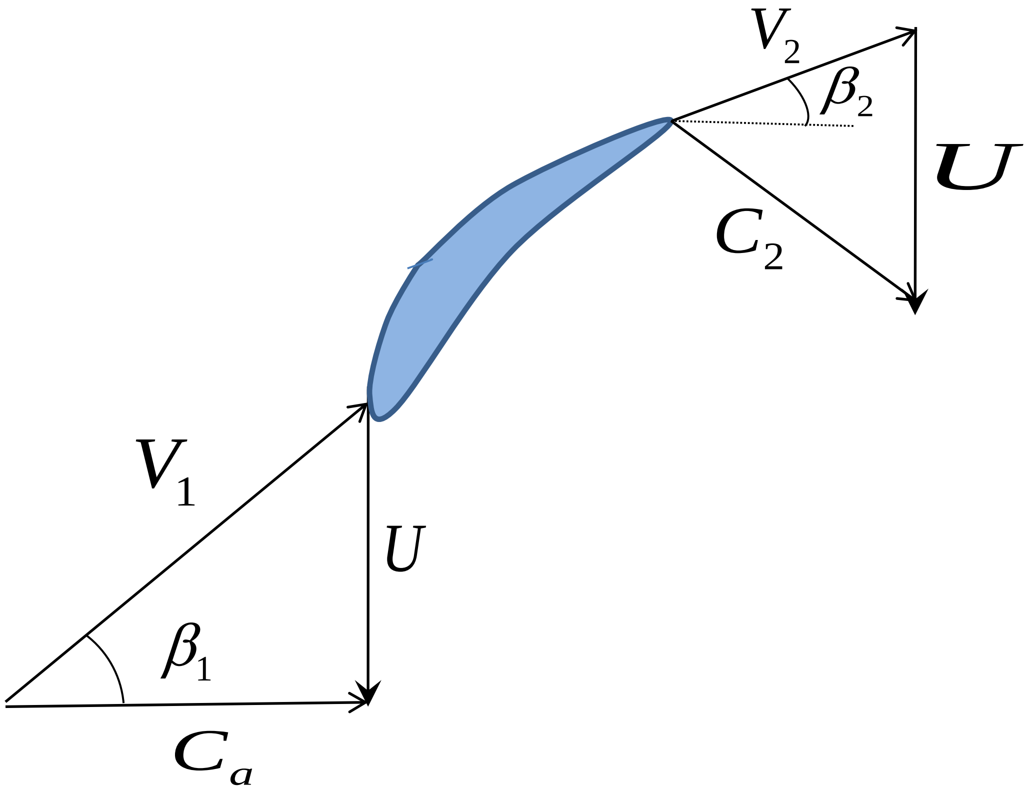
Problem 1

Air at 1.0 bar and 288 K enters an axial flow compressor with an axial velocity of 150 m/s. There are no inlet guide vanes. The rotor stage has a tip diameter of 60 cm and a hub diameter of 50 cm and rotates at 100 rps. The air enters the rotor and leaves the stator in the axial direction with no change in velocity or radius. The air is turned through 30.2^{o} as it passes through the rotor. Assume an overall pressure ratio of 6 and a stage pressure ratio of 1.2. Find a) the mass flow rate of air, b) the power required to drive the compressor, c) the degree of reaction at the mean diameter, d) the number of compressor stages required if the isentropic efficiency is 0.85.

Solution : Problem # 1



Solution: Problem # 1

$$U = \pi \times \left(\frac{d_t + d_h}{2} \right) \times N = \pi \times \left(\frac{0.6 + 0.5}{2} \right) \times 100 = 172.76 m/s$$

$$\beta_1 = \tan^{-1}\left(\frac{U}{C_a} \right) = 49.2°$$

$$\beta_2 = 49.2 - 30.2 = 19°$$

$$\tan\alpha_2 = \left(\frac{U - C_a \tan\beta_2}{C_a} \right) = 80.75$$

$$\alpha_2 = 38.92°$$

Solution: Problem # 1

$$\dot{m} = \frac{\pi}{4} \times \left(d_t^{\ 2} - d_h^{\ 2}\right) \times C_a \times \rho_2 \qquad \& \qquad T_1 = T_{01} - \frac{C_a^2}{2C_p} = 276.8\,K$$

$$T_{02} = T_{01} \times \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} \therefore \quad T_{02} = 303.41K$$

$$T_2 = 303.41 - \frac{C_2^{\ 2}}{2C_p} \& \quad \cos\alpha_2 = \frac{C_a}{C_2}$$

$$\therefore C_2 = \frac{C_a}{\cos\alpha_2} = \frac{150}{\cos 38.92} = 192.79 m/s$$

