## Quiz 1

## ME 651 Fluid Dynamics

Instructor: Rajneesh Bhardwaj

Date: 21 Aug, 2015

- Time: 5:30 pm to 6:45 pm OR 6:30 pm to 7:45 pm.
- Total: 50 marks.
- Closed book and closed notes examination.
- Clearly show your answers for different parts of a problem and write all parts of a question in one place.
- No clarifications and make suitable assumptions wherever necessary.

Problem 1 [8 marks]: Write mathematical expression for each if the following <u>in terms of del operator</u>:

- Vorticity
- Continuity equation for incompressible fluid
- Total acceleration of a fluid paricle
- Volumetric strain rate of a fluid particle

Problem 2 [6 marks]: Write mathematical expression of the following:

- Circulation in terms of velocity vector.
- Velocity vector in terms of velocity potential for a 3D fluid particle
- Velocity vector in terms of stream function for a 2D fluid particle

Problem 3 [6 marks]: Derive mass conservation equation for a fluid particle using Lagrangian approach.

Problem 3 [14 marks]: Consider a fluid (density $\rho$) flowing through an infinitesimal 2D Cartesian control volume (CV) of size $\delta x$ and $\delta y$ along x and y-axis, respectively. Consider velocity components as u and v across left and bottom faces, respectively.

a) [2 marks] Determine momentum flow rates in $x$ and $y$ direction at left and bottom faces of the CV in terms of $\rho$, $u$ and $v$.

b) [2 marks] Using Taylor series expansion and neglecting higher order terms, determine momentum flow rates in $x$ and $y$ direction at right and top faces of the CV.

c) [5 marks] Using balance statement for momentum for the infinitesimal CV and using results obtained in (a) and (b) determine the forces per unit volume, $f_x$ and $f_y$, acting on the CV in x and y direction, respectively (assume unit depth for CV in z direction).

d) [5 marks] Using results obtained in (c) and continuity equation, derive Newton's second law in terms of material derivative.

Problem 4 [10 marks]: Derive Bernoulli equation for a two-dimensional case using the Navier-Stokes equation given below. Use suitable assumptions as needed.

$$\frac{D\vec{V}}{Dt} = -\frac{1}{\rho}\vec{\nabla}p + \frac{\mu}{\rho}\vec{\nabla}^2\vec{V} + \vec{g}$$

Problem 5 [8 marks]:

a) [6 Marks] Consider the following dimensional governing (Navier-Stokes equation in x-direction) equation for flow past a circular cylinder of diameter $D$ and free stream velocity $U_\infty$ and pressure $P_\infty$. Obtain non-dimensional form of this equation taking $D$, $U_\infty$ and $P_\infty$ as characteristic distance, velocity and pressure, respectively.

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \rho g_x.$$

b) [2 Mark] Write the expressions of Euler number, Reynolds number and Froude number using the non-dimensional equation obtained in (a).

Quiz 2

ME 651 Fluid Dynamics

Date: 27 Oct, 2015

Instructor: Rajneesh Bhardwaj

Notes:

- Time: 6:30 pm to 8:00 pm. Total: 50 marks.
- Closed book and closed notes examination.
- Clearly show your answers for different parts of a problem AT ONE PLACE.
- No clarifications and make suitable assumptions wherever necessary.
- Useful equations are given in the Appendix (at the end of the question paper).

Problem 1 [20 marks] *Potential flow over a circular cylinder* :

a) [5 marks] Develop complex potential for free stream horizontal flow (free stream velocity = $U$) past a circular cylinder of radius R with superimposed clockwise (+) circulation $\Gamma$, such that dividing streamline is on the cylinder surface ($r = R$, where $R$ is radius of the cylinder). Use complex potentials for elementary potential flows as given in Appendix.

b) [2+2+1 marks] Derive velocity components (tangential and radial) in r-$\theta$ coordinates. Determine the magnitude and location of maximum velocity.

c) [2 + 2 + 2 marks] Find the location of the stagnation point(s) for the following three cases: $\frac{\Gamma}{4\pi UR}<1$ , $\frac{\Gamma}{4\pi UR}=1$ , $\frac{\Gamma}{4\pi UR}>1$

d) [3 marks] Determine the pressure on the surface of cylinder as a function of $\theta$.

e) [1 mark] Briefly describe d'Alembert paradox in context of answer obtained in (d).

