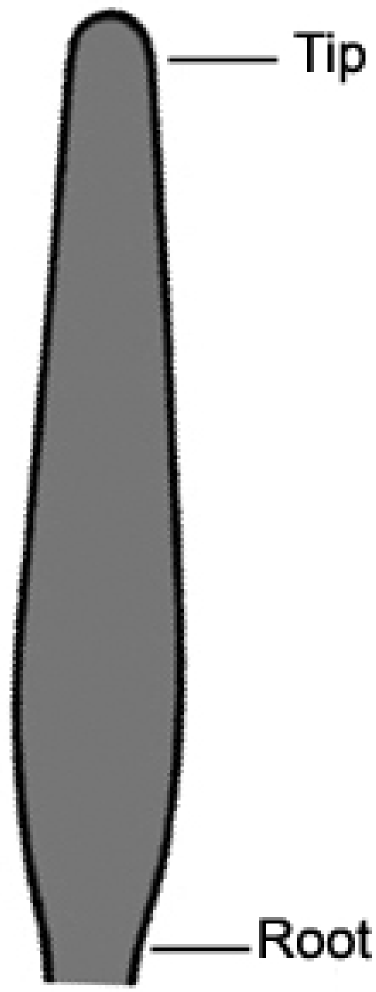
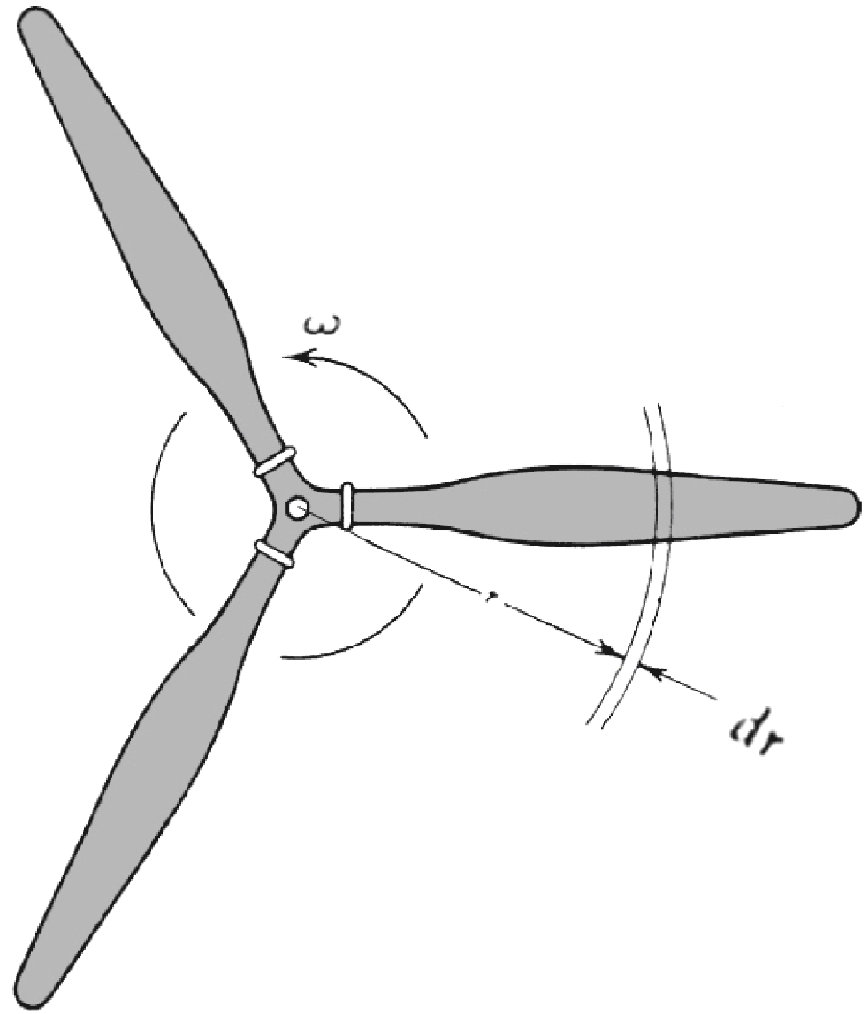
# AE 658

