Problem 1

Free vortex design is being advocated for design of an axial compressor rotor with high hub/tip radius ratio (0.9) – taken to be constant through the stage. At the rotor tip (1m dia) the flow angles are given as : $\alpha_1 = 30^0$, $\beta_1 = 60^0$, $\alpha_2 = 60^0$, $\beta_2 = 30^0$.
Also, RPM = 6000; $\rho = 1.5$ kg/m^3; Enthalpy, H(r) = constant and Entropy change, Δs (r)= constant - along blade length.

For such a rotor design determine the design point performance parameters :

i) Axial velocity C_a, constant from root to tip
ii) Mass flow rate, $\dot{m}$
iii) Ideal minimum power to be supplied for this rotor
iv) Flow angles at the rotor blade root w.r.t. axial direction.
v) Degree of reaction at the blade root

Solution: Problem 1

(i) Rotor angular velocity , $\omega = 2\pi N/60$ rad/s = 628.4 rad/s

Blade speed at rotor tip, $U_{tip} = \omega r_t$ = 314.2 m/s
and, Blade speed at root , $U_{hub} = \omega r_h$ = 282.5 m/s
and, Blade speed at mean, $U_{mean} = \omega r_{mean}$ = 298.5 m/s

Now from standard velocity diagram of a rotor inlet,

$$U_{tip} = C_w + V_w = C_a (\tan \alpha_1 + \tan \beta_1)_{tip}$$

From which, $C_a = 136$ m/s

(ii) Mass flow rate , $\dot{m}$ = Annulus area x density x axial velocity

$$= \pi (r_t^2 - r_h^2).\ \rho.\ C_a = 30.4 \text{ kg/s}$$

Solution: Problem 1

iii) At inlet to the tip , $C_{w1\text{-}tip} = C_a \tan \alpha_1 = 78.6$ m/s

By applying the Free Vortex Law

$$C_{w1\text{-}mean} = C_{w1\text{-}tip} . r_{tip} / r_{mean} = 82.73 \text{ m/s}$$

At the exit to the tip, $C_{w2\text{-}tip} = C_a \tan \alpha_1 = 235.6$ m/s

By applying the Free Vortex Law

$$C_{w2\text{-}mean} = C_{w2\text{-}tip} . r_{tip} / r_{mean} = 248 \text{ m/s}$$

Minimum Power to be supplied (with 100% efficiency) is the power absorbed by the rotor -- at any radial station , as per free vortex law:

$$W = \dot{m}. U_{mean}.(C_{w2\text{-}mean} - C_{w1\text{-}mean})$$

$$= 1512924 \text{ J/s} = 1.513 \text{ mW}$$

Solution: Problem 1

iv) Using Free vortex law :

$$C_{w1\text{-}hub} = C_{w1\text{-}tip} . r_{tip} / r_{hub} = 87.3 \text{ m/s}$$
$$C_{w2\text{-}hub} = C_{w2\text{-}tip} . r_{tip} / r_{hub} = 262 \text{ m/s}$$

The flow angles at the hub are :

$\tan \alpha_1 = C_{w1\text{-}hub} / C_a = 87.3/136 = 0.642;$ $\alpha_1 = 32.75^0$

$\tan \alpha_2 = C_{w2\text{-}hub} / C_a = 262/136 = 1.928;$ $\alpha_2 = 62.6^0$

$\tan \beta_1 = U_{hub} / C_a - \tan \alpha_1 = 1.436;$ $\beta_1 = 55.15^0$

$\tan \beta_2 = U_{hub} / C_a - \tan \alpha_2 = 0.152;$ $\beta_2 = 8.64^0$

Solution: Problem 1

v) Degree of Reaction at the hub :

$$R_{x\text{-hub}} = (\tan \beta_{2\text{-hub}} - \tan \alpha_{1\text{-hub}})\, C_a / 2U_{hub}$$
$$= 0.382$$

As one can see also from the answers (iv) the velocity triangles at hub would be asymmetric whereas the velocity triangles are symmetric at the rotor tip ($R_x = 0.5$). One can calculate the values at mean and it would be seen that velocity triangle at the mean also would be asymmetric. In free vortex design the velocity triangles can be symmetric at only one radial location along the blade length.

Problem 2

An axial flow compressor is originally designed with free vortex law, and has degree of reaction , $R_x = 0.6$ at the mean , with hub/tip radius ratio of 0.6. The flow angles at the mean radius are given as $\alpha_1 = 30^0$, $\beta_1 = 60^0$. Calculate the relative and absolute flow angles, at the hub and tip – both at the inlet and the exit of the rotor and the degree of reaction at both hub and tip.