Solution: Problem # 1

$$T_2 = 303.41 - \frac{192.79^2}{2010} = 284.91K$$

$$P_2 = 0.963bar$$

$$\rho_2 = \frac{0.963 \times 101325}{287 \times 284.9} = 1.193Kg/m^3$$

$$\dot{m} = 15.46Kg/s$$

$$P = U \times C_a \times \dot{m} \times (\tan\beta_1 - \tan\beta_2)$$

$$= 172.76 \times 150 \times 15.46 \times (\tan 49.2 - \tan 19) = 326KW$$

Solution: Problem # 1

$$R_X = \frac{C_a}{2U} \times \left(\tan\beta_1 + \tan\beta_2\right)$$

$$= \frac{150}{2 \times 172.76} \times \left(\tan 49.2 + \tan 19\right) = 0.65$$

$$\Delta T_{0s} = \frac{U \times C_a}{C_p} \times \left(\tan\beta_1 - \tan\beta_2\right)$$

$$= \frac{172.76 \times 150}{1005} \times \left(\tan 49.2 - \tan 19\right) = 20.99\,K$$

Solution: Problem # 1

$$\Delta T_{0overall} = \frac{T_{01}}{\eta_C} \times \left(\pi_C^{\frac{\gamma-1}{\gamma}} - 1 \right)$$

$$= \frac{288}{0.85} \times \left(6^{0.286} - 1\right) = 226.5K$$

$$n = \frac{226.5}{20.99} = 10.79 \approx 11$$

Therefore the number of stages required for the given pressure ratio is 11.0.

Solution: Problem # 1

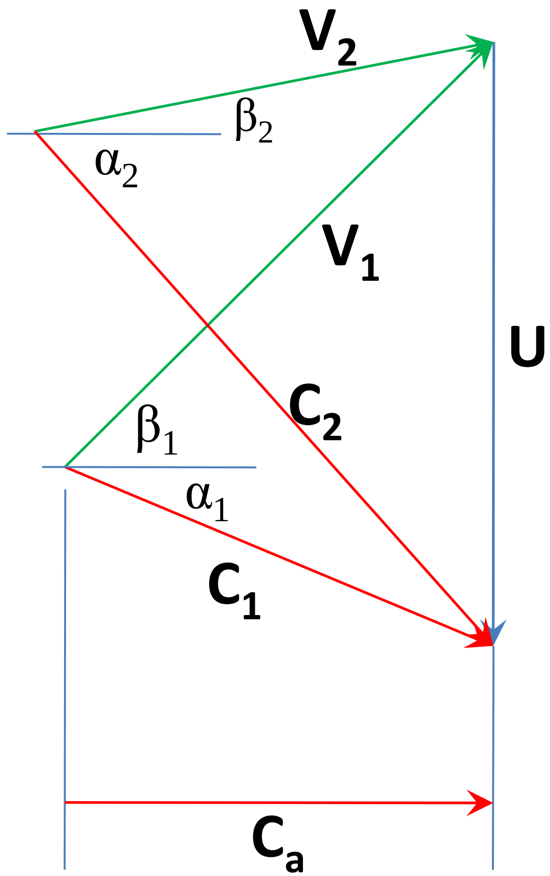
Some additional points:

1. You can also calculate the mass flow rate from the inlet density. How does this mass flow rate compare with mass flow calculated from rotor exit conditions? How do you explain this discrepancy?
2. The number of stages in this question can also be calculated from the overall pressure ratio and the stage pressure ratio. Is this number the same as that calculated from the temperature rises? Why the difference?

Problem # 2

An axial flow compressor is to be designed to generate a total pressure ratio of 4.0 with an overall isentropic efficiency of 0.85. The inlet and outlet blade angles of the rotor blades are 45 degree & 10 degree, respectively and the compressor stage has a degree of reaction of 50 percent. If the blade speed is 220 m/s and the work done factor is 0.86, find the number of stages required. Is it likely that the compressor will suffer from shock losses? The ambient air static temperature is 290 K and the air enters the compressor through guide vanes.

Solution: Problem # 2



Solution: Problem # 2

Axial velocity, $C_a = \dfrac{U}{\tan\beta_1 + \tan\beta_2} = 187 m/s$

Absolute velocity at inlet, $C_1 = \dfrac{C_a}{\cos\alpha_1} = 190 m/s$

The per stage temperature rise,

$$\Delta T_{0s} = \frac{\lambda \times U \times C_a \times \left(\tan\beta_1 - \tan\beta_2\right)}{C_p} = 29K$$

Solution: Problem # 2

Total temperature at compressor inlet,

$$T_{01} = T_1 + \frac{C_1^2}{2Cp} = 331.8K$$

Isentropic total temperature at compressor exit,

$$T_{02s} = T_{01} \times \pi_c^{\frac{\gamma-1}{\gamma}} = 493.9\,K$$

Actual total temperature at compressor exit,

$$T_{02} = T_{01} + \frac{\left(T_{02s} - T_{01}\right)}{\eta_c} = 522.5\,K$$

Solution: Problem # 2

Therefore total temperature rise across the compressor = $T_{02} - T_{01} = 190.74K$

The number of stages required=

$$\frac{\text{Overall temperature rise across the compressor}}{\text{Per stage temperature rise}}$$

$$= \frac{190.74}{29} = 6.6 \approx 7$$

Solution: Problem # 2

To determine whether the compressor will suffer from shock losses, we need to find the relative Mach number

$$M_{rel} = \frac{V_1}{\sqrt{\gamma R T_1}}$$

$$V_1 = \frac{C_a}{\cos \beta_1} = 264.5 m/s$$

$$\therefore M_{rel} = 0.77$$

Since relative Mach number is less than unity, the compressor is not likely to suffer from shock losses.

