AE 658

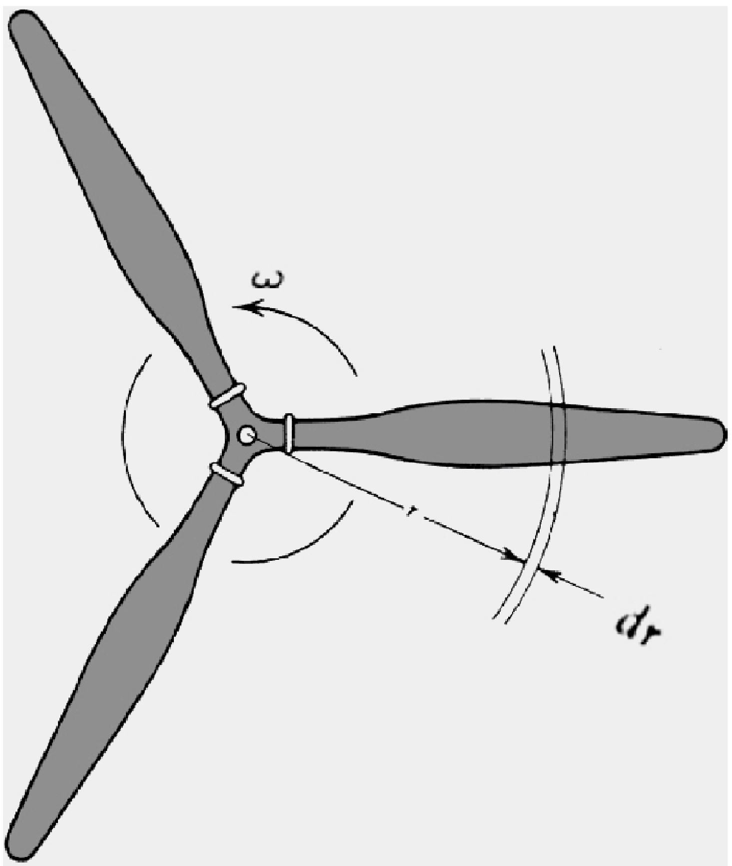
Chapter – 1

Propeller Theory -

Blade Element Theories

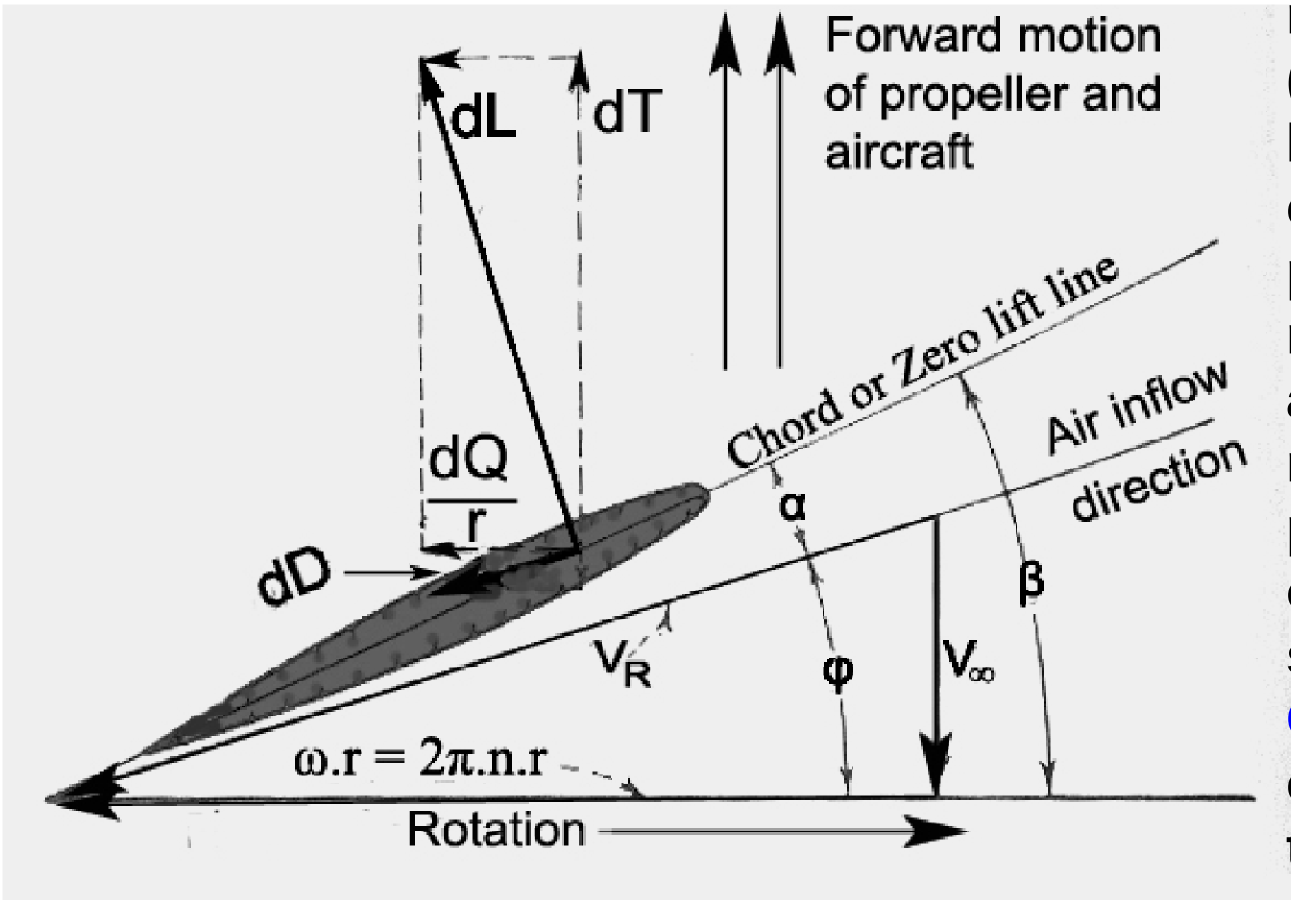
Propeller Blade Element

Assume that two cylindrical surfaces around the axis of rotation and radially only small distance *dr* apart, cut the propeller blade at a radius *r* from the axis. The flow is coming into this element with an axial velocity V, while the propeller blades are rotating with angular velocity ω rad/s



It is assumed that each blade element (airfoil shape) of a blade uses its proportional share of the power and the torque (tangential force) supplied to it and creates thrust for propelling an aircraft. Thus the aggregate amount of thrust can be designed and predicted by integration from the elemental considerations

