than the design flow coefficient; (b) less than the design flow coefficient.

c. From the first principles, show that the static pressure at the rotor exit increases from the hub to the tip when using a free vortex design. How does this affect the static performance?

d. Using a compressor map and throttle characteristic, explain operational stability.

e. Using velocity distributions on a blade surface, explain diffusion factor.

(5×4=20)

2. The volume flow rate through an axial flow compressor is 6 m^3/s. The compressor has IGVs and operates at a rotational speed of 10000 rpm. The un-twisted rotor blade has tip and hub diameters of 40 cm and 25 cm, respectively. The compressor stage develops a total pressure ratio of 1.2 with a stage efficiency of 0.85. If the rotor exit blade angle is 25^o, determine the IGV exit angle. Also determine whether this compressor would suffer from any shock losses. The inlet stagnation conditions are 303 K and 1 atm.

(10)

3. An axial compressor has a degree of reaction of 0.6, a flow coefficient of 0.5 and a stage loading coefficient of 0.35. If the flow exit angles for each blade row remain unchanged when the mass flow is throttled, determine the degree of reaction and the stage loading coefficient when the mass flow is reduced by 10% at constant blade speed.

(10)

4. The front stage of an axial compressor (with a hub-tip ratio of 0.5), is being designed using the following variation of tangential velocity before and after the rotor, respectively: $C_{w1} = ar - b/r$ and $C_{w2} = ar + b/r$, where, a and b are constants and r is the radius. The stage is designed for a loading coefficient of 0.3 based on the blade tip speed. At the mean radius of 0.3 m, the stage has symmetrical velocity triangles. The rotor is to operate at a speed of 3000 rpm when operating with a constant axial velocity of 100 m/s. Determine the blade angles of the stage at the mid-section.

(10)

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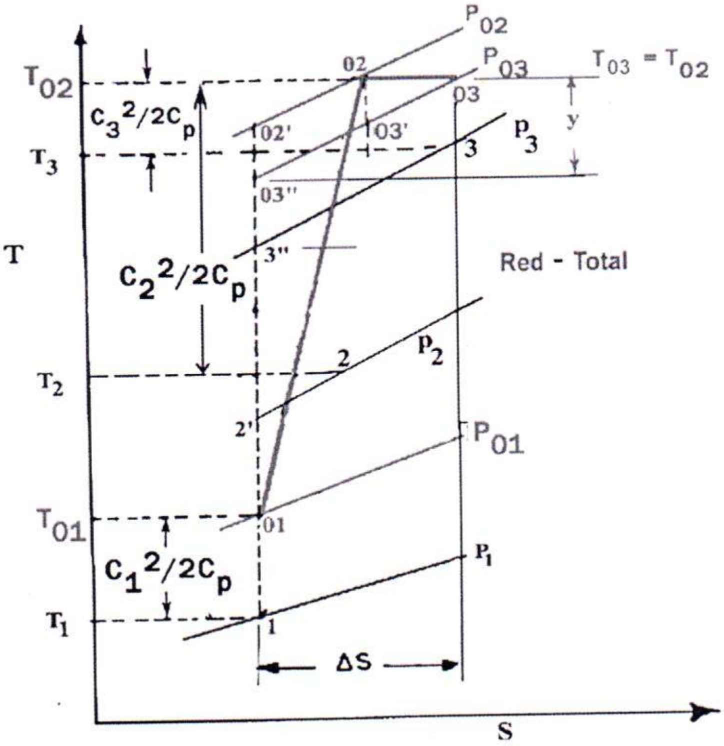
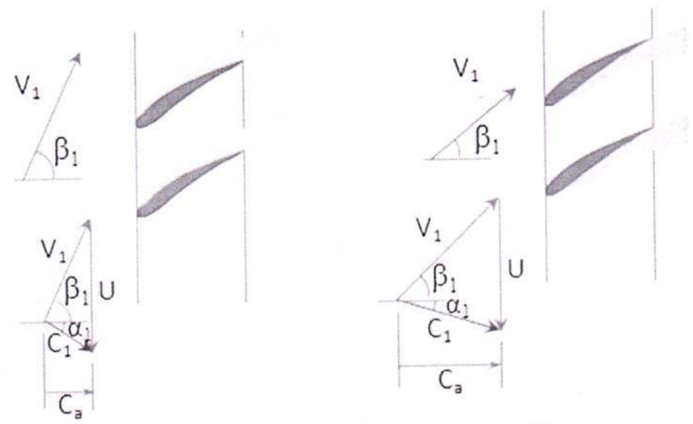
1 (a)



