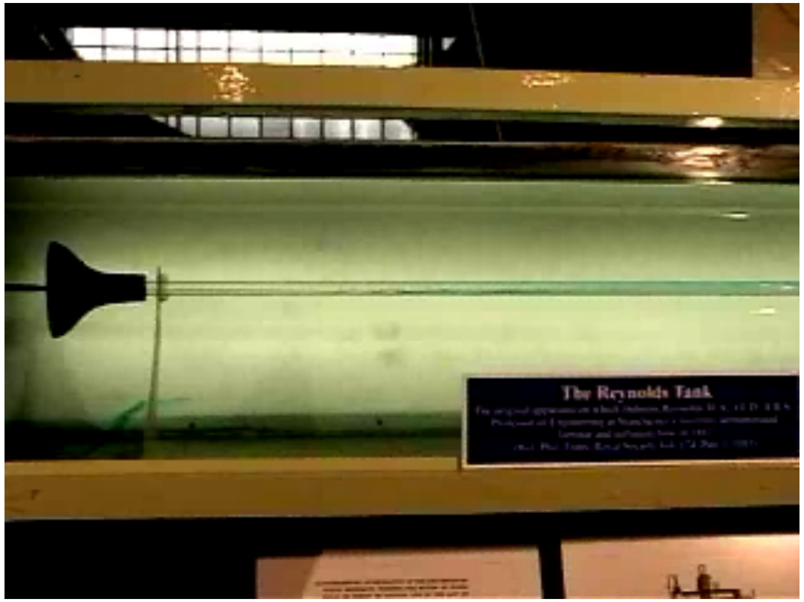
Lecture 1: Essentials of Turbulence
ME 724

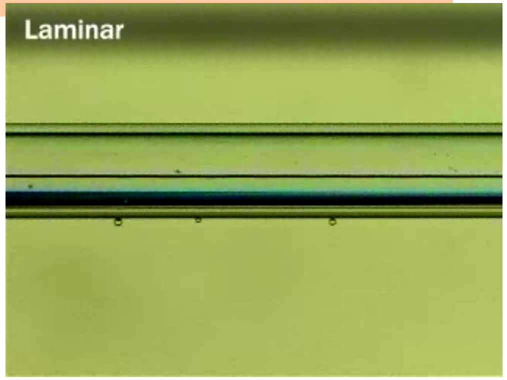
Amitabh Bhattacharya

What Is Turbulence ?

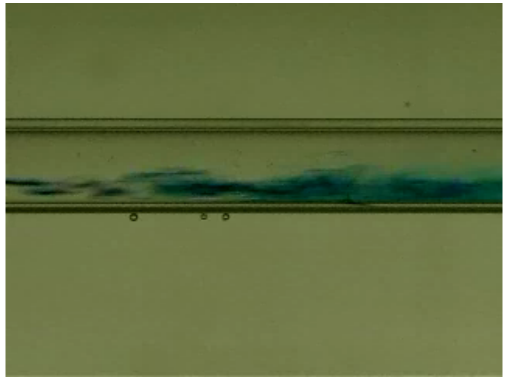
- First systematic study by Osborne Reynolds (1883)
  - Increase "Reynolds number" Re=UD/ν



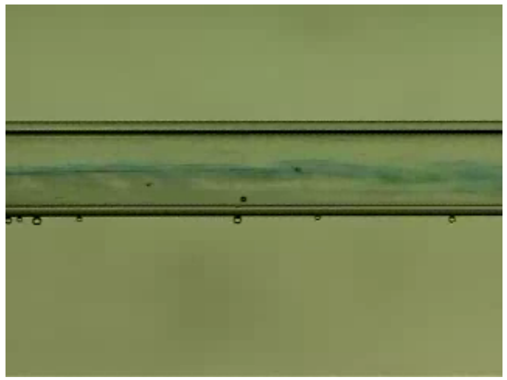
Laminar



Transition

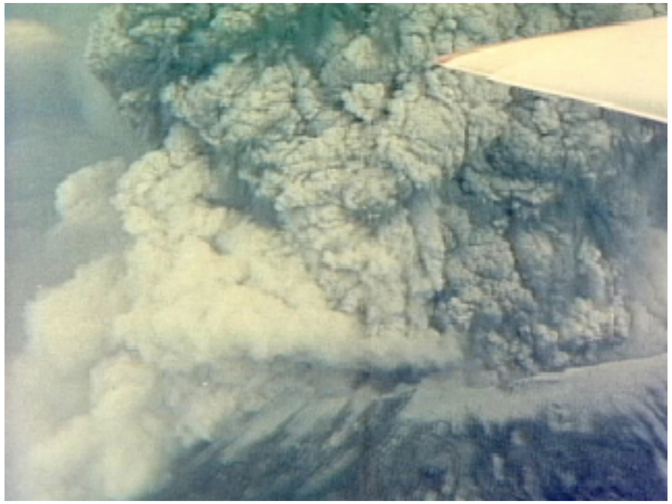


Turbulent



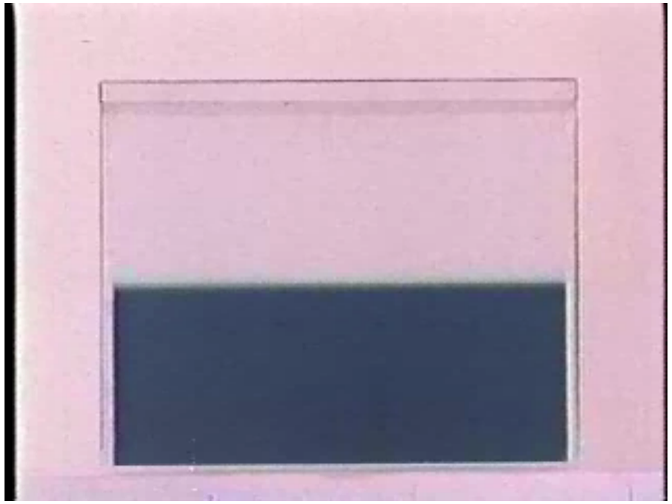
What Is Turbulence ?