Propeller Blade Element



The blade elements are assumed to be made up of airfoils (chord = c) of known lift, C_l and drag, C_d characteristics. In practice a large number of different airfoils are used to make up one propeller blade. Each of these elements shall have its own lift, C_l and drag, C_d characteristics. The thrust, dT of an element of radial length dr is made from an airfoil of lift, dL and drag, dD

From elemental considerations

Thrust produced,

$$dT = dL \cos\varphi - dD \sin\varphi$$

$$= \tfrac{1}{2}\rho V_R^2\, c.dr.\,(C_l \cos\varphi - C_d \sin\varphi)$$

Torque supplied ,

$$dQ = (dL \sin\varphi + dD \cos\varphi).\, r$$

$$= \tfrac{1}{2}\rho V_R^2\, c.dr.(C_l \sin\varphi + C_d \cos\varphi)$$

Substituting for resultant velocity,

$V_R = V_\infty / Sin\ \varphi$, and

for dynamic head, using $q = \tfrac{1}{2}\rho V_\infty^2$

Elemental Thrust $$dT = \frac{q.c.dr}{\sin^2\phi}(\mathrm{C_l} \cos \phi - \mathrm{C_d} \sin \phi)$$

Elemental Torque $$dQ = \frac{q.c.r.dr}{\sin^2\phi}\left(\mathrm{C_l} \sin \phi + \mathrm{C_d} \cos \phi\right)$$

Blade Thrust $$T = q.B.\int_0^R \frac{c.dr}{\sin^2\phi}(\mathrm{C_l} \cos \phi - \mathrm{C_d} \sin \phi)$$

Blade Torque $$Q = q.B\int_0^R \frac{c.r.dr}{\sin^2\phi}(\mathrm{C_l} \sin \phi + \mathrm{C_d} \cos \phi)$$

Where, B is the number of blades

Thus, the net thrust and the torque are seen to be directly proportional to the number of blades, B and the chord, c.

***This is not quite true in practice*, as more the number of blades and wider the chords it has greater surface area, flow blockage and higher aerodynamic losses.**

Thus optimum number of blades need to be found separately and <u>not from the blade element theory</u>.

The blade element efficiency,

η_{el} = Thrust power produced / Torque power supplied

$$\eta_{el} = \frac{v.dT}{2\pi n.dQ} = \frac{V}{2\pi nr}.\frac{C_l \cos\phi - C_d \sin\phi}{C_l \sin\phi + C_d \cos\phi}$$

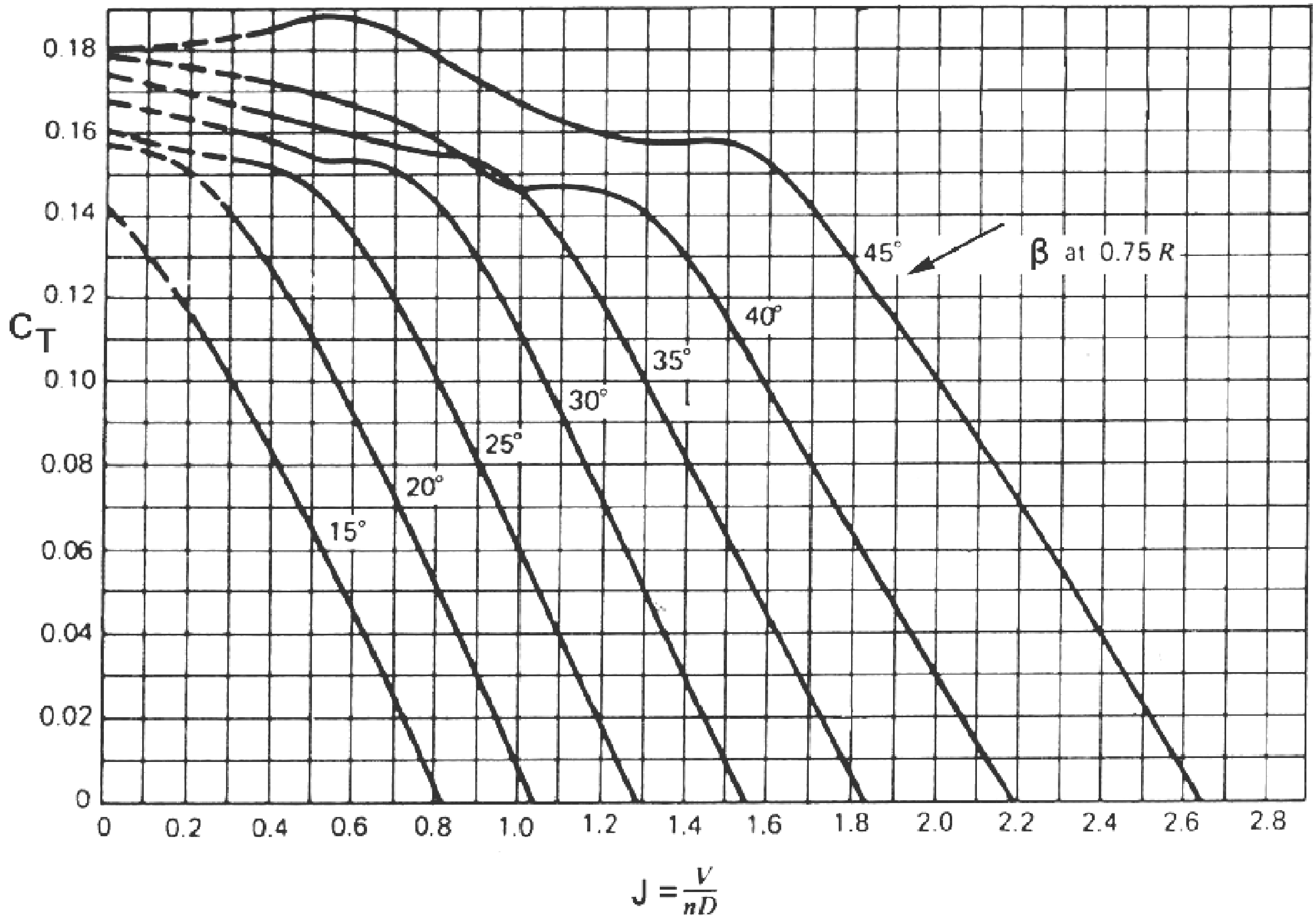
$$= \frac{C_l \cos\phi - C_d \sin\phi}{C_l \sin\phi + C_d \cos\phi}.\tan\phi$$