## Chapter – 1

# Propeller Theory and Design




Blade Planform

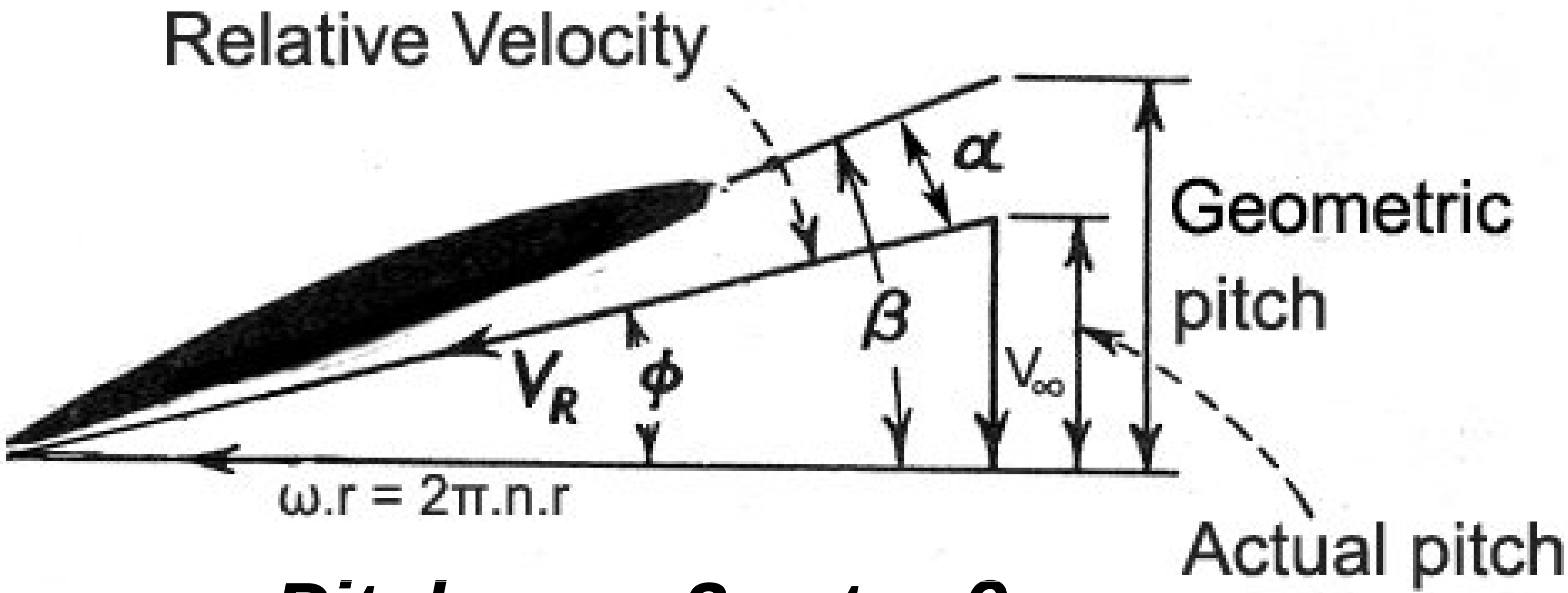



3-bladed propeller front view

**Two basic propeller theories**

1) **Blade Element Theory**

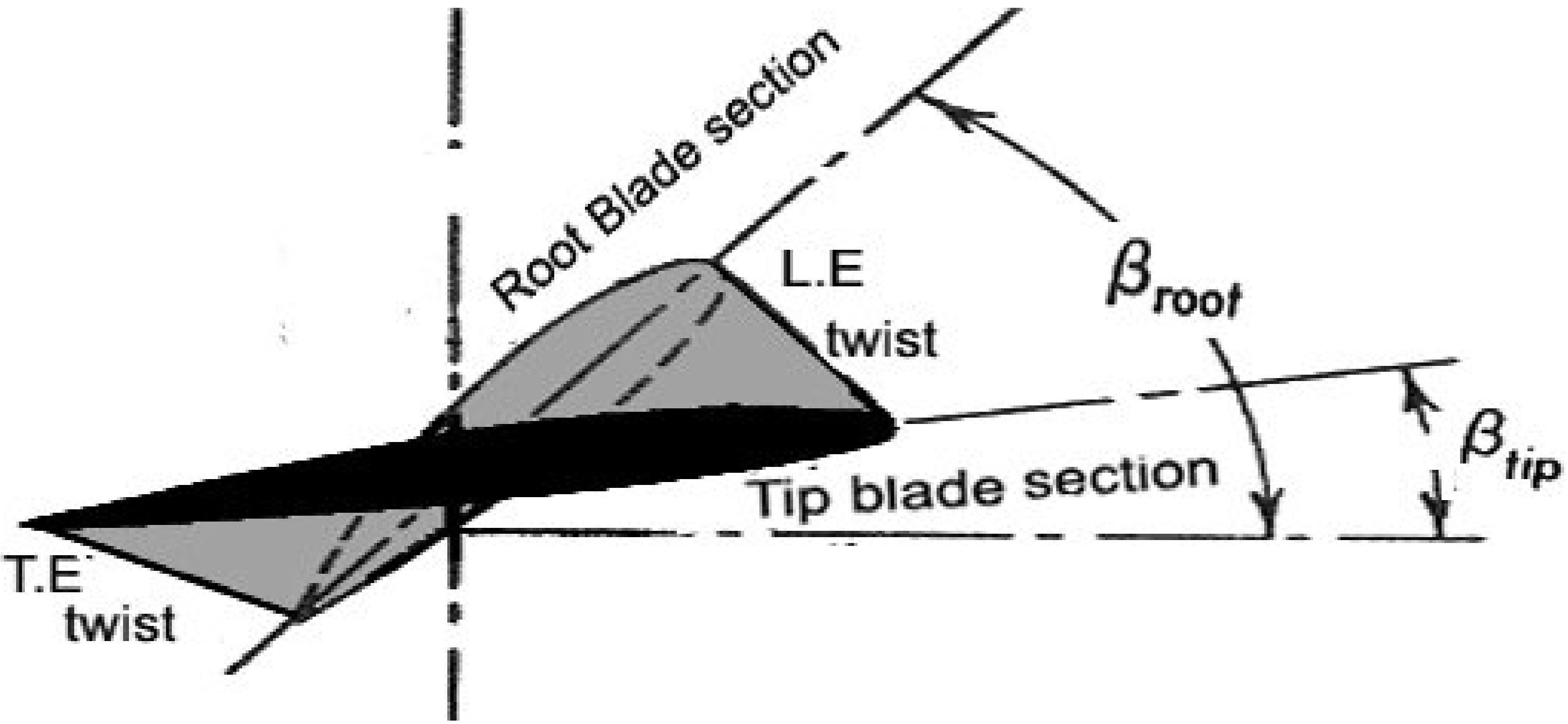
2) **Actuator Disk Theory**




***Pitch , p = 2πr.tanβ***

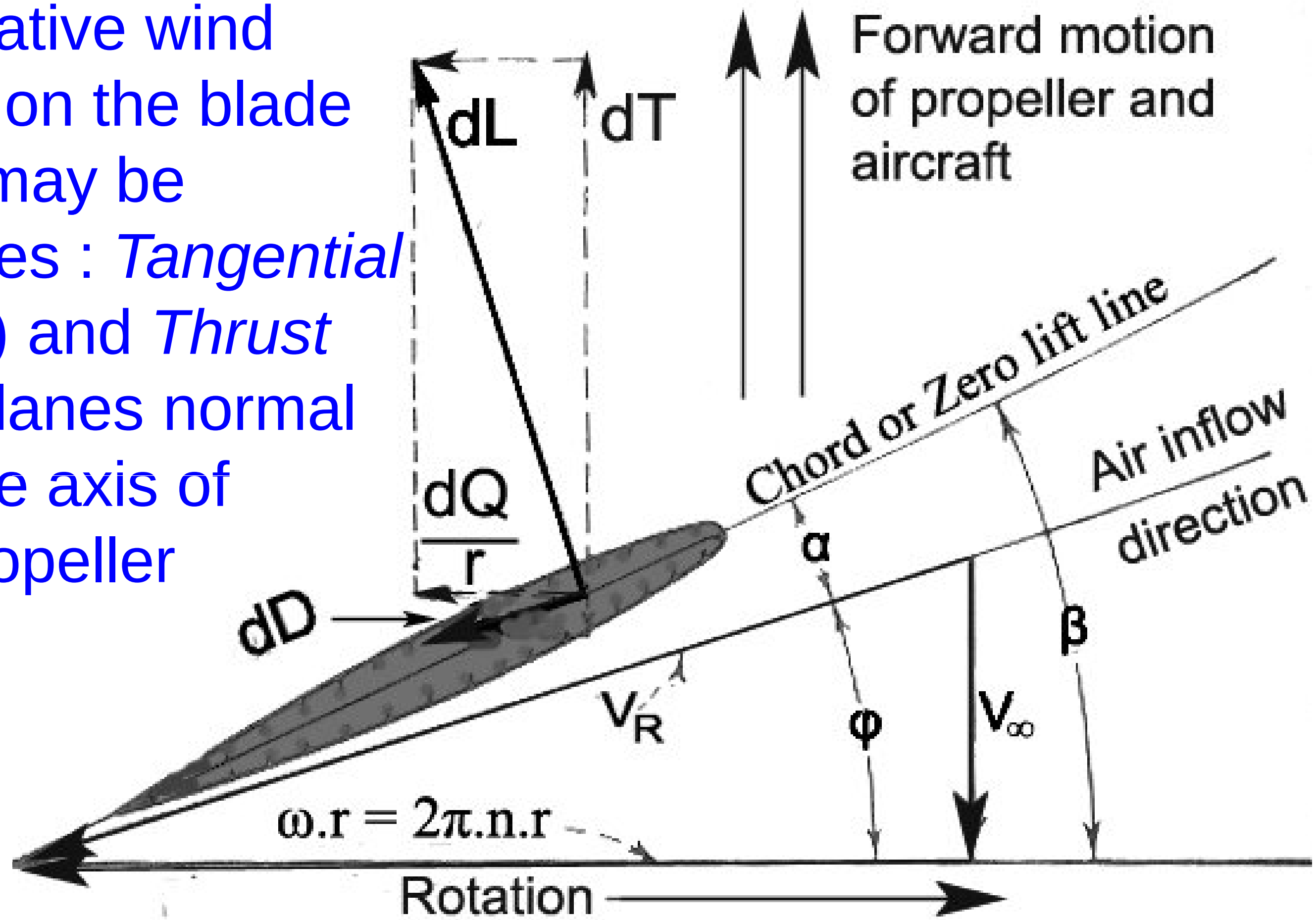
**The difference between the *geometric pitch*, p , as defined above, and the *actual or effective pitch*, as the entire solid body of the blade move in unison, is called Slip**

**Angle of Attack, α ; blade setting angle, β ; effective pitch angle (flow angle), φ ; forward speed, $V_\infty$,**




**The pitch setting needs to be varied from hub to tip so as to maintain the best AoA for each blade element**

The lift and the drag of a blade element are perpendicular and parallel to the relative wind direction coming on the blade element. These may be projected as forces : *Tangential force* (for Torque) and *Thrust* (axial force), in planes normal and parallel to the axis of rotation of the propeller respectively.