Problem # 3

- The conditions of air at the entry of an axial compressor stage are P_1=1 bar and T_1=314 K. The air angles are $\beta_1=51^o$, $\beta_2=9^o$, $\alpha_1=\alpha_3=7^o$. The mean diameter and peripheral speed are 50 cm and 100 m/s respectively. Given that the work done factor is 0.95, stage efficiency is 0.88, mechanical efficiency is 0.92 and the mass flow rate is 25 kg/s, Determine a) air angle at stator entry, b) blade height at entry and hub–tip diameter ratio, c) Stage loading coefficient, d) Power required to drive the stage.

Solution: Problem # 3

a)

$$\frac{U}{C_a} = \tan\alpha_1 + \tan\beta_1$$

$$\frac{100}{C_a} = \tan 7 + \tan 51 \qquad \therefore C_a = 73.65 m/s$$

$$\tan\alpha_2 + \tan\beta_2 = \frac{U}{C_a}$$

$$\tan\alpha_2 + \tan 9 = \frac{100}{73.65} \qquad \therefore \alpha_2 = 50.18^\circ$$

Solution: Problem # 3

b)

$$\dot{m} = \rho_1 \times C_a \times \left(\pi \times d_m \times h\right), \quad and \quad \rho_1 = P_1 / RT_1$$

Substituting known values in the above,

$$h = 0.19\,m$$

$$d_t = 50 + 19 = 69cm,$$

$$d_h = 50 - 19 = 31cm$$

$$\text{The hub-tip ratio is } \frac{d_h}{d_t} = 0.449$$

Solution: Problem # 3

c)

$$\Psi = \frac{w}{U^2} \quad \& \qquad w = \lambda \times C_a \times U \times (\tan\beta_1 - \tan\beta_2)$$

$$w = 0.95 \times 100 \times 73.65 \times (\tan 51 - \tan 9) = 7534.8 J/Kg$$

$$\Psi = \frac{7534.8}{100^2} = 0.7535, \text{ is the loading coefficient.}$$

d)

$$P = \frac{\dot{m} \times w}{\eta_m} = 204.75 KW \text{ is the power required.}$$

Exercise Problem # 1

- An axial flow compressor of 50 percent reaction design has blades with inlet and outlet angles at 45^o and 10^o respectively. The compressor is to produce a pressure ratio of 6:1 with overall isentropic efficiency of 0.85 when inlet static temperature is 37^oC. The blade speed and axial velocity are constant throughout the compressor. Assuming a value of 200 m/s for blade speed. Find the number of stages required if the work done factor is (a) unity and (b) 0.87.
- Ans: (a) 8 (b) 9

Exercise Problem # 2

- Air at 1 bar and 288 K enters an axial flow compressor stage with an axial velocity of 150 m/s. There are no inlet guide vanes. The rotor has a tip diameter of 60 cm and a hub diameter of 50 cm and rotates at 100 rps. The air enters the rotor and leaves the stator with no change in velocity or radius. The air is turned through 30^o as it passes through the rotor. Determine (a) the blade angles (b) mass flow rate (c) power required and (d) the degree of reaction.
- Ans: (a) 49^o, 19^o (b) 14.38 kg/s © 300.7 kW (d) 0.65

Exercise Problem # 3

- An axial compressor stage has the following data: Degree of reaction : 50%, Mean blade dia: 36cm, rotational speed: 18000 rpm, blade height at entry: 6 cm, air angles at rotor inlet and stator exit: 25^o, axial velocity: 180 m/s, workdone factor: 0.88, stage efficiency: 0.85, mechanical efficiency: 96.7%. Determine (a) air angles at rotor and stator entry (b) mass flow rate (c) power required (d) stage loading coefficient (e) pressure ratio developed by stage (f) relative Mach number at rotor entry.

- Ans: (a) 54.82^o, 25^o (b) 14.37 kg/s (c) 51.2 kJ/kg (d) 0.44 (e) 1.6 (f) 0.90

Exercise Problem # 4

- A 50% reaction axial flow compressor has inlet and outlet blade angles of 45^o and 12^o, respectively. The blade speed at the tip of the rotor is 320 m/s. If the inlet total temperature is 300 K, determine the tip relative Mach number.
- Ans: 1.146

Exercise Problem # 5

- A 10 stage axial flow compressor develops an overall pressure ratio of 8.0 with and isentropic efficiency of 0.85. The absolute velocity component of air enters the rotor at an angle of 27^o to the axial direction. The axial component of velocity is constant throughout the compressor and is equal to 150 m/s. The mean blade speed is 200 m/s. If the ambient air conditions are 15^oC and 1 bar, determine the angle which the relative component of velocity makes with the axial direction at the exit of the rotor.

- Ans: 14^o