Applying maxima condition it can be shown that maximum efficiency, $\eta_{el\text{-}max}$ occurs at

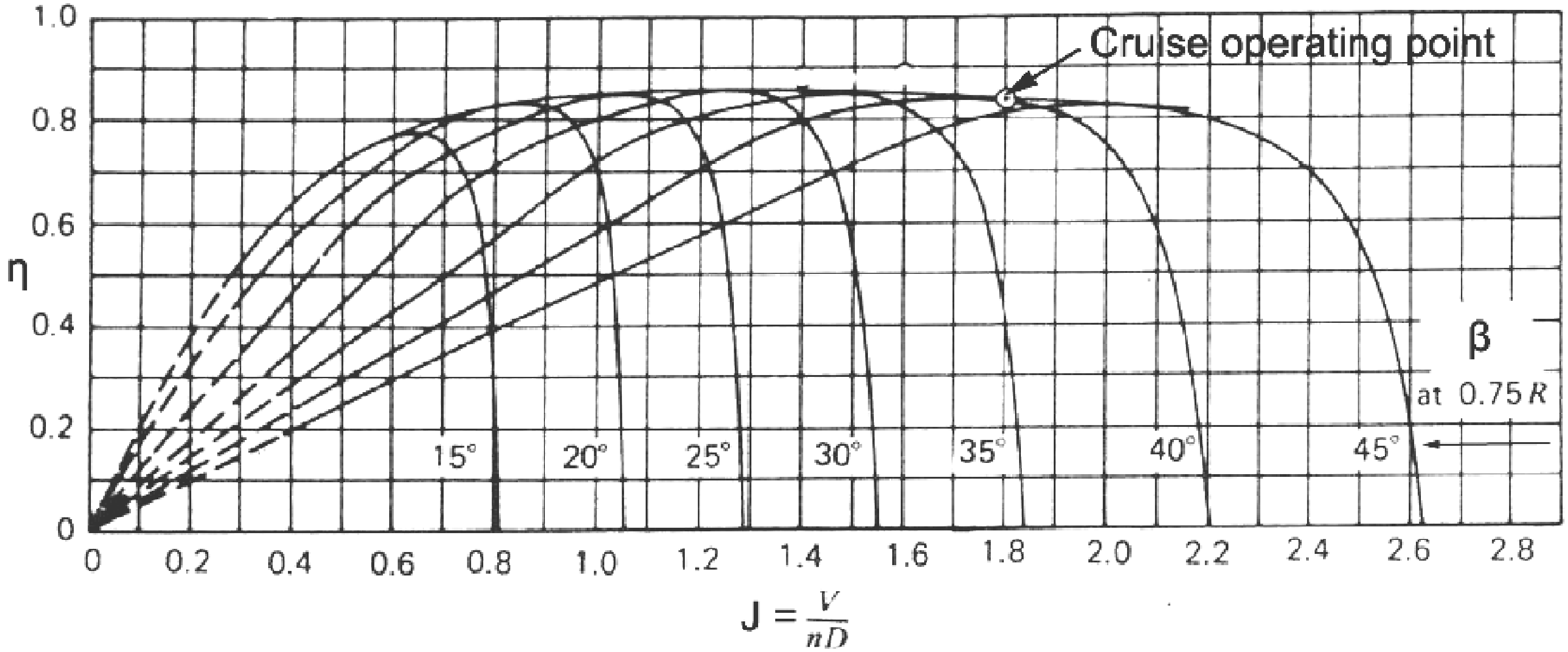
$$\phi = \frac{\pi}{4} - \frac{C_d}{2.C_l}$$

for a blade element airfoil characterized by its C_d & C_l

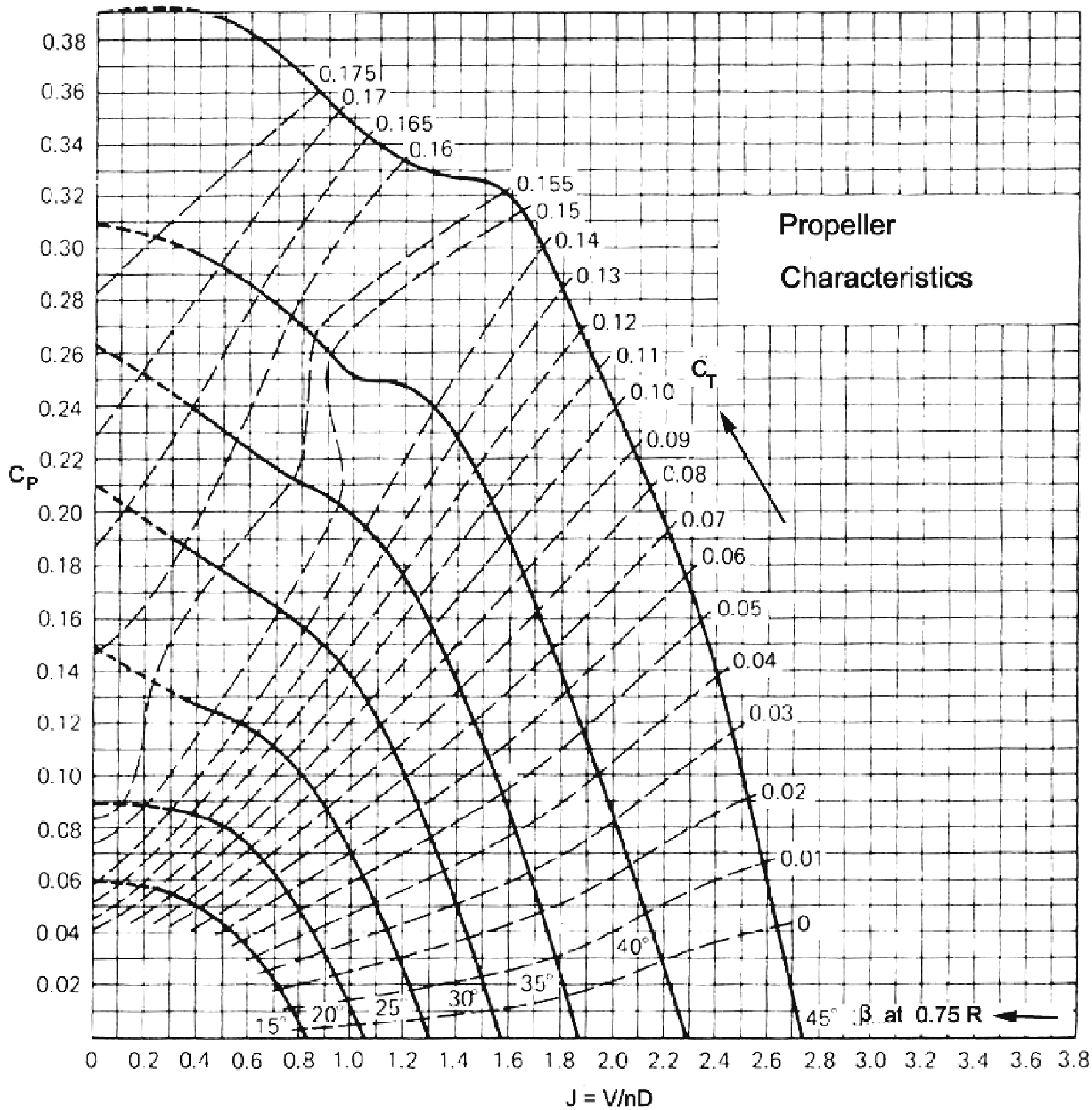
The estimations from simple 2-D blade element theory is within 10% of the actually obtained performances. The theory as stated above does not account for interaction between various cross-sections and does not incorporate secondary flow effects and tip flow, present in any propeller operations. However, it provides a quick approximate estimation of propeller performance, when propeller aspect ratios (span/chord) are quite high (>6).