Now if this axial compressor is to be re-designed with exponential law, than recalculate the relative and the absolute flow angles, at the hub and the tip – both at the inlet and at the exit of the rotor and, the degree of reaction at both hub and tip. Prescribed, $a = 100$; $b=40$

Solution: Problem 2

Following the procedure adopted in the 1st problem the solution for the original free vortex design may be found to be :

$\alpha_{1\text{-hub}} = 37.6^0$; $\beta_{1\text{-hub}} = 24.8^0$; $\alpha_{2\text{-hub}} = 66.6^0$; $\beta_{2\text{-hub}} = -30^0$

$\alpha_{1\text{-tip}} = 43.9^0$; $\beta_{1\text{-tip}} = 67.5^0$; $\alpha_{2\text{-tip}} = 54.2^0$; $\beta_{2\text{-tip}} = 56.3^0$

Using the degree of reaction relations developed

$$R_{x\text{-hub}} = 0.29$$

$$R_{x\text{-tip}} = 0.744$$

Solution: Problem 2

For exponential law re-design we apply the law :

upstream : $C_{w1} = a - b/R$

and, downstream: $C_{w2} = a + b/R$

where R is radius ratio, r/r_{mean}

And $a = 100$; $b=40$ expressed in m/s

$C_{w1-hub} = 46.7$ m/s ; $C_{w1-tip} = 68$ m/s

Solving the velocity triangles we get :

$C_{a1-hub} = 121.7$ m/s; and $C_{a1-tip} = 94.1$ m/s

Solution: Problem 2

Using the prescribed law --- in front and behind the rotor;

At the hub :

$C_{a2\text{-hub}} = 142$ m/s ; $C_{w2\text{-hub}} = 153$ m/s ;

$\tan \alpha_{1\text{-hub}} = C_{w1\text{-hub}} / C_{a1} = 0.384$; $\alpha_{1\text{-hub}} = 21^0$

$\tan \alpha_{2\text{-hub}} = C_{w2\text{-hub}} / C_{a2} = 0.93$; $\alpha_{2\text{-hub}} = 43^0$

$\tan \beta_{1\text{-hub}} = U_{hub} / C_{a1} - \tan \alpha_1 = 1.157$; $\beta_{1\text{-hub}} = 49.1^0$

$\tan \beta_{2\text{-hub}} = U_{hub} / C_{a2} - \tan \alpha_2 = 0.392$; $\beta_{2\text{-hub}} = 21.4^0$

Degree of Reaction at the hub : $R_{x\text{-hub}} = 0.59$

Solution: Problem 2

Using the prescribed law --- in front and behind the rotor;

At the tip : $C_{w2\text{-}tip} = 132$ m/s

$\tan \alpha_{1\text{-}tip} = C_{w1\text{-}tip} / C_{a1} = 0.722$; $\alpha_{1\text{-}tip} = 35.85^0$

$\tan \alpha_{2\text{-}tip} = C_{w2\text{-}tip} / C_{a2} = 1.755$; $\alpha_{2\text{-}tip} = 60.32^0$

$\tan \beta_{1\text{-}tip} = U_{tip} / C_{a1} - \tan \alpha_1 = 2.6$; $\beta_{1\text{-}tip} = 69^0$

$\tan \beta_{2\text{-}tip} = U_{tip} / C_{a2} - \tan \alpha_2 = 2.355$; $\beta_{2\text{-}tip} = 67.4^0$

Degree of Reaction at the tip : $R_{x\text{-}tip} = 0.734$

Solution: Problem 2

The values obtained for the Free Vortex and the Exponential Law designs permit us to conclude that :

1) The Degree of Reaction at hub for the exponential design is much higher than that of the free vortex design. That normally makes it a safe design

2) The rotor twist i.e. β_1 , β_2 variation from root to tip is much less for the exponential design. This means it will have less structural loading on the blades

Exercise Problem 1

1. A 3-D design of a rotor blade of axial flow compressor following vortex laws may be used :

$$C_{w1} = (aR-b)/R \qquad \text{and} \qquad C_{w2} = (aR+b)/R$$

Where $R = r/r_{mean}$

And following data may be used : hub/tip radius ratio,

$r_h / r_t = 0.6$; C_a = constant across the rotor at mean radius

At mean radius $C_{w1} = 60$ m/s ; $C_{w2} = 150$ m/s

Specific work = 21.6 kJ/kg

Calculate the following parameters :

i) at mean radius : ΔT, Rx, a, b

ii) at root and tip, Rx

iii) at root and tip : inlet and exit axial velocities, C_{a1} , C_{a2}

iv) Inlet and exit flow angles at root, mean and tip

Exercise Problem 2

2) The table here shows a few data of an axial flow compressor rotor designed with Free vortex theory.

i) Calculate all the data to complete the table
ii) Plot the entry and exit velocity triangles at root, mean and tip

Variable	Root Section	Mean Section	Tip Section
C_{W1} [m/s]	32	24	?
C_{W2} [m/s]	?	150	?
$R[r/r_m]$	?	?	?
U_1 [m/s]	?	?	?
U_2 [m/s]	?	?	?
R_x	?	?	?
C_{a1} [m/s]	109	?	?
C_{a2} [m/s]	?	?	?
ΔT_0 [K]	?	?	?
$U \Delta C_W$ [J/kg]	?	?	?
α_1 [°]	?	?	?
α_2 [°]	?	?	?
β_1 [°]	?	?	?
β_2 [°]	?	?	?
Chord [cm]	6.0	5.5	5.0

Exercise Problem 3

3. An axial flow compressor with hub/tip radius ratio of 0.4 and maximum diameter as 0.6m, has been designed with constant reaction of 50% from root to tip . The blade tip speed U_{tip} = 300 m/s. The stagnation temperature rise is 16^0 C. The axial velocity prescribed for the flow near the casing, upstream of rotor, is 120 m/s. For air c_p = 1.005 kJ/kgK.
Determine the axial velocity before and after the rotor such that radial equilibrium is maintained.