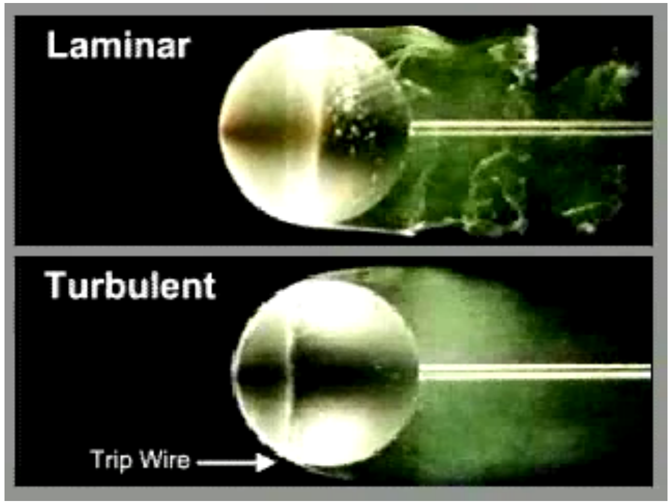
- It is *hard* to see laminar flow in large systems
  - At high Reynolds number (Re), fluid flows have a tendency to be "unstable" in their laminar state
- Instability: Tendency of a system to move away from its base state
  - Base state=laminar flow for fluids



What Is Turbulence ?

- ❑ Turbulence will
  - ➢ Enhance transport of momentum and energy
    - ❖Leads to high drag and heat transfer
  - ➢ Enhance mixing
    - ❖Important for combustion
  - ➢ Delay separation in boundary layers



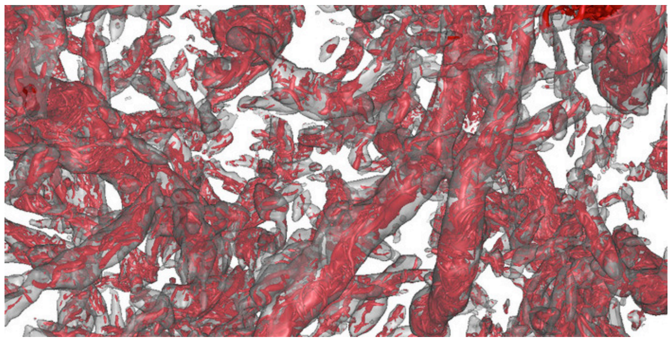


Common Properties of Turbulence

- ❑ Is a solution to the Navier Stokes equation
  - ➢ Incompressible N.S.:

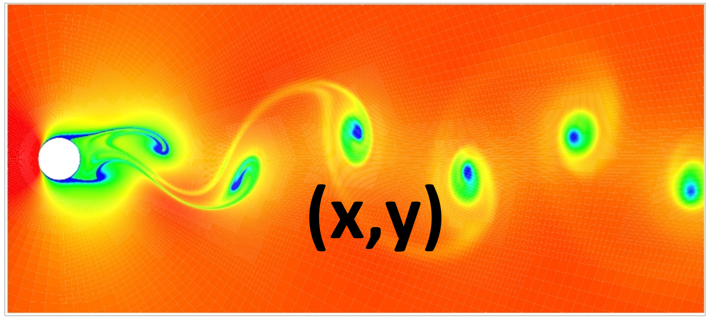
$$\partial \mathbf{u} / \partial t + \mathbf{u} \bullet \nabla \mathbf{u} = -(1/\rho)\nabla p + \nu \nabla^2 \mathbf{u}, \quad \nabla \bullet \mathbf{u} = 0$$

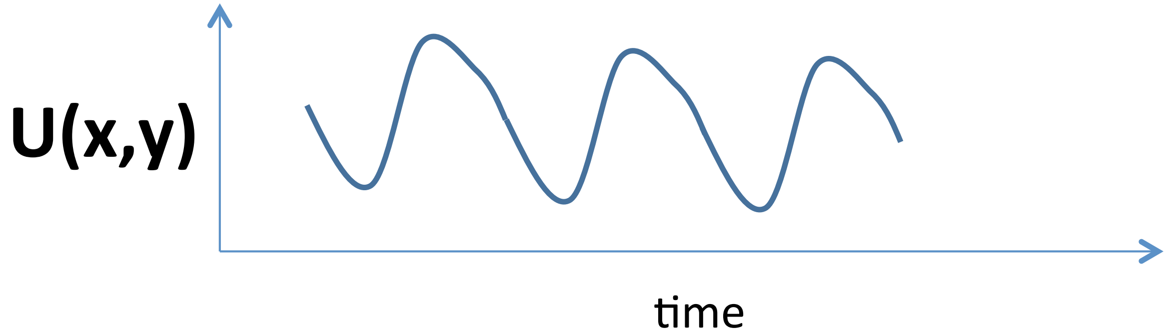
- ❑ 3 dimensional, infinite degrees of freedom
- ❑ Has a large range of "scales"
  - ➢ Small, fast "eddies" + large, slow "eddies"
- ❑ Details of velocity field are unpredictable and random
- ❑ Contains vortical structures
- ❑ Occurs at high Re flows
- ❑ Dissipates energy

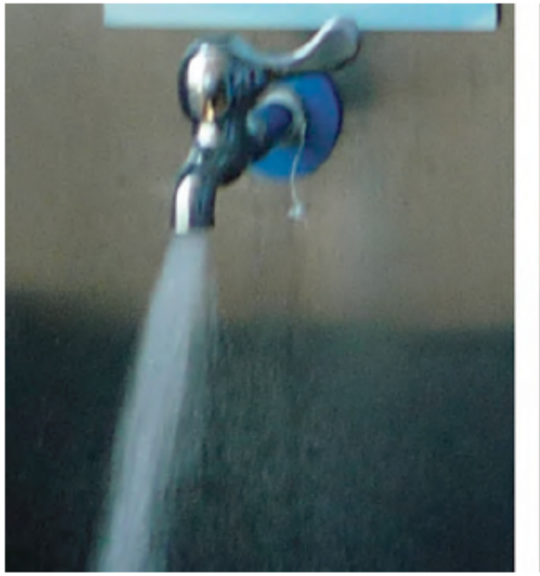


Burger *et. al*, Gallery of Fluid Motion, 65th APS-DFD Meeting

Are these Turbulent Flows ?