Problem 2 [15 marks]: *Flow in a thin liquid film*:

Consider steady, incompressible, viscous flow of a thin liquid (density $\rho$, viscosity $\mu$) film flowing over a wall, making an angle $\theta$ with the horizontal, under influence of gravity g. Consider two dimensional coordinates in which $x$-axis is along the wall, pointing downwards, and $z$-axis is normal to $x$ axis such that it points from wall to the fluid. The velocities $u$ and $w$ are in the respective $x$ and $z$ directions. Assume that the velocity along x-axis is much larger that along z-axis (u >> w).

a) [3 marks]: Starting with Navier-Stokes equations, write the governing equations for above problem.

b) [3 marks]: Assume that film thickness (say $e$) is uniform and pressure at the liquid-gas interface is atmospheric pressure ($P_{atm}$), derive and sketch pressure profile along the thickness of the film as function of $P_{atm}$, g, $z$ and $\theta$.

c) [5 marks]: Simplify equation derived in (a) using results obtained in (b) and solve for velocity profile $u(z)$. Assume that there is no shear force at the liquid-gas interface. Sketch $u(z)$.

d) [2 marks]: Find the average velocity in the film $U$ by integrating the velocity $u$ from 0 to the thickness of the film $e$.

e) [2 marks]: Express the flow rate $Q = eU$ as a function of the gravity g, film thickness $e$ and dynamic viscosity $\mu$. Determine how much the flow rate Q increases if the film thickness doubles as $e'=2e$ keeping all other parameters same.

Problem 3 [15 marks] : *Velocity profile of two immiscible fluids*:
Consider flow of two distinct immiscible fluids 1 and 2 parallel to each other in a channel of height $2H$ jointly driven by a pressure gradient. Their rates of flow are so adjusted that each fills one half of the channel height. The dynamic viscosities of fluid-1 and fluid-2 are $\mu_1$ and $\mu_2$, respectively. Take origin at the middle of the channel.

a) [6 marks] Simplify the momentum equations for the two-fluids and write boundary conditions. Show that the pressure gradient along x is constant say, $\lambda = dp/dx$.
b) [4+3 marks] Derive the expression for the profile of shear-stress and u-velocity with y-coordinate.
c) [2 marks] Draw the u-velocity and shear stress profiles.

Appendix: Useful equations

| Case | Complex potential |
|---|---|
| Uniform flow at an angle α, | $w = Ue^{-i\alpha} z$ |
| Source(+) / sink(-) | $w = \pm \dfrac{m}{2\pi} \ln z$ |
| Doublet | $w = \mu / z$ |
| Irrotational vortex(clockwise(+), anticlockwise(-)) | $w = \pm \dfrac{i\Gamma}{2\pi} \ln z$ |

Continuity Equation for steady, incompressible flow in Cartesian coordinates:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Navier Stokes Equations for incompressible flow in Cartesian coordinates:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left\{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right\}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left\{\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right\}$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left\{\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right\}$$

End-Semester Examination
ME 651 Fluid Dynamics

Date: 10 Nov, 2015

Instructor: Rajneesh Bhardwaj

- Time: 2 pm to 5 pm.
- Total: 100 marks.
- Closed book and closed notes examination.

**IMPORTANT NOTES**:

- Clearly show your answers for different parts of a problem and write all parts of a question in one place.
- No need to ask clarifications around and if something is unclear, make suitable assumptions and add justification for the same wherever necessary.
- Refer Appendix for useful equations.
- Good luck!

Problem 1 [5 marks]: *Basics*: Consider the steady, planar flow of an inviscid, incompressible fluid (density $\rho$) in a right-angle corner as given by the stream function $\psi = Axy$ where $A$ is a constant.

(a) [1 marks] Show that this flow is irrotational.

(b) [2 marks] Find an expression for the pressure, $p$, at any point in the flow assuming that the pressure at the origin, $p_0$, is known. The $y$-axis is vertically upward and the only body force is that due to gravity, $g$.

(c) [2 marks] If the $x$-axis is a thin wall with a uniform pressure, $p_0$, on its underside, find the vertical force on that portion of the wall between $x$=0 and $x$=1. Assume unit depth perpendicular to the page.