**The *Advance ratio*, J is defined as:**

$$J = V_\infty / (n D).$$

**where,**

**$V_\infty$ - Forward speed (of propeller /aircraft), m/s,**
**$n$ - Rotational speed, rps; and**
**$D$ - Propeller diameter, m.**

**The main performance parameters thrust T, (produced by the propeller), torque, Q and power, P (required to be supplied to maintain operation at a given rpm, n and atmospheric air density, *ρ*), may be defined as follows,**

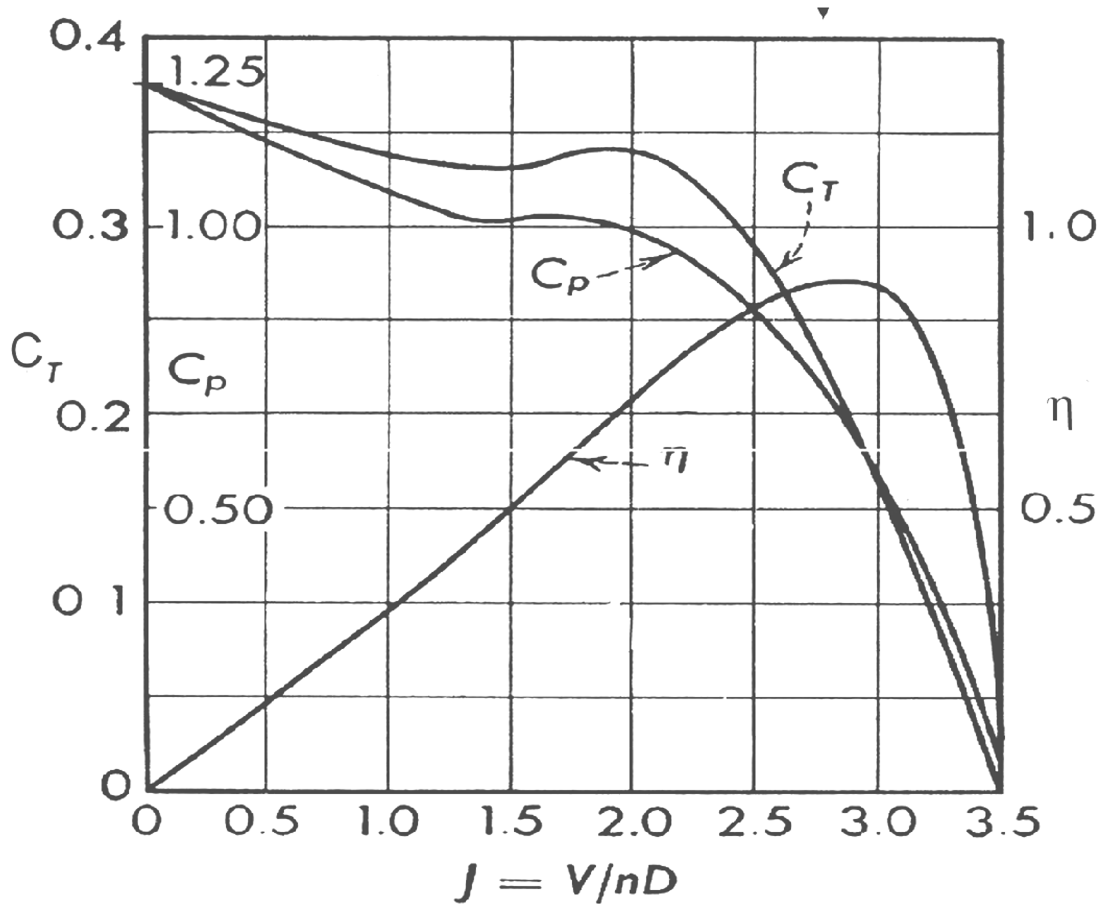
$$T = \rho . n^2 . D^4 . C_T$$

$$Q = \rho . n^2 . D^5 . C_Q$$

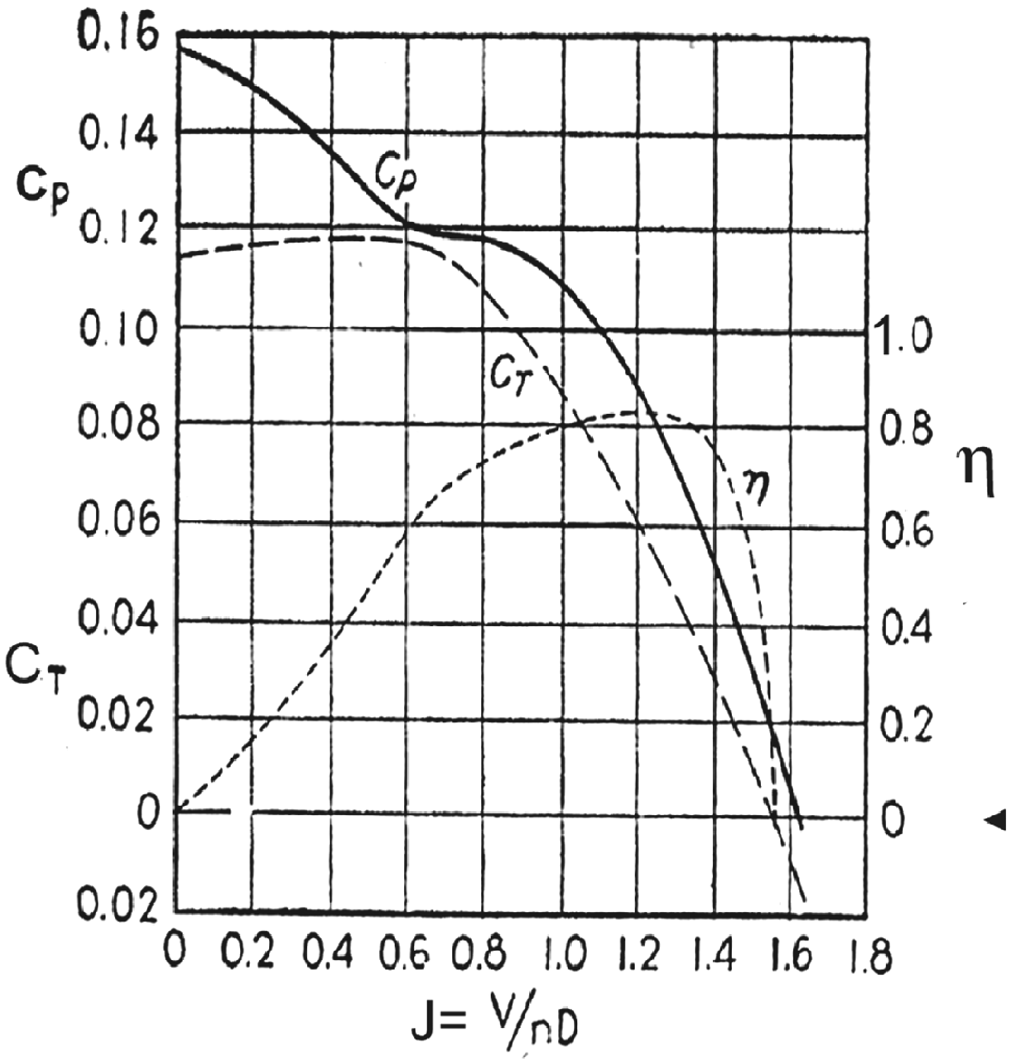
$$P = \rho . n^3 . D^5 . C_P$$