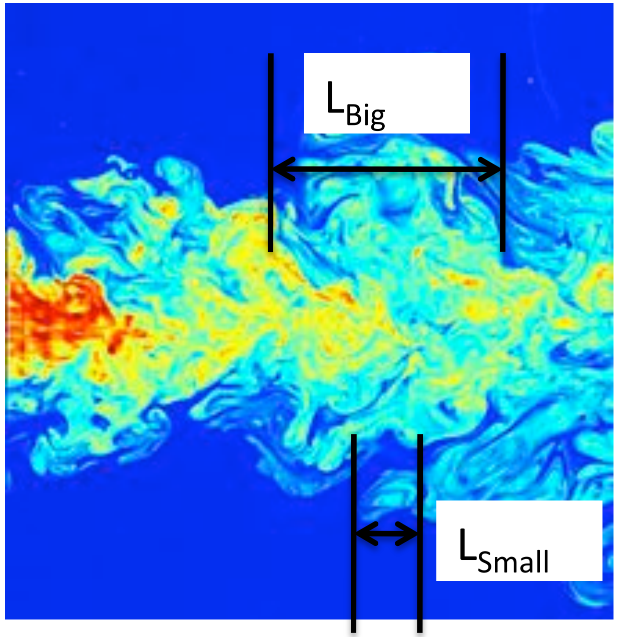


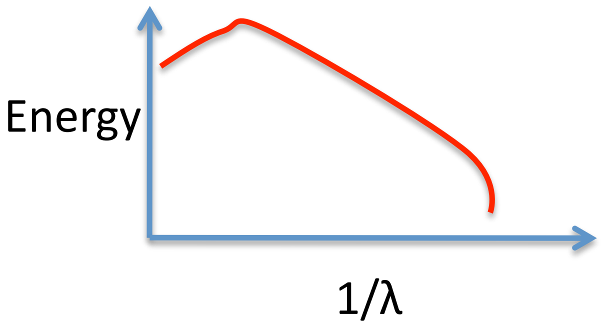


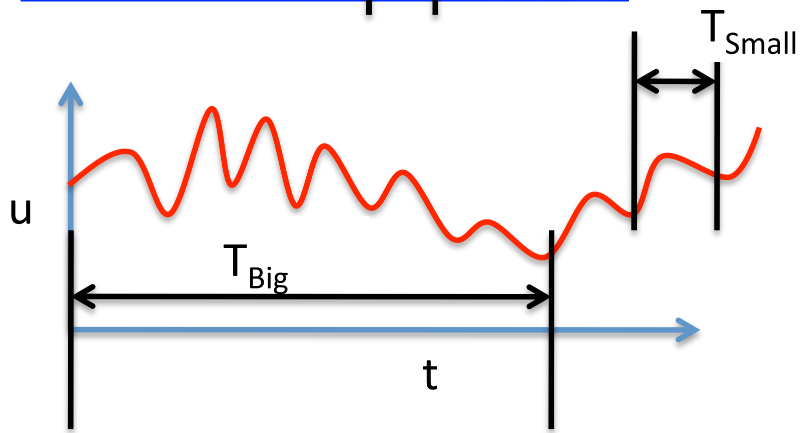


Length Scales and Time Scales in Turbulence

- ❑ Range of length scales
  - ➢ Different wavelengths
- ❑ Range of time scales
  - ➢ Different frequencies
- ❑ No separation of scales
  - ➢ Spectrum of energy is continuous

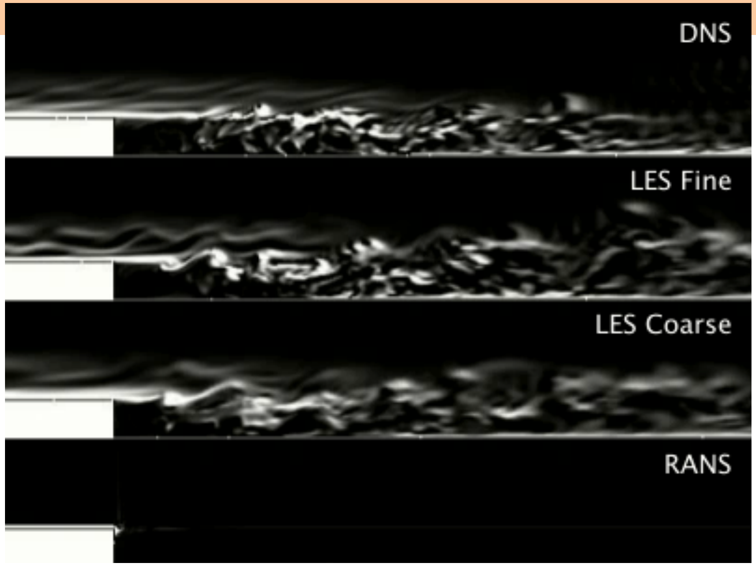






Our Goal as Engineers

- ❑ Predict the effects of turbulence
  - ➢ Mixing and Dispersion
  - ➢ Drag
  - ➢ Heat transfer
- ❑ Control or modify the effects of turbulence
  - ➢ E.g. via drag reduction



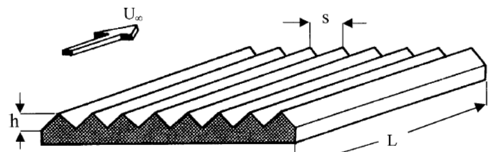


Fig. 1. Sketch of riblet geometry (taken from [7]).

PERGAMON

Progress in Aerospace Sciences 38 (2002) 571–600

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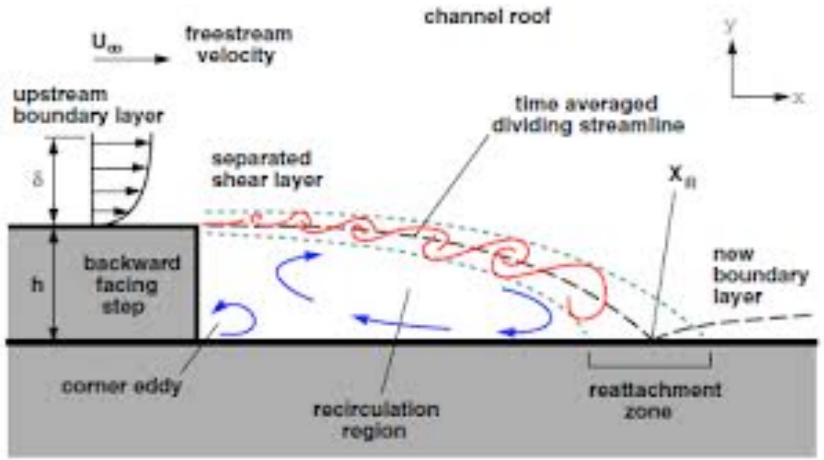
Aircraft viscous drag reduction using riblets

P.R. Viswanath*

Experimental Aerodynamics Division, National Aerospace Laboratories, Bangalore 560 017, India