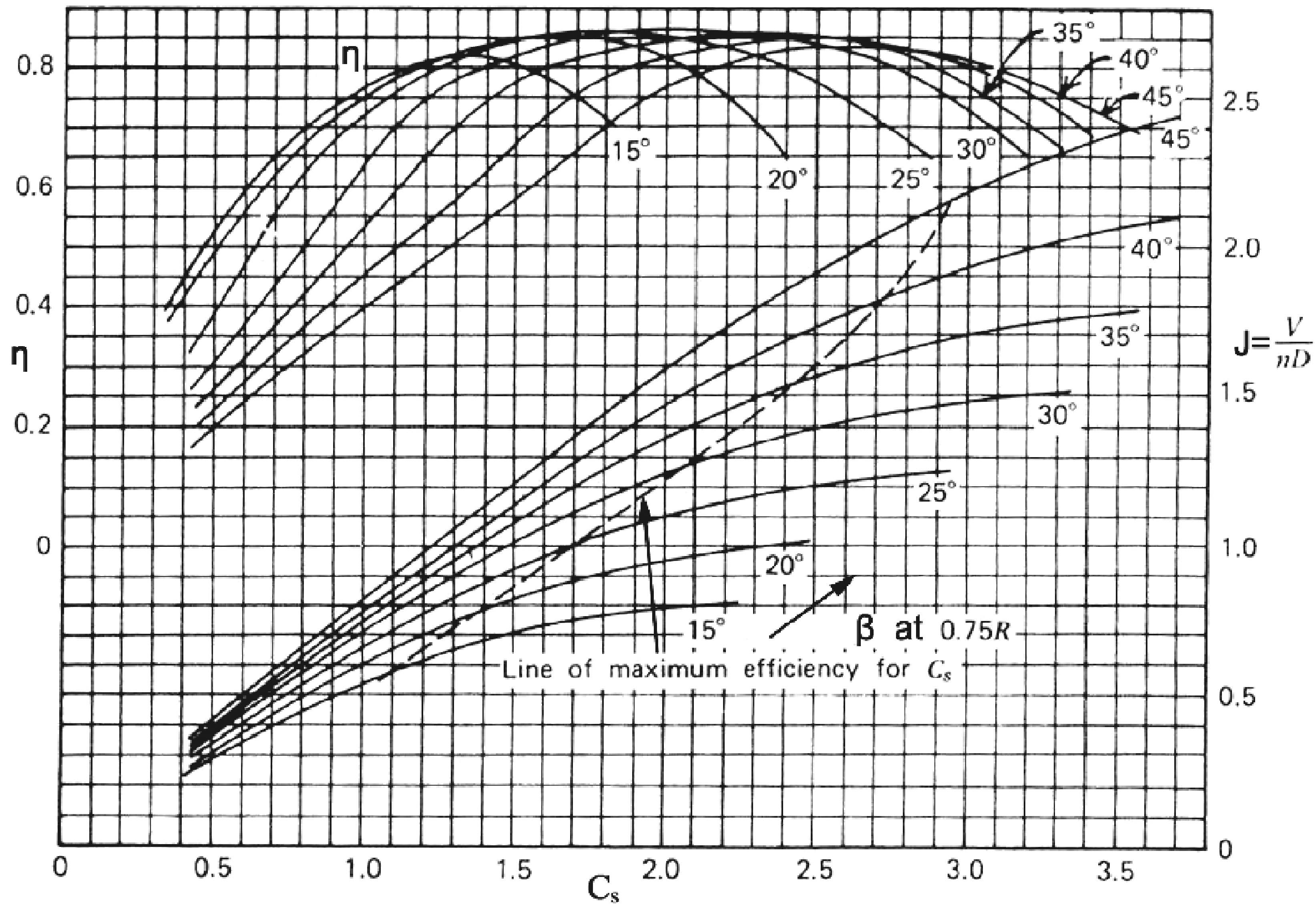
Thrust coefficient curves for a NACA propeller



Efficiency curves for a typical propeller with Three Blades.