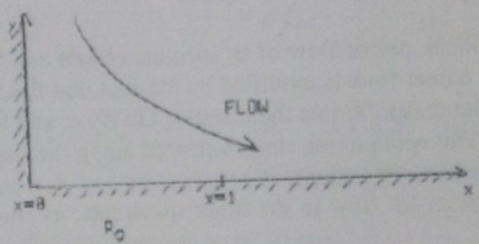


Figure for problem 1

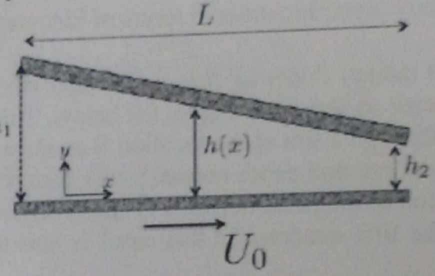


Figure for problem 2

Problem 2 [26 marks]: *Flow in a slider bearing*: Consider steady, incompressible, viscous flow of a liquid with viscosity $\mu$, flowing in a slider bearing as shown in the above figure. Consider two dimensional coordinates in which $x$-axis is along the bottom plate. The upper plate is inclined to the moving surface and has a projected length of $L$. The plate at $x$ = 0 moves with velocity $U_0$. The distances between the two plates are $h_1$ and $h_2$ at the two ends. The velocities $u$ and $v$ are in the respective $x$ and $y$ direction. Assume that the $v$ velocity along y-axis is negligible ($v \sim 0$).

a) [1 mark] Neglect gravity and assume fully-developed flow in the bearing (this is realized when the angle between the plates is very small), simplify Navier-Stokes equations and write the governing equations for above problem.

b) [3 marks] Solve for velocity $u$ with appropriate boundary conditions using solution obtained in (a).

c) [8 + 6 Marks] The slider block is completely submerged in the fluid so that the upstream and downstream pressures are equal ($p(0) = p(L) = p_0$). This is known as the flooded condition. Using the velocity profile obtained in (b) and assuming pressure inside the bearing as a function of $x$, obtain pressure profile and flow rate in the bearing.

d) [2 + 3 + 3 marks] Plot pressure profile in the bearing, and find the maximum pressure and its location in the bearing.

Problem 3 [10 marks]: *Internal flow in a liquid*: Consider an explosion in a incompressible (density $\rho$) and inviscid liquid, which creates a purely radial flow in the liquid surrounding a bubble whose radius, denoted by $R(t)$, is increasing with time. Given that the bubble radius increases linearly with rate $\partial R / \partial t$.

a) [5 marks] Determine the radial velocity field in the liquid.

b) [5 marks] Find the pressure field in the liquid. Neglect body forces and surface tension effects. Assume the pressure far from the bubble is known ( $p_\infty$, a constant).

Problem 4 [12 marks]: *Squeeze flow between two disks*: A thin liquid film (dynamic viscosity $\mu$) is trapped between two circular disks of radii $R$ with initial gap $H_0$. They are squeezed quasi-steadily together with a constant force $F$. Consider fully developed along the radial direction, as shown in a axisymmteric geometry in the figure.

a) [8 marks] Use scaling analysis in the region far away from the centre of the disk and obtain a ODE for separation distance $H(t)$ as a function of time $t$ in terms of given variables.

b) [4 marks] Solve the ODE to obtain $H(t)$ in terms of $t$ and given variables.

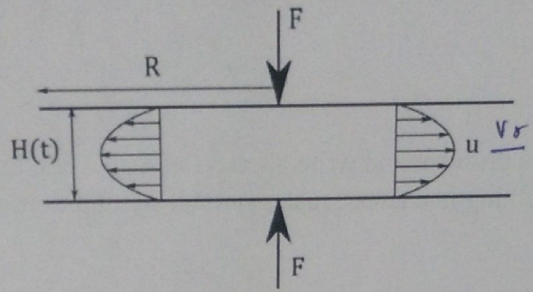


Figure for problem 4

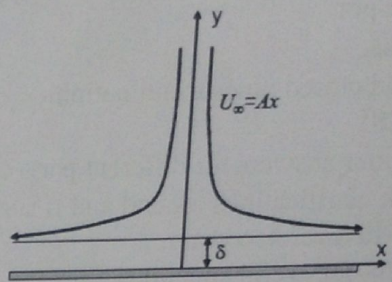


Figure for problem 5

Problem 5 [7 marks] *Boundary layer theory*:

a) [2 marks]: Write definition of displacement and momentum thickness in not more than three lines each. (No credit for writing mathematical expressions).