Canonical Flows

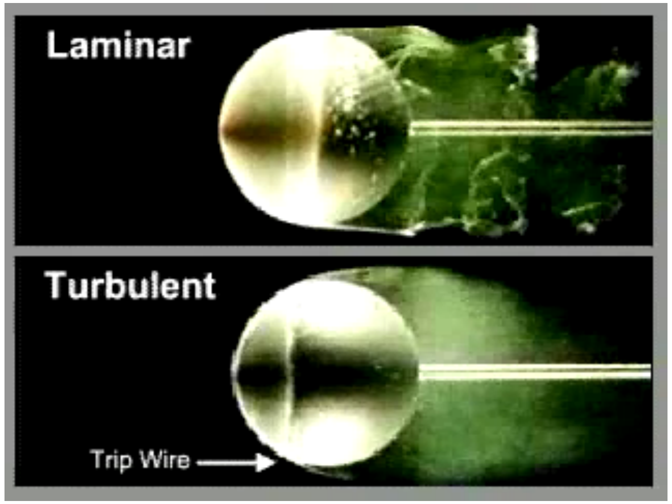
- ❑ Any complex flow may be divided into simpler (canonical) flows
- ❑ Flow over a backward-facing step
  - ➢ Fully developed channel flow
  - ➢ Mixing layer
  - ➢ Boundary layer
- ❑ We need to understand how each canonical flow behaves



Turbulence In a Box

- What is the range of scales if we give a certain input power ?
  - Dependence on viscosity
  - Energy spectrum
- How does the smallest length scale change with increasing Re ?

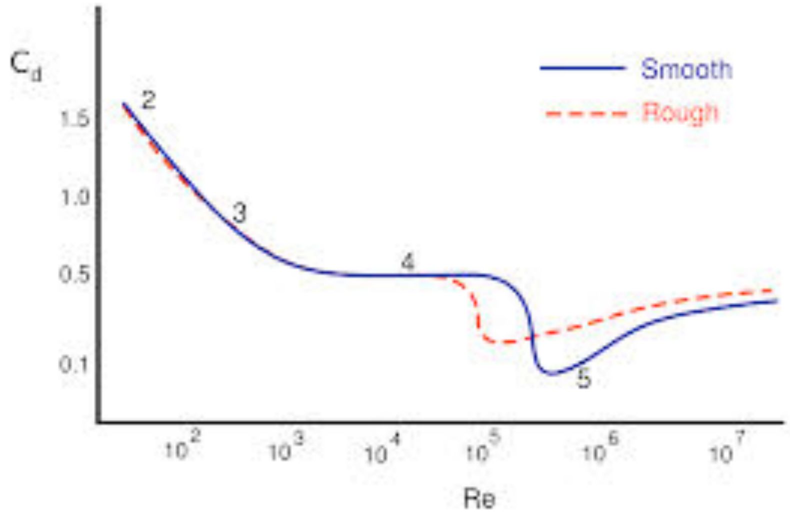
Drag Law For Sphere



- For a sphere of radius R:

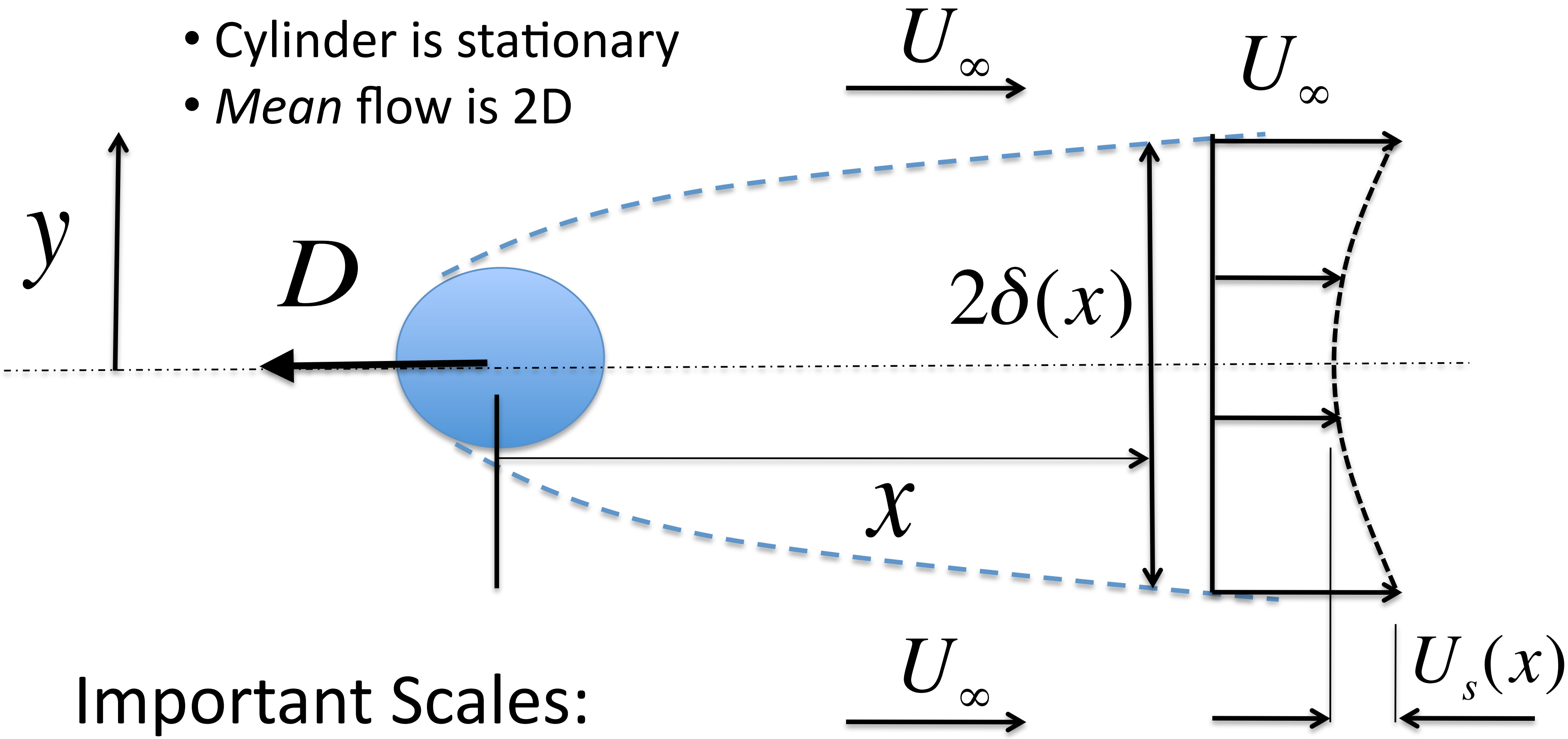
$$F = \frac{1}{2} C_D \rho \pi R^2 U^2$$

