ME724 – Tutorial 1

1. For the following types of flow, list the directions in which the flow is statistically homogeneous, and whether or not it is statistically stationary in time. Clearly define your coordinate system in each case, and state any additional assumptions clearly if you think that the information given is insufficient.

   (a) Fully developed turbulent channel flow.

   (b) Turbulent round jet.

   (c) Turbulent flow around a stationary cylinder. Cylinder axis is perpendicular to the incoming flow.

   (d) Plane developing turbulent boundary layer over a flat plate.

   (e) Turbulent flow over an oscillating cylinder. The mean incoming flow is in the x direction and the axis of the cylinder is located at $x = x_0$, $y = A\sin(\omega t)$, where ω is an angular frequency, t is time. The cylinder is infinite in z direction.

2. Consider an *irrotational* random velocity field $\mathbf{u}(\mathbf{x}, t)$, for which vorticity is zero (true for flow of water waves). Starting with with $\langle u_i(u_{i,j} - u_{j,i})\rangle = 0$, show that $\frac{\partial\langle u_i u_j\rangle}{\partial x_i} = \frac{\partial K}{\partial x_j}$, where K is the turbulent kinetic energy. What is the implication of this relationship for RANS equation ?

3. The dissipation spectra of 3D turbulence is given by $\hat{\epsilon}(k) = \nu k^2 E((k))$. Assuming a Kolmogorov spectra $E(k) = C_K \epsilon^{2/3} k^{-5/3}$:

   (a) Calculate the fraction of dissipation that takes place between wavenumbers $k_0 = 2\pi\alpha/\eta$ and $k_\eta = 2\pi/\eta$, where $0 < \alpha < 1$, and η is the Kolmogorov scale

   (b) Find the value of α for which this fraction is 0.9

   (c) For turbulence in a 3D periodic box at $Re_T = k^2/(\nu\epsilon) = 1000$, approximately how many Fourier modes are required to represent 90% of the dissipation ? You can assume that wavelength of the smallest mode is equal to η.

4. Suppose we have a box of size L (m) with a fluid having *kinematic* viscosity ν (m^2/s) and density ρ (kg/m^3). We stir this box with a spoon of size a (m). While stirring the box, we put in a power P (J/s). Assuming that while moving the spoon this power is being homogeneously distributed in the box, what is the length scale of the smallest eddy inside the box ? Suppose $a \ll L$, then what is the approximate size of the "integral length scale" ?

5. The Fourier representation of a statistically homogeneous turbulent field is given as $\mathbf{u}(\mathbf{x}) = \sum_{\mathbf{k}} \hat{\mathbf{u}}(\mathbf{k}) \exp[i\mathbf{k} \cdot \mathbf{x}]$. Show that:

$$R_{ij}(\mathbf{r}, t) \quad = \quad \langle u_i(\mathbf{x}, t) u_j(\mathbf{x} + \mathbf{r}, t) \rangle = \sum_{\mathbf{k}} \langle \hat{u}_i^*(\mathbf{k}) \hat{u}_j(\mathbf{k}) \rangle \exp[i\mathbf{k} \cdot \mathbf{r}] \qquad (1)$$