1 (b)



Off - design condition :	Off - design condition :
Positive incidence flow separation	Negative incidence flow separation
$\left(\frac{C_a}{U}\right) < \left(\frac{C_a}{U}\right)_{design}$	$\left(\frac{C_a}{U}\right) > \left(\frac{C_a}{U}\right)_{design}$

1 c

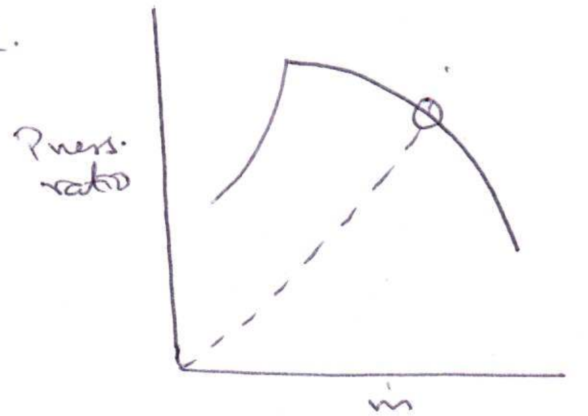
$$R_x = \frac{C_a(\tan\beta_1 + \tan\beta_2)}{2U}$$

$$= 1 - \frac{(K_1+K_2)/2\Omega}{r^2}$$

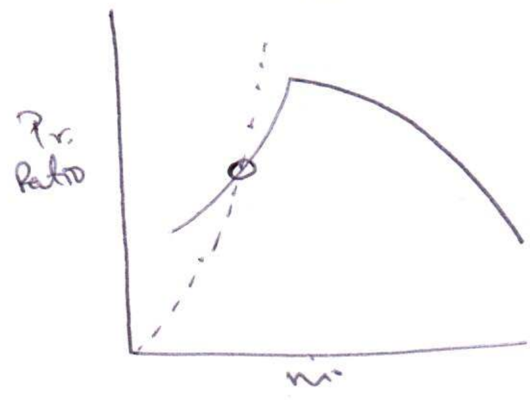
$\frac{C_w^2}{r} > 0$ ∴ P increases from root to tip

∴ ΔP across the stator needs to be higher from root to tip as R_x increases from root to tip.

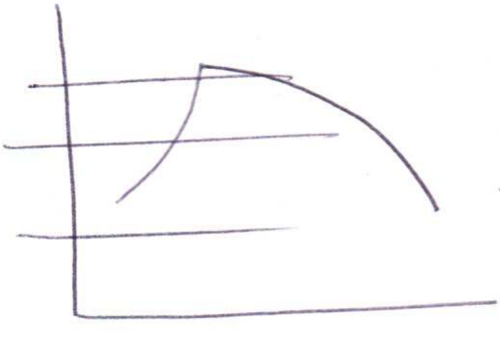
1d.

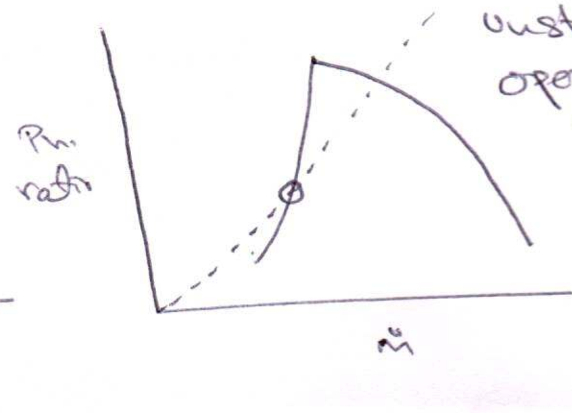


Stable operating point. perturbation in the world eventually result in the system returning to its equilibrium state