- $C_D(\mathrm{Re})$=constant
  - $\mathrm{Re} = UR/\nu$



Free Shear Flows (Wake)

- Cylinder is stationary
- *Mean* flow is 2D



Important Scales:

$U_\infty, U_s(x), L(x), \delta(x), q^2$

P=constant

Free Shear Flows (Wake)

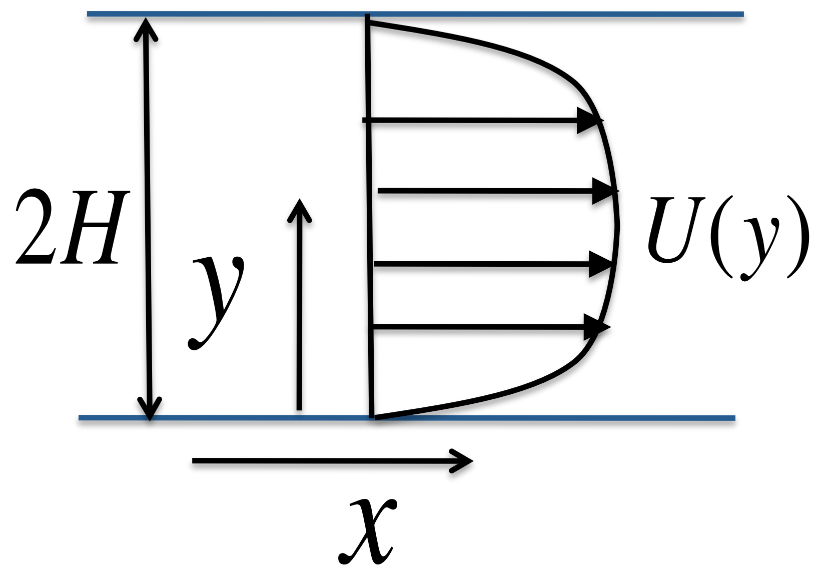
- Relevant engineering question:
  - How far does the wake affect the surrounding ?
- For a wake, we will derive:

$$\boxed{\delta(x) \propto \sqrt{\frac{Dx}{U_\infty^2 \rho}}, \quad u_0(x) \propto \sqrt{\frac{D}{\rho x}}}$$

  - Exact spreading rate depends on turbulent mixing of momentum

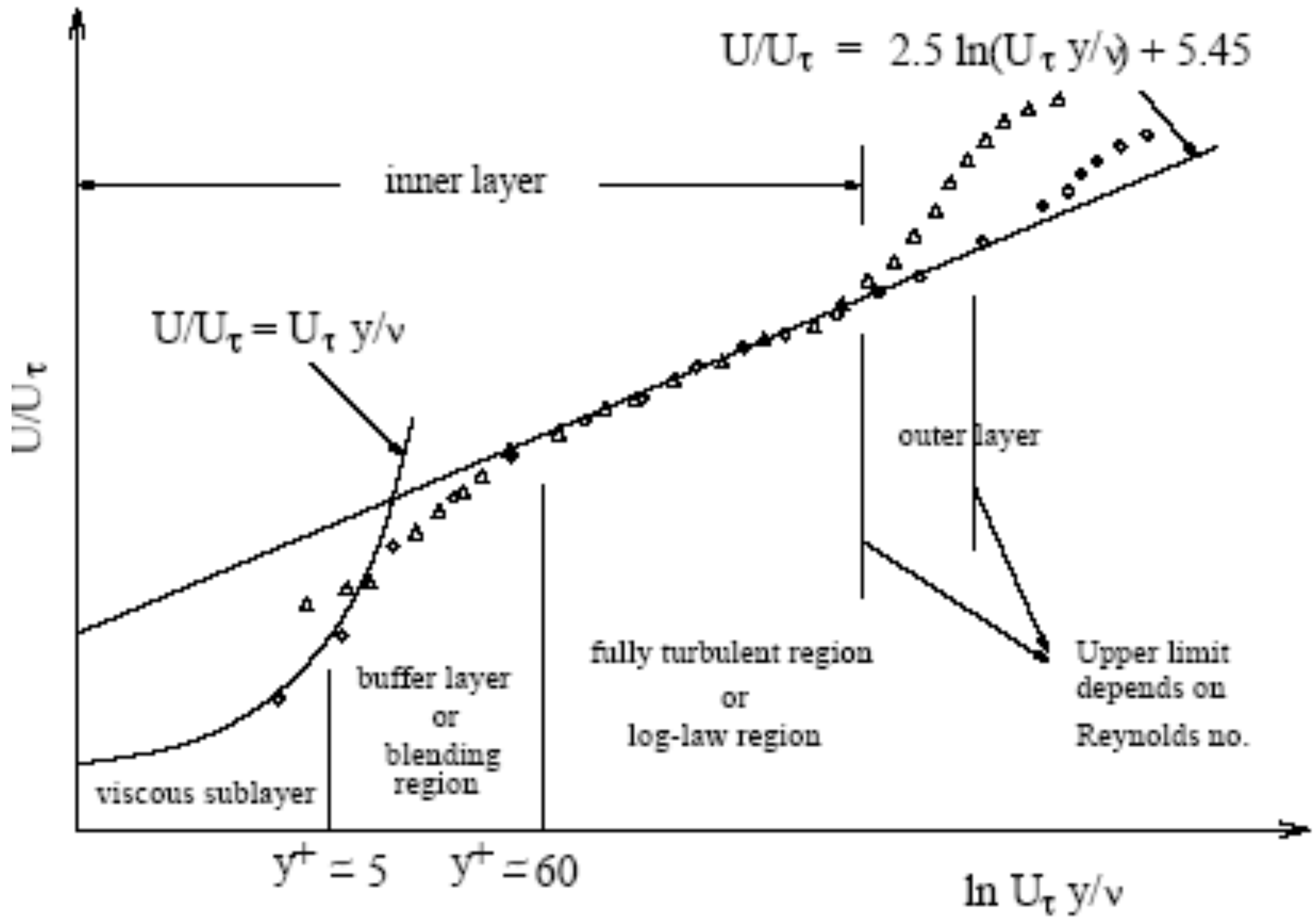
Channel/Pipe Flow

- **Aim:** Find pressure gradient required to pump turbulent flow at a given flow-rate