**where, $C_T$, $C_Q$, $C_P$ are the *thrust, torque and power coefficients* of the propeller**

**Variation of the coefficients $C_T$, $C_Q$, $C_P$ with advance ratio, $J$ and blade pitch angle, β are plotted to make the 'characteristic maps' of a propeller**

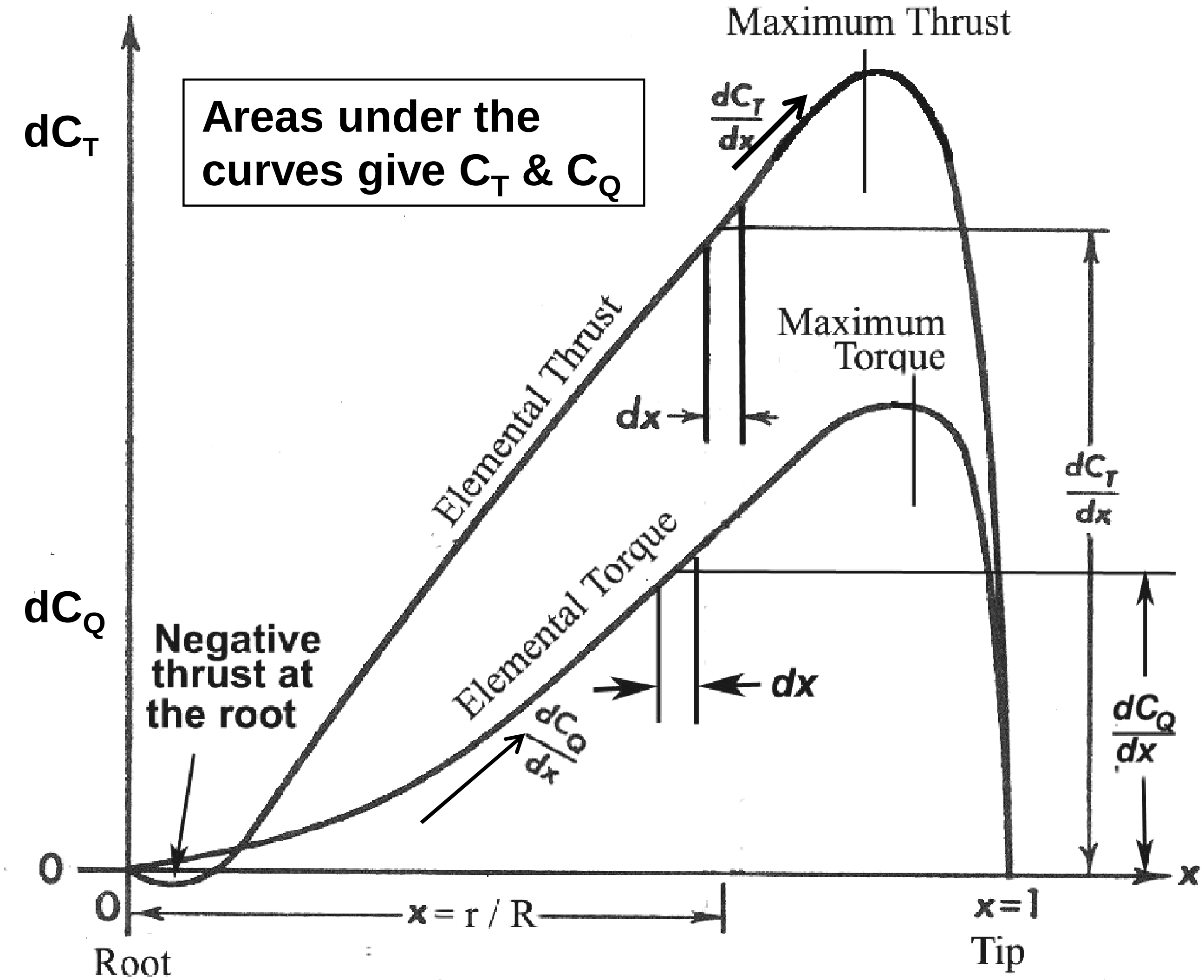



**Typical High speed Aircraft (M=0.65) Propeller Characteristics Map**




**Typical Low Speed (M<0.4) Aircraft Propeller Characteristics Map**

**The maps may be used for <u>propeller selection</u>, <u>performance prediction</u> and for <u>in-flight propeller control</u>, for which it is built into the control system of the propeller**