Power coefficient curves for a typical propeller



Cs, the speed power coefficient is used for first cut propeller selection

Cs, the speed power coefficient, defined by,

$$C_s = (\rho.V^5/P.n^2)^{1/5}$$

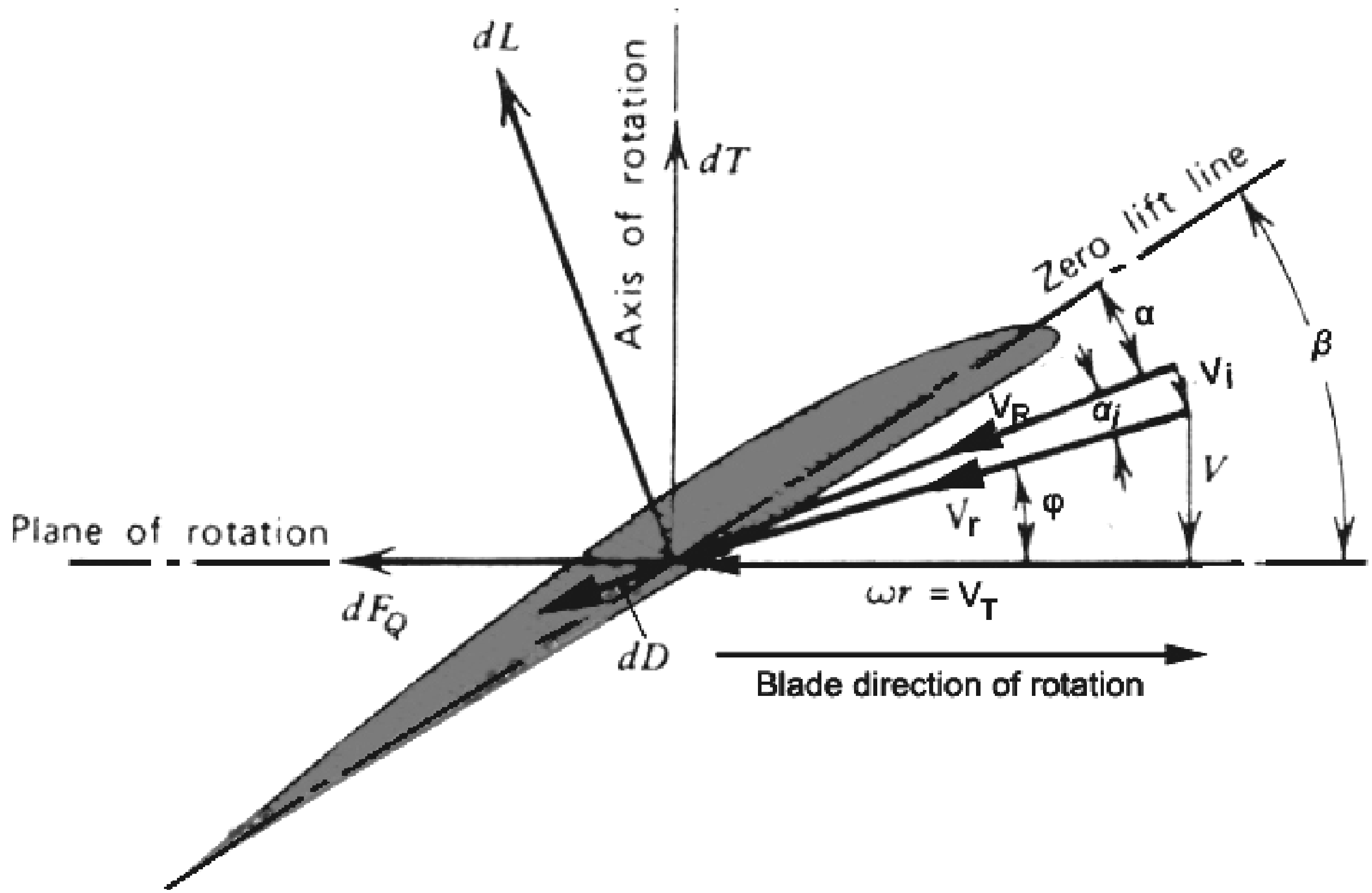
Knowing C_p as a function of J , C_s can be calculated from

$$C_s = J/C_p^{1/5}$$

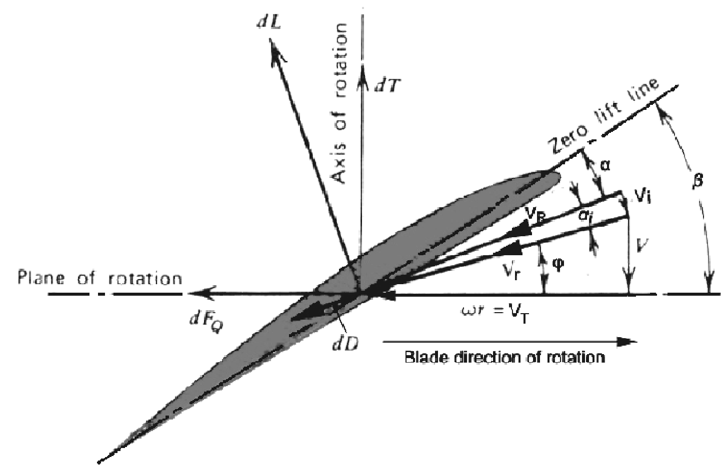
The *usefulness of C_s is in the process of defining it -- the diameter was eliminated*

Advanced (or Revised) Blade element theory

Flow over airfoil shaped bodies experience an induced flow that finally is seen at the trailing edge as an effect popularly known as *downwash.* Overall effect of the induced flow changes the lifting strength, L, and hence the strength of the vortex (or circulation) around the airfoil. The effect of this induced flow is felt immediately upstream in flow past the blades. This results in change in both the direction by α_i from $(\alpha+\alpha_i)$ to α , and the magnitude of the inlet resultant velocity, from V_r to V_R by an induced velocity v_i (considered perpendicular to V_R). Thus, final angle of attack, $\alpha <(\alpha+\alpha_i)$ and final velocity, $V_R < V_r$. Inflow angle changes from φ to $\varphi + \alpha_i$. These changes are all captured in the model presented below (Fig.).



Revised blade element velocity and force diagram



From the figure the contribution of a blade element to the blade thrust T & the torque Q, are

$$dT_{el} = dL \cos(\varphi + \alpha_i) - dD \sin(\varphi + \alpha_i)$$

$$dQ_{el} = r.\, dF_Q = r.\,[dL \sin(\varphi + \alpha_i) + dD \cos(\varphi + \alpha_i)]$$

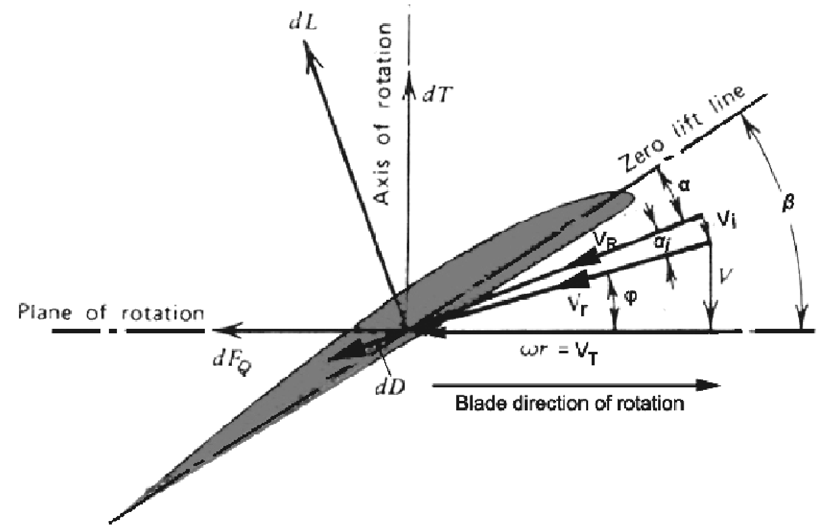
Where,

$$dL = \tfrac{1}{2}\,.\rho.V_R^{\,2}.c.C_l\,.dr \quad ; \quad dD = \tfrac{1}{2}\,\rho\, V_R^{\,2} c.C_d\, dr$$

$$C_l = a\,(\beta - \varphi - \alpha_i)\,,$$

where, *a* is the lift-curve slope (C_l *vs* α) of the aerofoil.