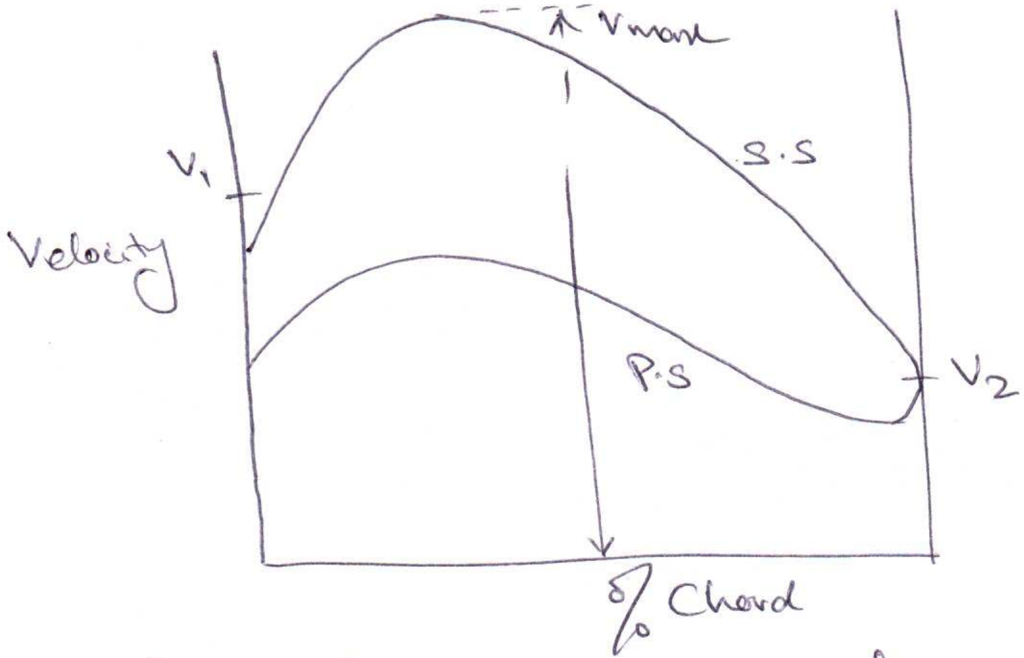
Stable operating point.





unstable operating point.

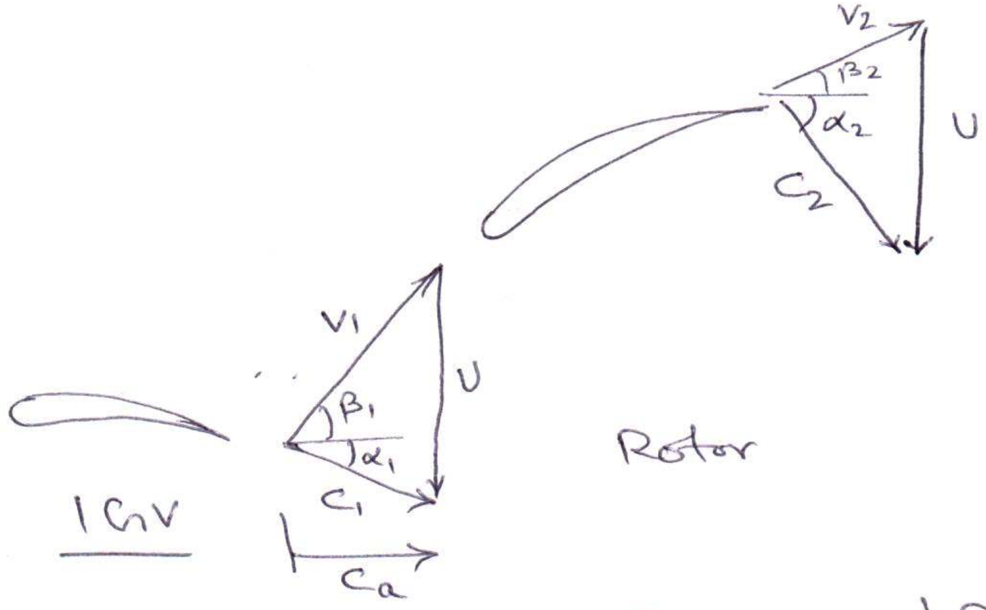
1. e.