**Maximum Thrust, Power, Torque or Efficiency do not occur at the same operating condition**

**• Matching of the propeller with the 'driver' engine is required to be achieved for both the torque and the power, to maintain the desired rotating speed.**

**•Only when the desired power and torque are supplied by the engine that the desired thrust production is ensured.**

**•At low advance ratio during take off and climb the power supply requirement is high, but due to fine pitch setting the torque requirement is low – and for both the reasons it is scheduled to run at higher speed.**

**•On the other hand during cruise the power supply requirement is often much lower, and runs at lower rpm but due to high pitch setting torque requirement is high,.**

The propeller efficiency is given by the usual output power to input power ratio,

$$\eta_P = (T\,V_\infty)\,/\,P = (T\,V_\infty)\,/\,(2\pi\, nQ)$$

Thus, $$\eta_P = J.C_T\,/\,C_P$$

Where, $$C_P = 2.\pi.C_Q$$

From equation (a) it can be shown that for a constant power input and a constant efficiency, the thrust generated by the propeller is proportional to inverse of the velocity. This means that the thrust would, mathematically, go to infinity as the forward velocity nears zero. Conversely, efficiency approaches zero at near static condition, typical of aircraft take-off condition. These singularities do not happen in real operation, and the theoretical ‘possibilities’ are overruled by practical real flow situations.

**In general the larger the propeller diameter is, the greater the propeller efficiency. But the larger the propeller diameter is, the higher the propeller tip speed for a given rpm and air speed. The propeller tip speed is given by,**

$$V_{tip,\ helical} = \sqrt{[(\pi nD)^2 + V_\infty^{\ 2}]}$$

**At high tip speeds, compressibility effects are pronounced and eventually shocks may be present; the shock losses reduce the lift and increase the drag for those blade section elements, unless the blade elements at the tip are designed to operate at transonic relative air speeds. The structural load in terms of centrifugal load and aerodynamic shear and bending loads also increase with local flow Mach number.**

- **The *design advance ratio* is normally chosen after careful matching of the propeller and the aircraft at a *design point*, where the flight conditions (flight Mach number, altitude, flight condition e.g. cruise / climb etc.) and the engine operating condition (engine speed, power, torque) are known.**

- **For maximum propeller efficiency at the design point AoA is set for nearly maximum lift-to-drag ratio.**

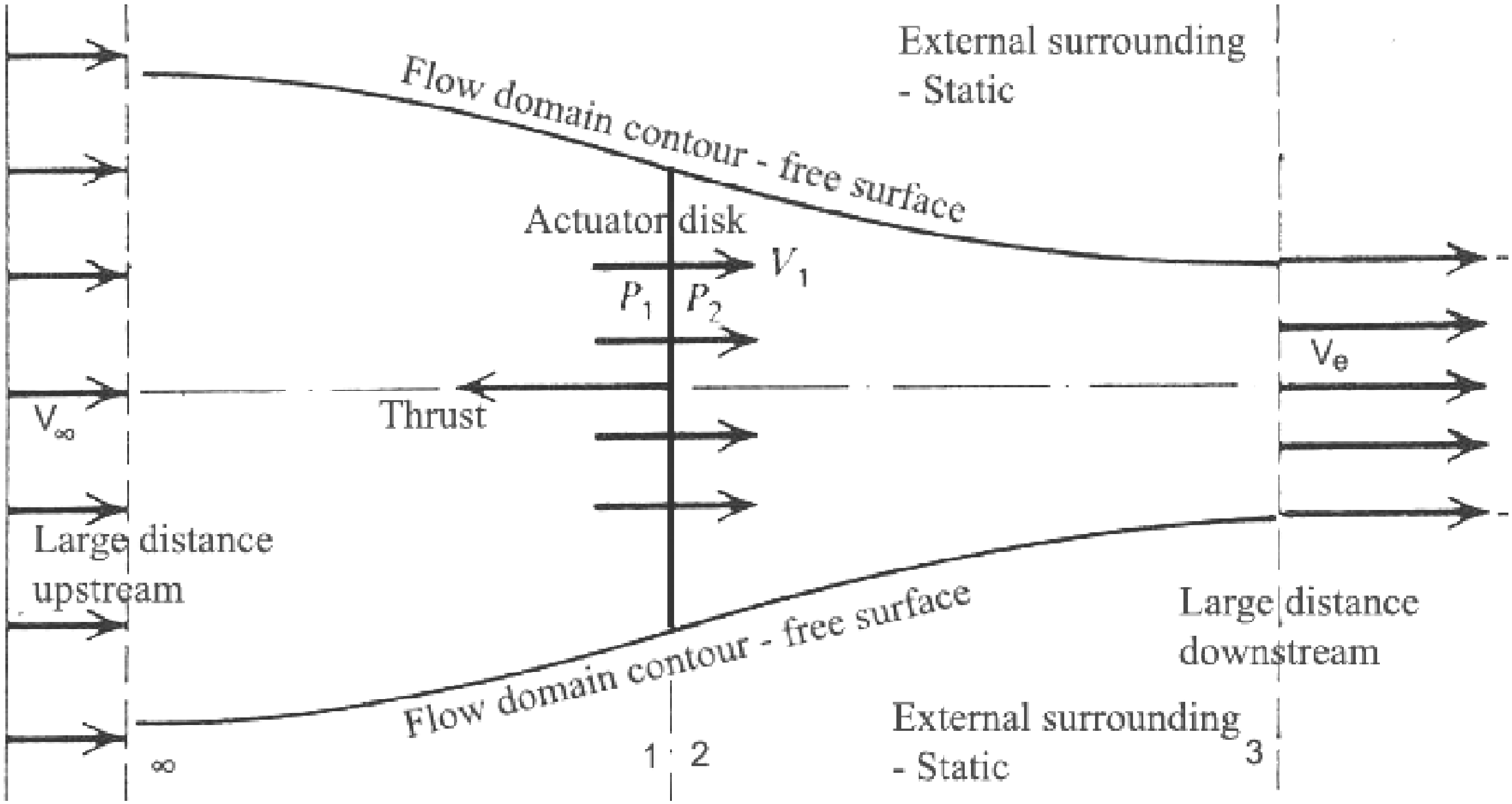
- **If the operating advance ratio is much lower than the design value, the propeller blades are likely to be partially or totally stalled; whereas, at much higher advance ratios, the propeller blades operate at very low efficiencies and may even produce negative thrust. <u>This is true if the propeller blade pitch setting angles are *fixed* or invariable during operation</u>.**