- **Effect of the induced flow is the induced angle α_i.**



- **<u>Induced flow angle *α_i* is unknown quantity</u>, without which the thrust and the torque estimation, (i.e. revised from the *simplified blade element theory)*, is not possible.**

- **Thus the first task at hand is to find out a method of estimation of the induced flow effects. To do that we need to adopt a few simplifications first.**

- **If we assume α_i and the drag to lift ratio, C_d / C_l are very small, then $V_R = V_r$,**

then

$$dT_{el} = B.\ \rho\ /2.\ V_r^{\ 2}\ .c.\ a.\ (\beta - \varphi - \alpha_i\ ).\ \cos\varphi.\ dr$$

Applying momentum principles to the differential annulus and letting $V_i = V_r.\ \alpha_i$

$$dT_{el} = \rho\ (2\pi.r\ .r)(V+V_r\ \alpha_i\ .\cos\varphi\ )2.v_i\ .\cos\varphi$$

Equating the two above equations yields the general equation involving the induced angle at an element,

$$\alpha_i^2 + \alpha_i\big(.\lambda\ /x + (\sigma.a.V_r/8.x.2V_T)\big) - \sigma.aV_r/8.x2.V_T.(\beta - \varphi) = 0$$

where, by definition

$\lambda = V/\omega.R$

$V_r = V_T\sqrt{(x^2+\lambda^2)}$

$v_{ia} = V_i\ Cos\ (\varphi + \alpha_i)$

$\sigma = B.c/\pi.R$

$\tan\varphi = \lambda/x$

$v_{it} = V_i\ Sin\ (\varphi + \alpha_i)$

$V_T = \omega.R$

$x = r/R$

$\tan\alpha_i = V_i/\ V_R$

Induced angle of attack on the element

$$\alpha_i = \frac{1}{2}\left\{-\left(\frac{\lambda}{x}+\frac{\sigma.a.V_r}{8.x^2.V_T}\right)+\sqrt{\left(\frac{\lambda}{x}+\frac{\sigma.a.V_r}{8.x^2.V}\right)^2+\left(\frac{\sigma.a.V_r}{2.x^2.V_T}\right)(\beta-\varphi)}\right\}$$

Now by definition

Thrust coefficient, $C_T = T/\rho n^2 D^4$

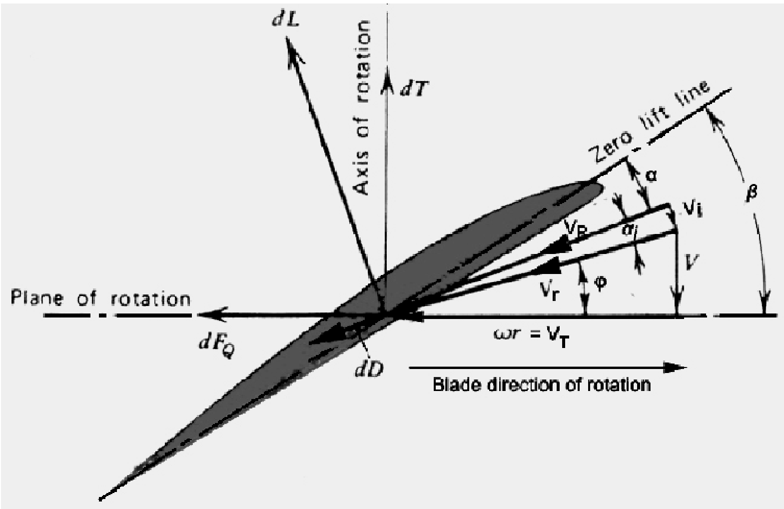
Power coefficient, $C_P = P/\rho\, n^3 D^5$

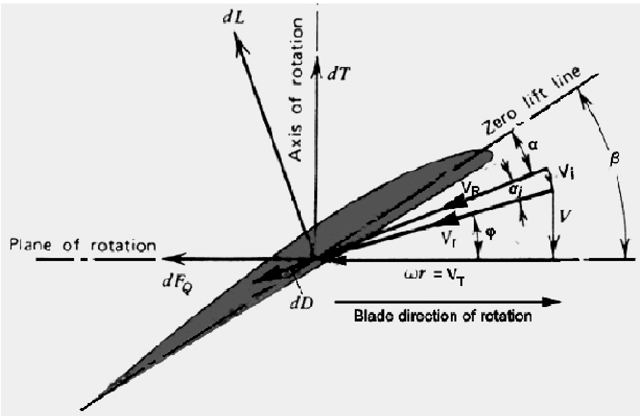
where, nD is called *reference vel.*

(*in lieu of* V_∞), and

D^2 is the *reference area*

(*in lieu of* A)





The angle of the resultant flow,

$\varphi \;\; = \tan^{-1}(V/\omega.r) = \tan^{-1}(J/\pi.x)$

where, *J is the advance ratio*, $J = V/nD$

Now, if we write $\varphi_o = \varphi + \alpha_i$; *and*

$V_e = V_r.\cos\alpha_i = 2\pi r.n.\cos\alpha_i / \cos\varphi$; *and*, $q = ½\ \rho.\ V_e^2$

For B no. of blades , the elemental thrust and torque are

$$dT_{el} = q.c.B.dr.(C_l \cos\varphi_o - C_d \sin\varphi_o)$$

$$dQ_{el} = q.c.B.dr.(C_l \sin\varphi_o + C_d \cos\varphi_o)$$

if we also substitute, (to encapsulate the flow parameters)

$\Psi_T = (\cos\alpha_i / \cos\varphi)^2.(C_l \cos\varphi_o - C_d \sin\varphi_o)$ *and*,