Diffusion factor : indication of an impending flow separation.

$$D^* = \frac{V_{max} - V_2}{V_1} \rightarrow \text{Diffusion on the suction surface.}$$

2.



$$\eta_{stage} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} \quad \therefore \Delta T_{0\,stage} = \frac{1}{\eta_{st}} T_{01} \left(\pi_{stage}^{\frac{\gamma-1}{\gamma}} - 1 \right)$$

$$= 19.05\,K$$

$$U_m = \frac{\pi d_m N}{60} = \frac{\pi (0.4 + 0.25)/2 \times 10000}{60} = 170\,m/s$$

$$Q = A C_a \text{ or } C_a = \frac{Q}{A} = 6 \Big/ \pi (0.4^2 - 0.25^2)/4$$

$$= 78.35\,m/s$$

$$\Delta T_{0\,stage} = \frac{U C_a}{C_p} (\tan\beta_1 - \tan\beta_2)$$

$$\therefore \underline{\underline{\beta_1 = 62.26^\circ}}$$

$$\tan\alpha_1 = \frac{U - C_a \tan\beta_1}{C_a} \quad \text{or} \quad \underline{\underline{\alpha_1 = 15.09^\circ}}$$

$$C_1 = \frac{C_a}{\cos\alpha_1} = 81.15\,m/s$$

$$T_1 = T_{01} - \frac{C_1^2}{2c_p} = 299.7\,K$$

$$V_1 = \frac{C_a}{\cos\beta_1} = 168.3\,m/s$$

$$\therefore M_{in\,relative} = \frac{V_1}{\sqrt{\gamma R T_1}} = 0.48 < 1$$

Hence no shock losses

3. $R_x = 0.5$, $\phi = 0.5$, $\psi = 0.35$

$$R_x = \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2)$$

$$0.6 = \frac{0.5}{2}(\tan\beta_1 + \tan\beta_2)$$

or $\tan\beta_1 + \tan\beta_2 = 2.4$

$$\psi = \frac{\Delta C_w}{U} = \frac{C_a}{U}(\tan\beta_1 - \tan\beta_2)$$

$\tan\beta_1 - \tan\beta_2 = 0.7$

$\therefore \beta_1 = 57.17^\circ$ and $\beta_2 = 40.36^\circ$

Similarly, $\tan\alpha_1 = \frac{1}{\phi} - \tan\beta_1$

$\tan\alpha_2 = \frac{1}{\phi} - \tan\beta_2$

$\therefore \alpha_1 = 24.22^\circ$ and $\alpha_2 = 49^\circ$

when $\dot{m}$ is reduced by 10%

$$\phi = \frac{C_{a\,new}}{U} = 0.9 \times 0.5 = 0.45$$

$$\therefore R_x = \frac{C_{a\,new}}{U}(\tan\beta_1 + \tan\beta_2)$$

$$= 0.59$$

And $\psi = \frac{\Delta C_w}{U} = \frac{C_{a\,new}}{U}(\tan\beta_1 - \tan\beta_2)$

β_2 and α_1 remain unchanged

$\tan\beta_1 = \frac{U}{C_{a\,new}} - \tan\alpha_1$ or $\beta_1 = 60.56^\circ$

$\therefore \psi = 0.414$

4. $r_m = 0.3\ m$, $\frac{r_h}{r_t} = 0.5$, $\therefore r_t = 0.4 m$

$r_h = 0.2 m$

$C_{w1} = ar - \frac{b}{r}$ and $C_{w2} = ar + b/r$

$\therefore \Delta C_w = \frac{2b}{r}$

At mean radius, $\Delta W = U_m \Delta C_w$

$= U_m \frac{2b}{r}$

$\because \varphi = 0.3 = \frac{W}{U_t^2}$

$\therefore W = \varphi U_t^2 = 0.3 \left(\frac{\pi \times 0.4 \times 2 \times 3000}{60}\right)^2$

$= 4737.4\ J/kg$

Also $b = \frac{W r_m}{2 U_m} = \underline{7.54}$

$R_{xm} = 0.5 = 1 - \frac{C_a (\tan \alpha_1 + \tan \alpha_2)}{2 U_m}$

$= 1 - \frac{a r_m}{U_m}$

$0.5 = 1 - \frac{a\ 0.3}{94.25}$ or $a = \underline{157.08}$

$C_{w1} = ar - \frac{b}{r} = 157.08 \times 0.3 - \frac{7.54}{0.3}$

$= 21.99\ m/s$

$C_{w2} = ar + \frac{b}{r} = 72.26\ m/s$

$\beta_{1m} = \tan^{-1}\left(\frac{U_m - C_{w1}}{C_a}\right) = \underline{\underline{35.85^\circ}}$

$\beta_{2m} = \tan^{-1}\left(\frac{U_m - C_{w2}}{C_a}\right) = \underline{\underline{12.4^\circ}}$