- **A *fixed pitch propeller*, in which the geometric pitch cannot be varied, must therefore be well matched to the various operating conditions of the engine on one hand and to the aircraft on the other hand.**

- **A *variable pitch propeller,* either variable on ground (manually) or in flight (through hydro-mechanical control system), offer two or more blade settings, (fine and course), so as to maximize the propeller efficiency according to the operating flight condition. For low advance ratios from take off to climb conditions fine pitch (low pitch setting angle), and for high advance ratios such as cruise conditions, course pitch (high pitch setting angle) are used.**

- **A *constant speed propeller* automatically changes propeller pitch according to a built in control law (floating pitch) so as to maintain proper torque such that the speed of the propeller shaft is maintained constant with the help of a governor and a electro-hydro-mechanical control system. This maximizes and stabilizes the propeller performance by allowing shaft carrying the propeller to rotate at a constant rpm. Most modern propellers are constant speed propellers.**

# Actuator Disk Theory (Momentum Theory)



1. **The propeller is replaced by an '*actuator disk*', which is assumed to be a flow energizer.**

2. **The 'disk' is assumed to be of infinitesimally small thickness and is conceptually a continuous but 100% porous body of no mass, having a total projected frontal area 'A' (known as *swept area*) equal to the area covered by the rotating propeller blades.**

3. **There is no 'resistance' (i.e. no drag) for the air passing through the 'actuator disk', (since there are no physical entities like blades offering resistance)**

4. **The axial velocity, $V_1$ through the 'disk' is uniform over the 'actuation' area and is considered to be smooth across the disk i.e. no abrupt changes are 'experienced'.**

**5. The received energy manifests itself in the working medium (i.e. air) finally in the form of differential pressure ($p_2$ – $p_1$), uniformly distributed across the disk surface.**

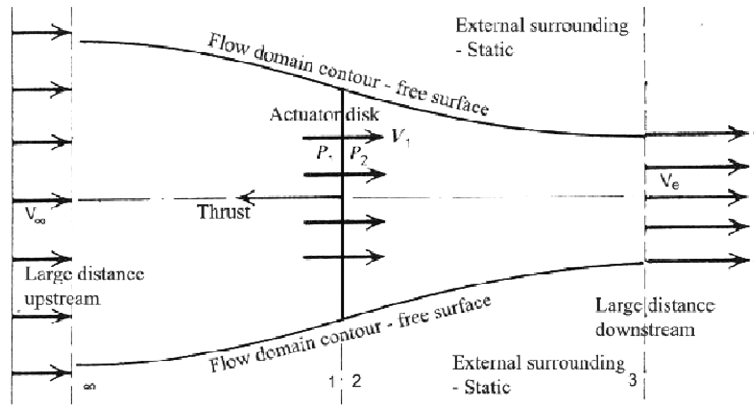
**6. The fluid medium, air, is a considered to be a perfect incompressible fluid.**

**7. Flow is ‘irrotational’ in front of and behind the disk, but not through it.** ***and***

**8. The static pressures far from the disk, i.e. far upstream and far downstream, are both assumed equal to the atmospheric pressure. The corresponding velocities are independent values to be determined separately.**

The mass flow passing thru' the disk (i.e. propeller) is given from continuity, as

$$\dot{m} = \rho.A.V$$




The thrust produced by the disk is found from Newton's 2nd and 3rd laws of motion from the effective change in momentum in air ---- resulting in reaction force, thrust.