$$\frac{\partial P}{\partial x} < 0 \quad \text{(Drives the flow)}$$

Different Regions in Channel Flow Profile



C_f vs Re

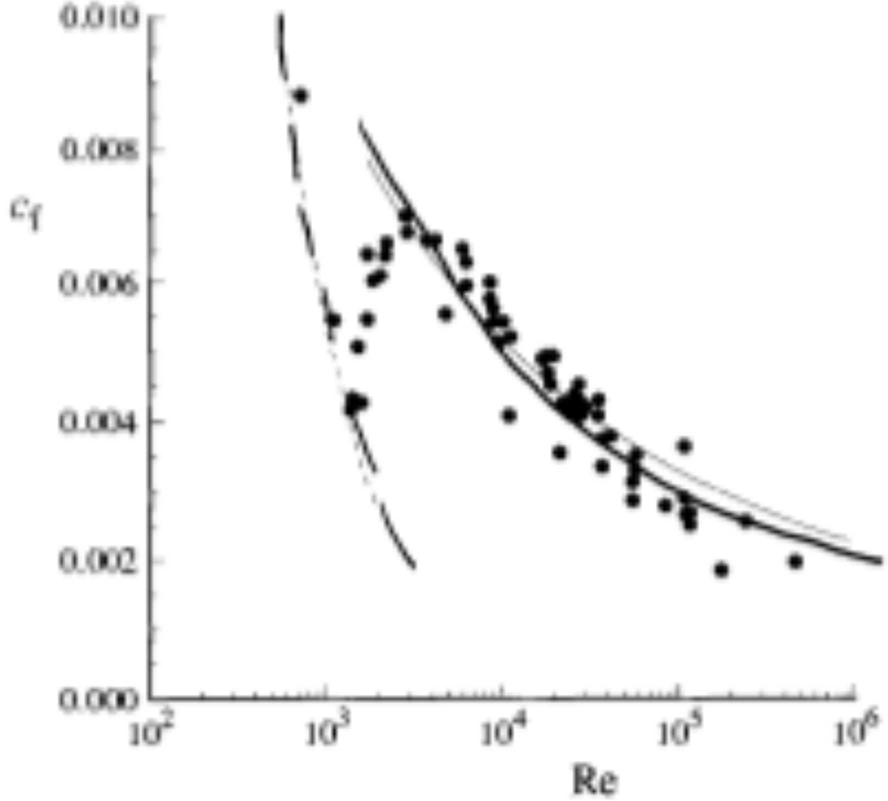


Fig. 7.10. The skin-friction coefficient $c_f = \tau_w/(\frac{1}{2}\rho U_0^2)$ against the Reynolds number ($\mathrm{Re} = 2\bar{U}\delta/\nu$) for channel flow: symbols, experimental data compiled by Dean (1978); solid line, from Eq. (7.55); dashed line, laminar friction law, $c_f = 16/(3\mathrm{Re})$.

Boundary Layers

- Relation for C_f can be approximated as:

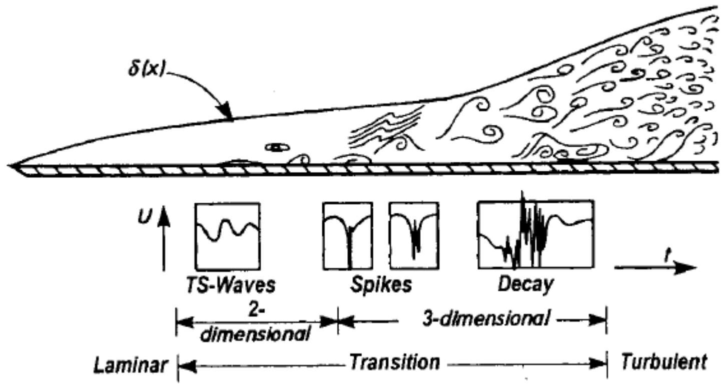
$$C_f = 0.02\,\mathrm{Re}_\delta^{-1/6}$$

- Using momentum conservation:

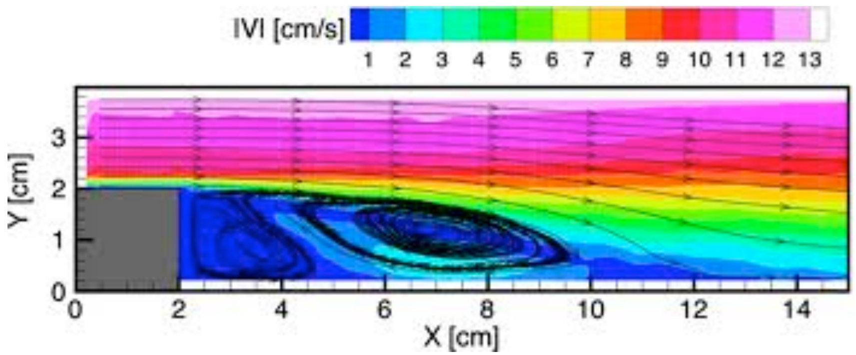
$$C_f = 2\frac{d\theta}{dx}, \quad \theta = \text{Momentum Thickness}$$

- Assuming : $$\frac{U}{U_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$$