b) [5 marks] Consider boundary layer of an incompressible fluid on a flat plate as shown in figure, with the approximation of free stream velocity as, $U_\infty = Ax$, where $A$ is a constant. Assume that the velocity profile in the boundary layer is of the approximate form, $u/U_\infty = 2(y/\delta) - (y/\delta)^2$ for $0 \le y \le \delta$ and $u/U_\infty = 1$ for $y > \delta$. Find boundary layer thickness in terms of kinematic viscosity ($\nu$) and the constant $A$.

Problem 6 [20 marks] *Potential flows*: Consider the irrotational, planar flow of an incompressible and inviscid fluid in a right-angle corner as shown in Figure (a) below. The basic corner flow is modified by the fact that fluid is being injected into the flow through a slot at the location B as shown in Figure (a). Denote the distance OB by $a$, and the volume rate of injection of fluid per unit depth normal to the sketch by $Q$. The width of the slot (measured along the $x$-axis) is negligible. Find the location of the point between O and B on the wall where the velocity is zero (in terms of $A$, $a$ and $Q$). [*Hint:* flow in a corner (the first quadrant in this case) is also the solution for flow in the other quadrants, as shown in Figure (b) below].

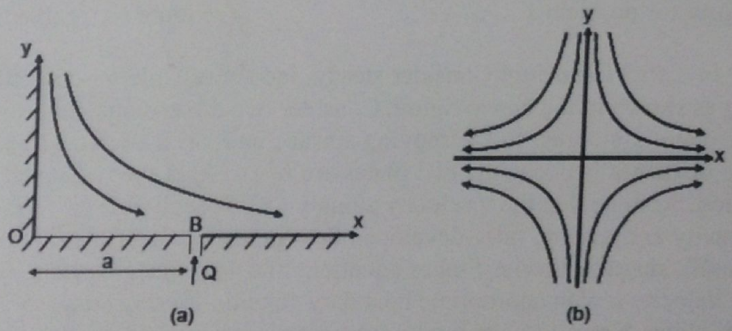


Figure for problem 6

Problem 7 [20 marks] *Vorticity equation*: Consider the two-dimensional Navier-Stokes equations in Cartesian coordinates for the planar flow of an incompressible viscous fluid (constant density $\rho$ and constant viscosity $\mu$) under the action of conservative body forces. Using continuity equation, derive vorticity equation in two-dimensional Cartesian coordinates [Hint: eliminate the pressure in Navier-Stokes equations].

**Appendix:**

Continuity Equation for steady, incompressible flow in Cartesian coordinates:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$\frac{c}{3}t^{-2/3}$

$\theta = ct^{1/3}$



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ME 651- Fluid Dynamics
Quiz 3 - Autumn 2015

Date: 3 Nov, 2015

Instructor: Rajneesh Bhardwaj
Notes:

- Time: 6:30 pm to 8:00 pm
- Total: 50 marks
- No clarification during examination. Make suitable assumptions wherever necessary.
- Write all parts of a question in one place.

Problem 1(15 marks): Order of magnitude analysis for boundary layer flow:
Consider the free stream flow (velocity $U_\infty$) over a surface (length $L$) with $x$ axis along the surface and $y$ axis being normal to the surface. Start with non-dimensional, 2D, steady-state Navier Stokes equations given below:

Continuity Equation:
$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0$$

X-Momentum Equation:
$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)$$

Y-Momentum Equation:
$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)$$

where U = u/ $U_\infty$, V = v/ $U_\infty$, $X = x/L$, $Y = y/L$. Use order of magnitude analysis and considering that u velocity scales as $U_\infty$ , x scales as L and y scales as δ (boundary layer thickness).