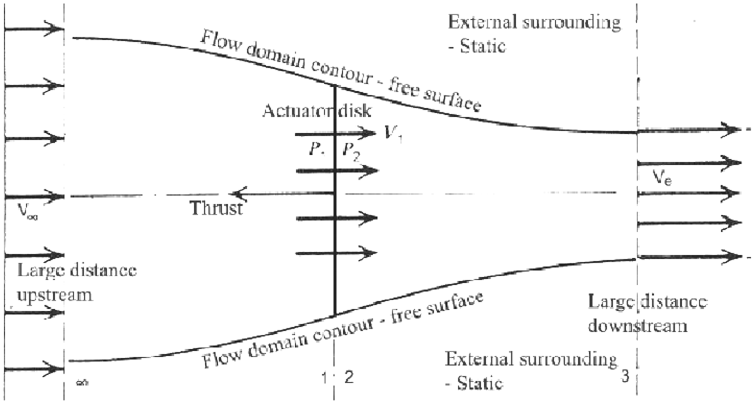
$$T = \dot{m}.\ \partial V \ = \ \rho.A.V.(V_e - V_\infty)$$

From simple physics, thrust is also produced by the differential static pressure on either side of the disk when multiplied by its projected surface area (swept area)

$$T = A\,(P_2 - P_1)$$

**Applying Bernoulli's equation separately on either side of the disk, but not through it, gives**

***[note : Bernoulli's theory is not valid if any energy is added within the flow domain.]***




$$P_\infty + \tfrac{1}{2}\rho V_\infty^2 = P_1 + \tfrac{1}{2}\rho V_1^2$$ -- **upstream of the actuator disk**

$$P_2 + \tfrac{1}{2}\rho V_2^2 = P_\infty + \tfrac{1}{2}\rho V_e^2$$ -- **downstream of the actuator disk**

**Therefore,**

$$P_2 - P_1 = \tfrac{1}{2}\rho.(V_e^2 - V_\infty^2)$$

**And from the above equations**

$$V_1 = \tfrac{1}{2}(V_e + V_\infty)$$

**thus, Thrust** $T = \tfrac{1}{2}\rho.(V_e^2 - V_\infty^2).A$

**The velocity at the disk comes out to be the *free stream axial velocity, $V_\infty$ + induced (axial) velocity (v)*, whereas, the *far downstream velocity is equal to the free stream velocity plus two times the induced velocity, v.***

$$V_1 = V_\infty + v \quad ; \quad \& \quad V_e = V_\infty + 2.v$$

**Therefore ,** $T = \rho A (V_\infty + v)\, 2.v = 2.\dot{m}.v$

**From the equation, the *induced velocity*, *v*, can be found as,**

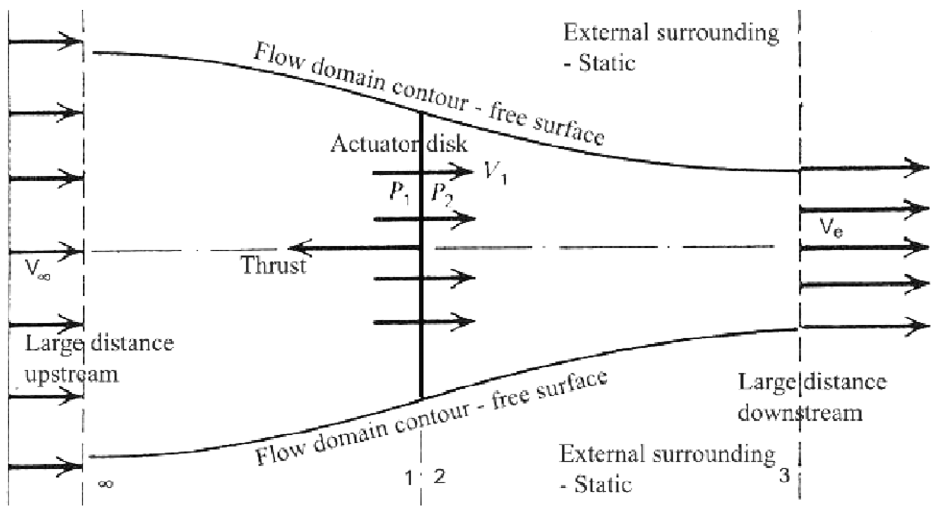
$$v = \frac{\left[ -V_\infty + \sqrt{\left\{ V_\infty^2 - \left(\frac{2T}{\rho.A}\right)\right\}} \right]}{2}$$

**For a static thrust, where the propeller is not in forward motion (at take off), $V_\infty$ is zero,**

$$v = \sqrt{\frac{T}{2\rho.A}}$$

**hence, power input for static thrust production**

$$P_{in} = T^{\frac{3}{2}} \sqrt{2\rho A}$$



**The ideal efficiency can be calculated by using classical definition of efficiency, $P_{out} / P_{in}$ . Power output is equal to thrust generated by the disk multiplied by velocity of the actuator disk through the air medium (flight velocity of the aircraft). The power input is thrust generated by the disk multiplied by the airflow velocity through the disk at the disk plane,**

$$P_{out} = T\,V_{\infty} \qquad \text{and} \qquad P_{in} = T.V_1$$

**Therefore,** $\eta_i = P_{out} / P_{in} = T.V_{\infty} / T.V_1$

$$= V_{\infty} / [\,\tfrac{1}{2}(V_{\infty} + V_e)] = 2V_{\infty} / (V_e + V_{\infty})$$

**Therefore,** $\eta_i = 1/ [1 + (v / V_{\infty})]$

**Efficiency from momentum theory is "*induced efficiency*"**

- **The induced efficiency is zero for zero forward velocity and approaches 1.0 as induced velocity, *$v$*, tends towards zero.**
- **The induced efficiency reaches a maxima but does not show any fall with increasing J (compared to η in Fig).**
- **Induced efficiency cannot be realized because of energy lost in rotational motion acquired by the flow in passing through the propeller.**
- **Losses due to non uniform thrust loading over the blade length.**
- **Blade interference losses due the interaction of flows over the neighboring blades.**
- **Propeller profile drag losses, stacked up over all the blade sections, *and***
- **Changes in flow properties due to compressibility not accounted for.**