- We can get: $$\frac{\delta}{x} = 0.16\,\mathrm{Re}_x^{-1/7}$$



Modeling Complex Flows



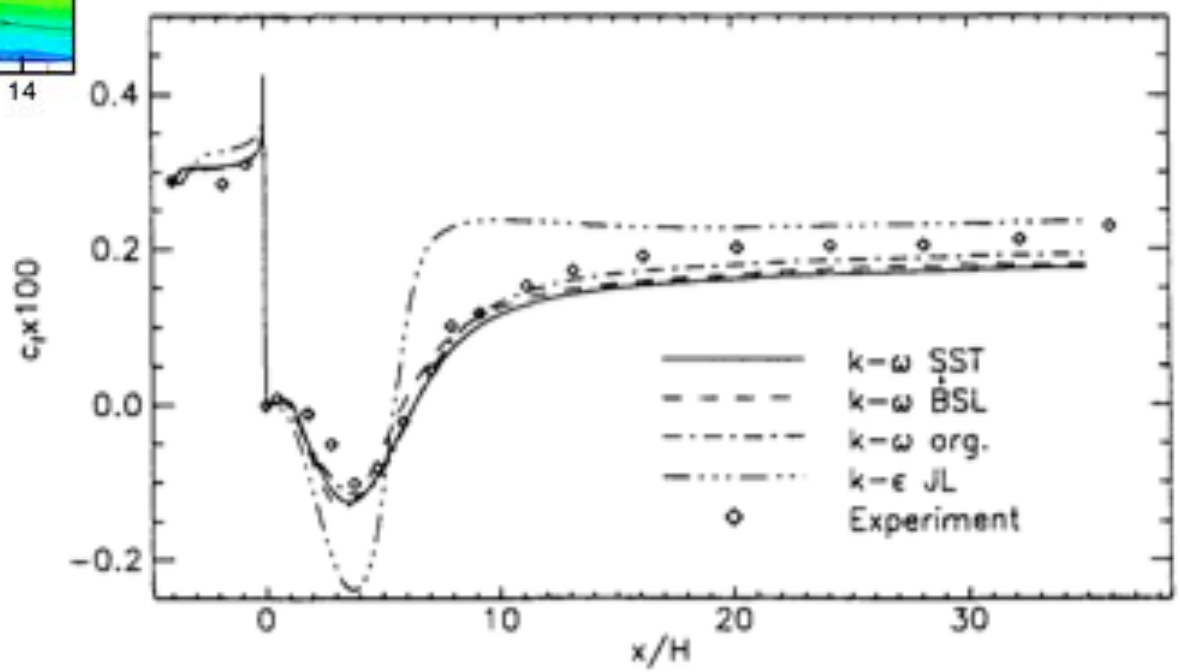


Fig. 17 Wall shear-stress distribution for backward-facing step flow.

The “Chaotic” Nature of Turbulence

- Turbulence is random and unpredictable
- But it is still a solution to deterministic Navier Stokes (N.S.) equations
  - Given initial and boundary conditions, we expect unique solution to N.S. equation
- Solution is very sensitive to initial and boundary conditions
  - Similar to rolling of dice, weather systems
- For such systems, we can only predict statistics

Lorenz Attractor: A Simple Chaotic System

- Lumped-mass model for thermal convection

$$\dot{x} = \sigma(y - x)$$

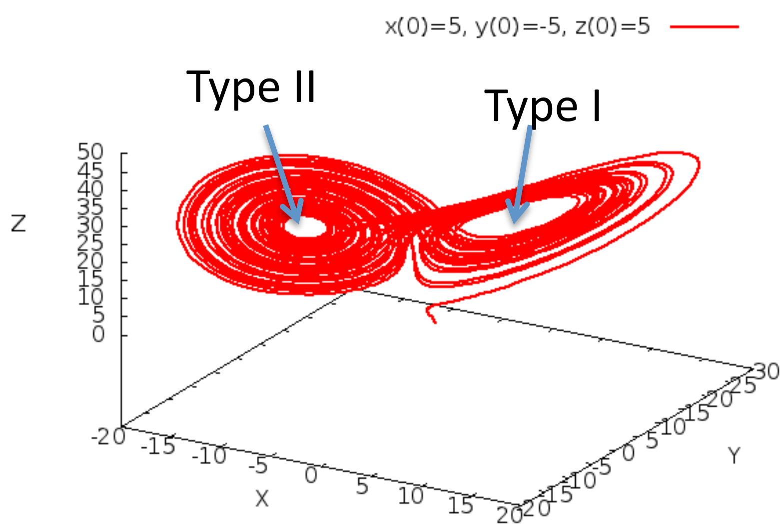
$$\dot{y} = -xz + Rx - y$$

$$\dot{z} = xy - bz$$

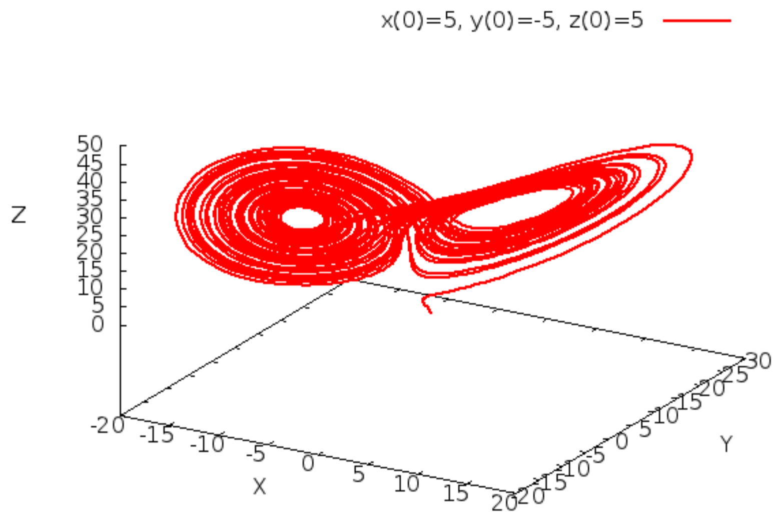
  - R=Raleigh number, σ=Prandtl number, b=coupling constant

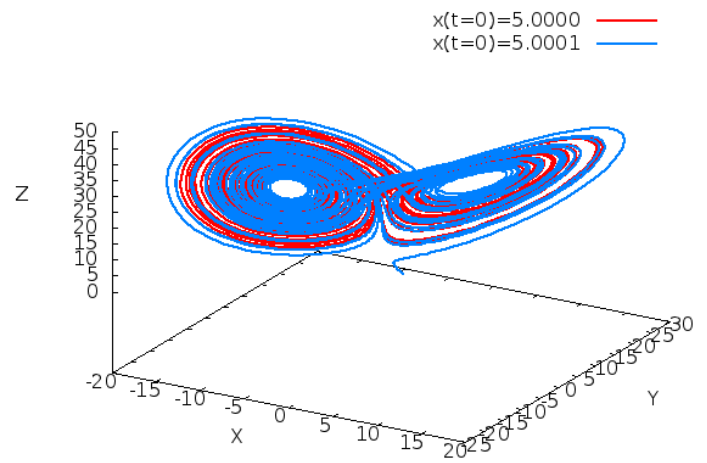
- Fixed points: $\dot{x} = \dot{y} = \dot{z} = 0$
  - Type I : $x = y = z = 0$
  - Type II : $x = y = \pm\sqrt{b(R-1)}$ $\quad z = R - 1$
    - Exists only for $R>1$