a) [2 marks] Comment on the relative magnitudes of inertia and viscous forces in the following regions: (i) near plate, (ii) inside boundary layer and (iii) outside boundary layer.
b) [3 marks] Using results of part (a, ii) and order of magnitude analysis, show that the non-dimensional boundary layer thickness ε (ε = δ/L) varies as inverse of square root of $Re_L$.
c) [3 marks] Using order of magnitude analysis on continuity equation, show that non-dimensional y-velocity $v$ scales as the non-dimensional boundary layer thickness ε (ε = δ/L).
d) [3 marks] Using order of magnitude analysis on X-momentum equation, show the term $\frac{\partial^2 U}{\partial X^2}$ can be neglected in x-momentum equation.
e) [2 marks] Using order of magnitude analysis on Y-momentum equation, show that the pressure gradient in y-direction can be neglected.
f) [2 marks] Using results of parts c, d and e, simplify non-dimensional, 2D, steady-state Navier Stokes equations (given above) to obtain Prandtl boundary layer equations.

Problem 2 (20 marks): Blasius Equation for boundary layer flow over a flat plate:

a) [2 marks] Write governing equations and boundary conditions for boundary layer flow over a flat plate in free stream flow (velocity $U_\infty$). Start with Prandtl boundary layer equations obtained in Problem 1 and assume that the free stream velocity $U_\infty$ is constant (no variation along x).

b) [15 marks] Consider similarity variable $\eta$ = y/δ, where $\delta = \sqrt{\nu x / U_\infty}$. Start with velocity profile $\frac{u}{U_\infty} = F(\eta)$, where $f(\eta) = \int F(\eta) d\eta$, derive Blasius equation :

$$f'''(\eta) + \frac{1}{2} f(\eta) f''(\eta) = 0$$

c) [3 marks] Convert boundary conditions in part (a) in terms of similarity variable $\eta$.

Problem 3 (15 marks): Boundary layer theory:

a) [6 marks] Plot the velocity as well as shear stress profile in the boundary layer for the following three cases of flow over a surface:
- No flow separation
- Onset of flow separation
- After flow separation

b) [2 marks] The stream-wise velocity component of a steady, incompressible, laminar, flat plate boundary layer (of boundary layer thickness δ) is approximated by the simple linear expression $u = u_\infty y/\delta$ for $y<\delta$, and $u = u_\infty$ for $y>\delta$. For the linear approximation, generate expressions for momentum thickness $\delta^{**}$ as functions of δ.

c) [7 marks] Using answer in (b), definition of local skin friction coefficient ($C_{f,x}=2\tau_w/(\rho u_\infty^2)$) and the Kármán integral equation ($2d\delta^{**}/dx=C_{f,x}$), derive expression for $\delta/x$.

**Midterm - Autumn 2015**
**ME 651 Fluid Dynamics**

Instructor: Rajneesh Bhardwaj                                        Date: 12 Sep, 2015

- Time: 3:00 pm to 5:00 pm
- Total: 100 marks.
- Closed book and closed notes examination.
- Clearly show your answers for different parts of a problem and write all parts of a question in one place.
- No clarifications and make suitable assumptions wherever necessary.

**Problem 1** [8 marks]: Consider a fluid (density $\rho$) flowing through an infinitesimal 2D Cartesian control volume (CV) of size $\delta x \times \delta y$, with velocity components as $u$ and $v$ across left and bottom faces, respectively.

a) [2 marks] Let $\dot{m}_x$, $\dot{m}_{x+\delta x}$, $\dot{m}_y$ and $\dot{m}_{y+\delta y}$ be the rate of mass flow at left, right, bottom and top faces of the CV, respectively. Express these mass flow rates in terms of the given variables (Hint: Use Taylor series expansion given in Appendix).

b) [3 marks] Using balance statement for mass for the infinitesimal CV and mass flow rates obtained in (a), derive the mass conservation equation for this CV in the generic form.

c) [3 marks] Express the equation obtained in (b) in terms of material derivative.

**Problem 2** [22 marks]: Consider a fluid (density $\rho$) flowing through an infinitesimal 2D Cartesian control volume (CV) of size δx and δy along x and y-axis, respectively. Consider velocity components as $u$ and $v$ at northwest corner of the CV, respectively.

a) [4 marks] Show that the volumetric strain rate of the particle is equal to the divergence of velocity vector.

b) [10 marks] Derive expression of shear strain rates using Taylor series.

c) [8 marks] Consider state of stress at O (centre of particle) as $\{\sigma_{xx}, \sigma_{yy}\ \tau_{xy}, \tau_{yx}\}$, where symbols and subscripts have meanings as discussed in class. Show that the stress tensor is symmetric for this system.