$\Psi_Q = (\cos\alpha_i / \cos\varphi)^2.(C_l \sin\varphi_o + C_d \cos\varphi_o)$ *and*,

using all the equations and definitions contained in the above Eq.s we get the elemental thrust and torque as:

$$dT_{el} = ½.\ \rho.(2.\pi.n)^2\ c.B.R^3.x^2.dx.\Psi_T$$

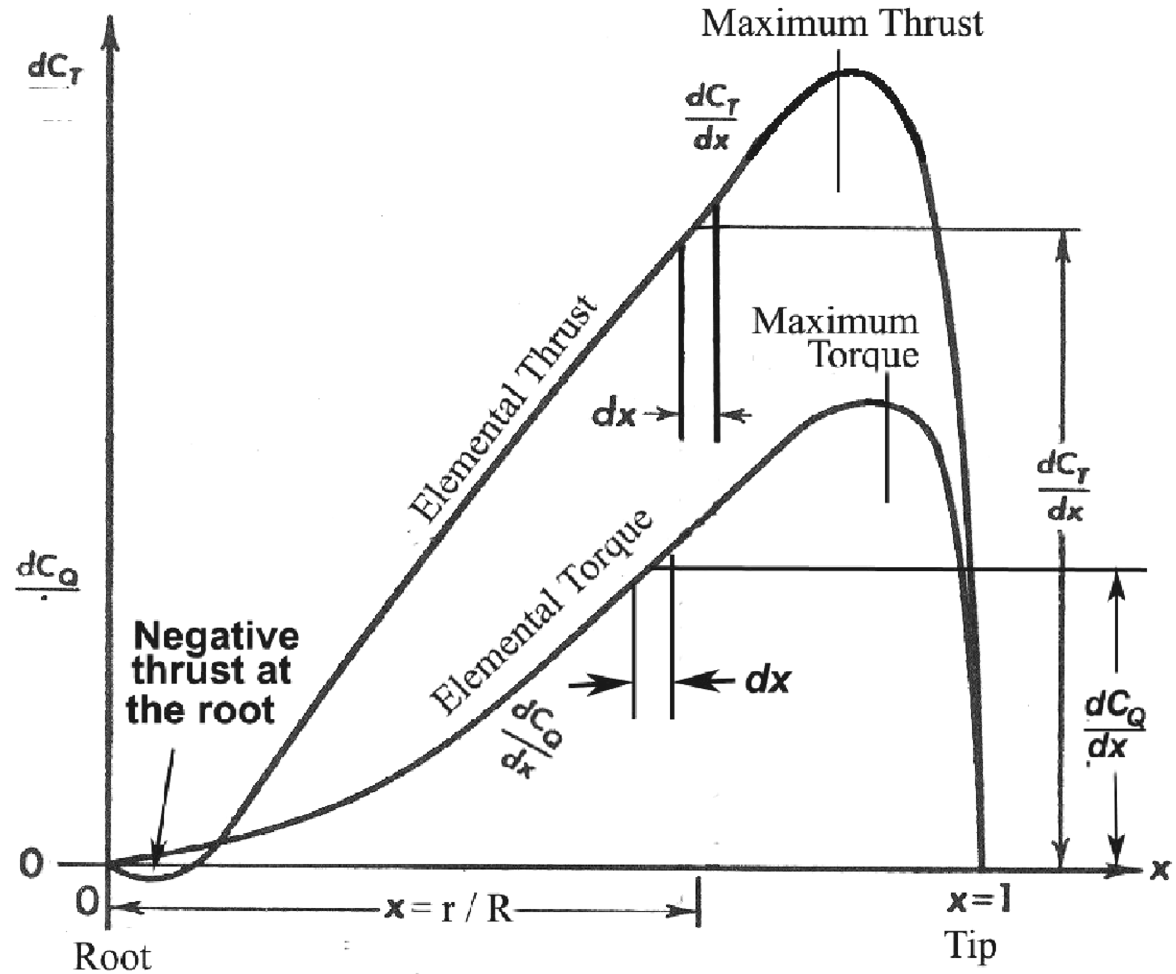
$$dQ_{el} = ½\ .\rho.(2.\pi.n)^2 c.B.R^4.x^3.dx.\Psi_Q$$

The elemental thrust and the elemental torque coefficients can now be written down in terms of geometric and flow parameters as

$$\frac{dC_T}{dx} = .\frac{c.B.\pi^2.x^2}{8R}\Psi_T = 3.88.x^2.\sigma.\Psi_T$$

$$\frac{dC_Q}{dx} = \frac{c.B.\pi^2.x^3}{16.R}\Psi_Q = 1.94.x^3\sigma.\Psi_Q$$

Which give the radial variation of the Thrust & Torque coefficients . Every propeller blade shape is characterized by such radial variation. In modern propeller blade designs such radial variations are arrived at first as design requirement and then the blade shape is found, often using CFD techniques. Such a design, is known as '**direct solution****'.**


Maximum Thrust
$\frac{dC_T}{dx}$
$\frac{dC_T}{\cdot}$
Elemental Thrust
Maximum Torque
dx
$\frac{dC_T}{dx}$
$\frac{dC_Q}{\cdot}$
Elemental Torque
Negative thrust at the root
dx
dC_Q/dx
$\frac{dC_Q}{dx}$
0
0
$x = r / R$
x
$x=1$
Root
Tip

Integrating them along the blade length gives the propeller Thrust and Torque Coefficients.