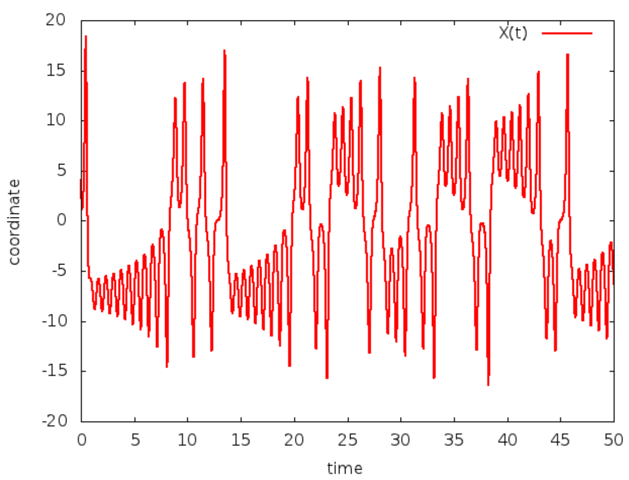
Phase map of Lorenz attractor

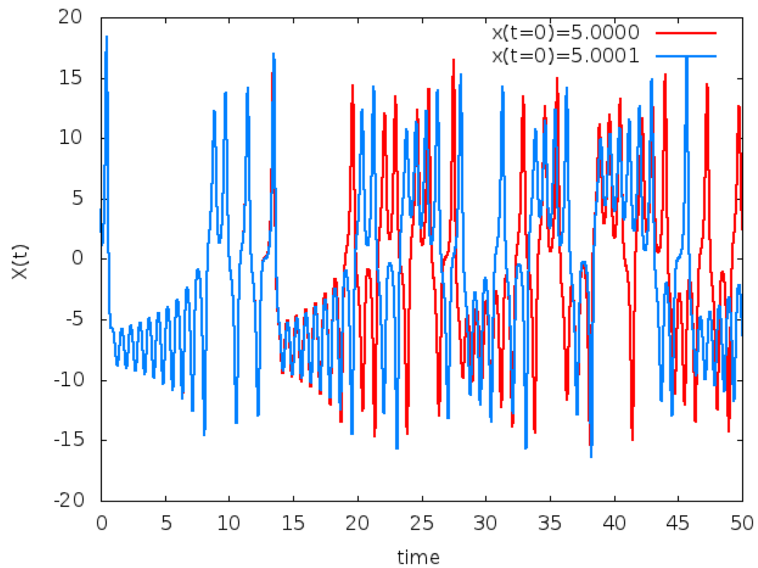


Lorenz Attractor: Sensitivity to Initial Conditions





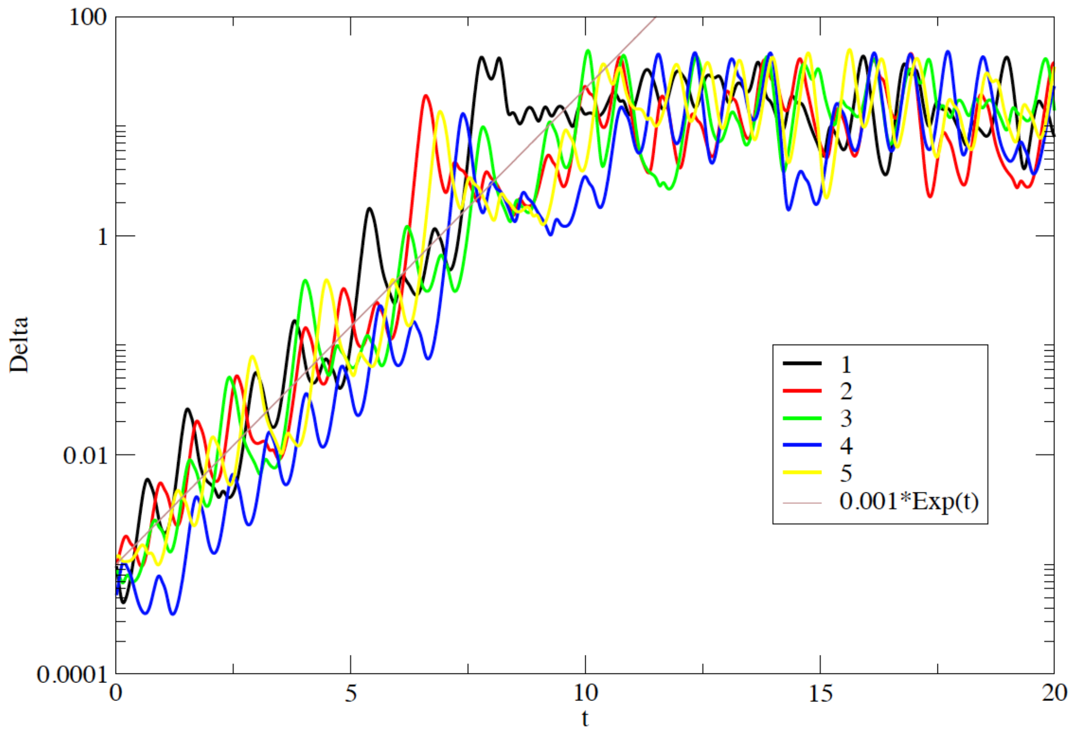




Lyapunov Exponent

- For any initial separation $\delta\mathbf{x}(0) \doteq \delta\mathbf{x}_0$

$$|\delta\mathbf{x}(t)| = |\delta\mathbf{x}_0|\exp[\lambda t]$$



Stability of Fluid Flows

- N.S. equations for u(x,*t*):

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{N}(\mathbf{u})$$

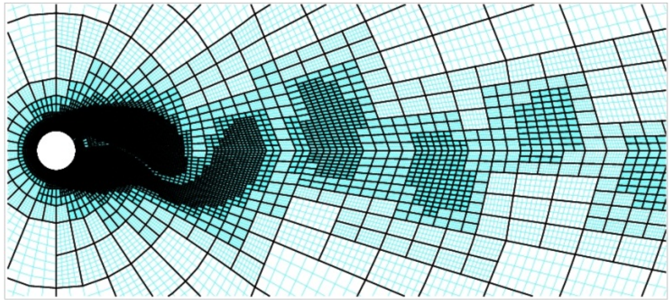
- Choose laminar solution U(x,*t*) as fixed point
  - E.g. Parabolic profile for channel flow

$$\frac{\partial \delta \mathbf{u}}{\partial t} = \nabla \mathbf{N}(\mathbf{U})\delta \mathbf{u}$$

- Find eigenvalues, eigenvectors of $\nabla \mathbf{N}(\mathbf{U})$
  - Above $Re > Re_{cr}$, we will find $Re(\lambda) > 0$

Lorentz Attractor vs Turbulence

- For 3D flows, turbulent flows are 2N degree of freedom (DOF) systems
  - N grid points
  - Only 3 DOF in Lorentz attractor !
- Turbulence occurs via instability of flow
  - "Fixed points" correspond to laminar solution
- Turbulence is very sensitive to initial condition
- Large scale flow features remain the same
  - "flow features" corresponds to attractors in Lorentz equations



N grid points