**Problem 3** [20 marks]: Consider 2D, steady, incompressible flow between two parallel plates (density $\rho$, viscosity $\mu$) in a channel of height $h$. Consider two dimensional coordinates in which $x$-axis is along the channel and $y$-axis is normal to $x$ axis. Origin of the coordinate system lies on the bottom channel wall. The bottom plate is fixed with time while top plate is moving with velocity $U$. The velocities $u$ and $v$ are in the respective $x$ and $y$ directions:

a) [3 marks] Simplify continuity and Navier-Stokes equations given in the appendix for the present case. Neglect gravity.

b) [4 marks] Prove that pressure gradient, $dp/dx$, along $x$ direction, is constant.

c) [5 marks] Derive expression for $u$.

d) [3 marks] Draw velocity profiles for the following three cases:

(i) $dp/dx < 0$ (ii) $dp/dx = 0$ (iii) $dp/dx > 0$

e) [5 marks] Find the critical value of $dp/dx$ at which backflow will start near the bottom plate.

**Problem 4** [25 marks]: Consider flow of a Newtonian, incompressible and viscous fluid (density $\rho$, viscosity $\mu$) over a horizontal plate in two-dimensional coordinates in which $x$-axis is along the plate and $y$-axis is normal to $x$ axis. Consider velocity components as $u$ and $v$ along and normal to the plate, respectively. The free stream velocity is $u_0$ and pressure is uniform above the plate. Consider the flow as fully-developed along the length of the plate. The fluid is uniformly sucked with a vertical velocity applied at the plate, $v_{\text{wall}}(x) = -v_0$, where $v_0$ is a positive constant. Assume gravity is negligible.

a. [8 marks] Simplify continuity and Navier-Stokes equations given in the appendix for the present case. Neglect gravity. Find velocity component, $v$.

b. [12 marks] Find velocity component, $u$, using answer obtained in (a).

c. [5 marks] Determine wall shear stress using result in (b).

**Problem 5** [25 marks]: The fluid (density $\rho$) in direct contact with a stationary solid boundary has zero velocity; there is no slip at the boundary. Thus, the flow over a flat plate adheres to the plate surface and forms a boundary layer, as depicted in Fig 1. The flow ahead of the plate is uniform with velocity $U$. The velocity distribution within the boundary layer ($0 \le y \le \delta$) along $cd$ is approximated as $u/U = 2(y/\delta) - (y/\delta)^2$. Assuming the plate width perpendicular to the paper to be $w$, calculate the mass flow rate across surface $bc$ of control volume $abcd$.

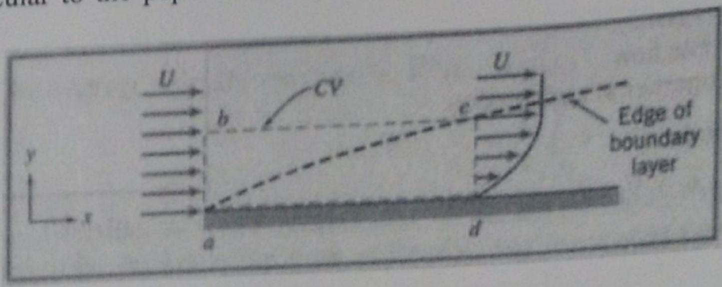


Fig. 1

**Appendix**

Taylor series expansion:

$$f(x+\delta x) = f(x) + \frac{df}{dx}(\delta x) + \frac{d^2 f}{dx^2}\frac{(\delta x)^2}{2!} + \ldots.$$

Continuity Equation for steady, incompressible flow in Cartesian coordinates:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Navier Stokes Equations for incompressible flow in Cartesian coordinates:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left\{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right\}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left\{\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right\}$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left\{\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right\}$$

Solution of second order, homogeneous, linear ODE: $y'' = ky'$ is

$$y = c_1 e^{-\lambda_1 y} + c_2 e^{-\lambda_2 y}$$

where $\lambda_1$ and $\lambda_2$ are the roots of characteristics equation.