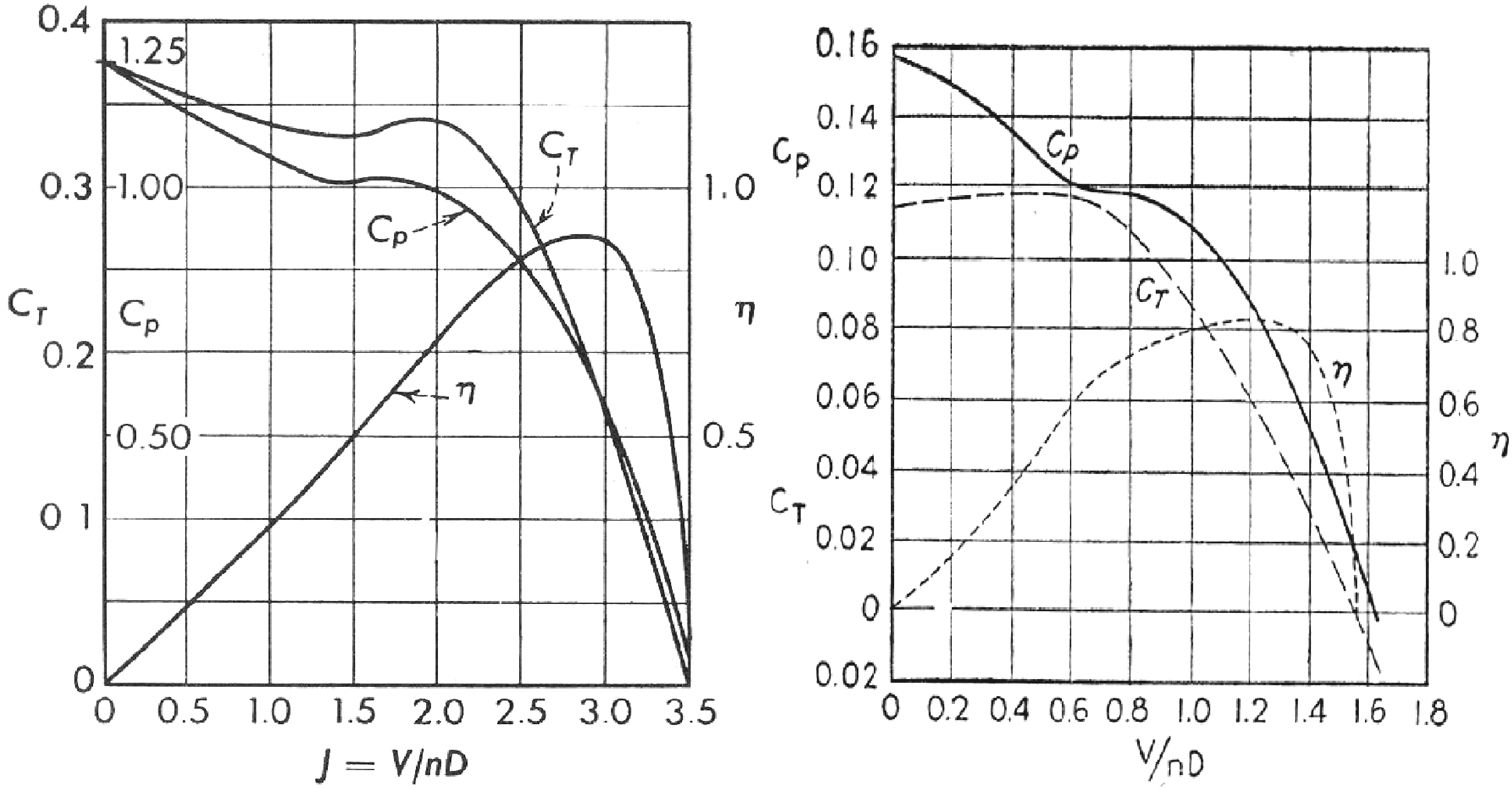
$$C_T = \int_0^1 \frac{dC_T}{dx} dx \qquad C_Q = \int_0^1 \frac{dC_Q}{dx} dx$$

In dimensionless full form the thrust and the torque coefficients are related to C_l and C_d

$$C_T = \pi/8 \int (J^2+\pi^2 x^2).\ \sigma.\ [C_l .\cos(\varphi + \alpha_i) - C_d .\sin(\varphi + \alpha_i)].dx$$

$$C_P = \pi/8 \int (J^2+ \pi^2 x^2)\ \sigma.\ [C_l .\sin(\varphi + \alpha_i) + C_d .\sin(\varphi + \alpha_i)].dx$$

Typical variations of C_T and C_P, as determined by controlled experiments for specific propellers



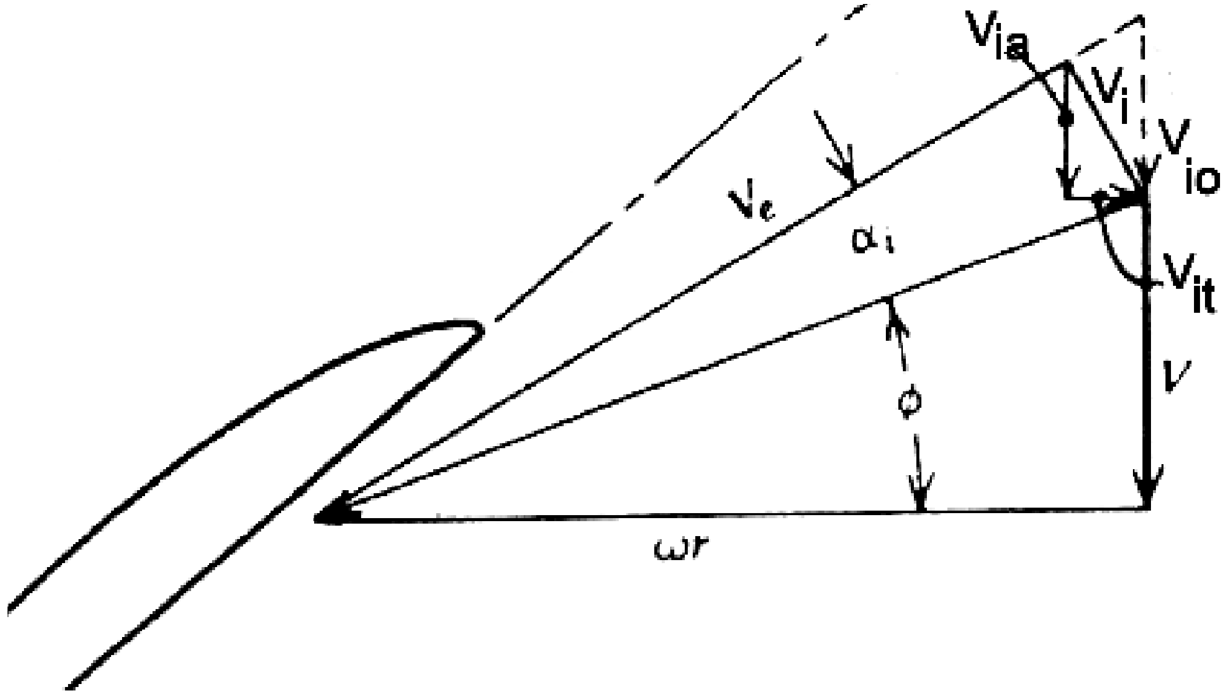


Fig. shows the induced velocity at the propeller plane in more detail. v_i is normal to the resultant velocity Vr and has a tangential component, v_{it} and an axial component, v_{ia}. From the eqn.s we can also solve for v_i the induced velocity, and α_i the induced angle of attack

The induced effects are thus captured as :

$$\boldsymbol{V_i = \frac{B.c.C_l.V_R}{8\pi.r.\sin(\phi + \alpha_i)}}$$

$$\alpha_i = \frac{\beta - \phi}{1 + \dfrac{8.x.\sin\phi}{\sigma.a}}$$

--- assuming α_i to be small